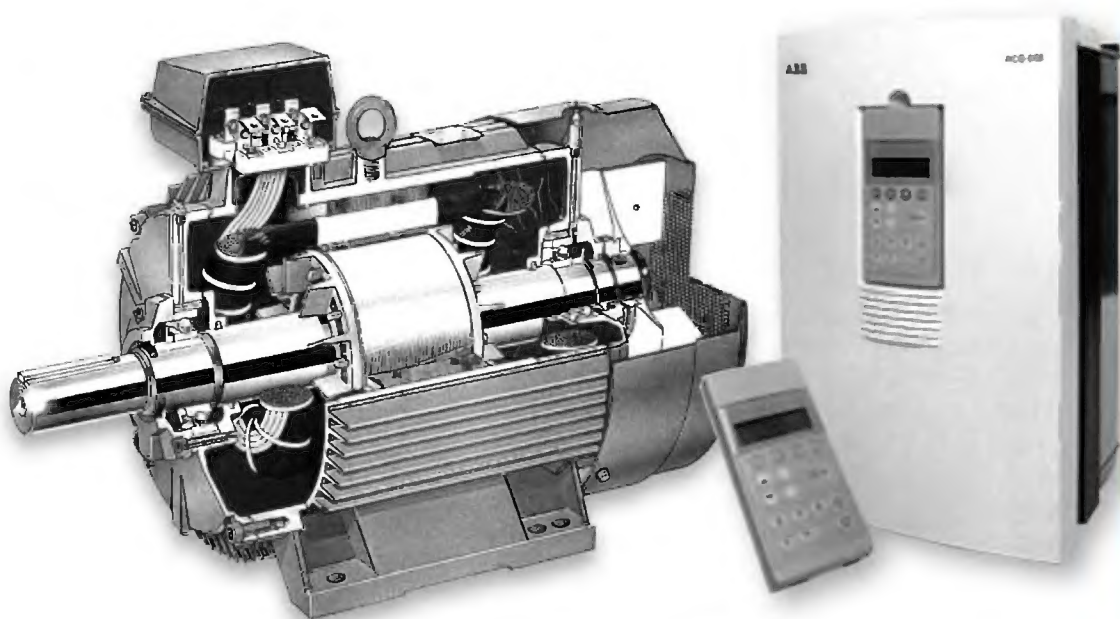


Induction Machines Handbook

THIRD EDITION

Transients, Control Principles,
Design and Testing



Ion Boldea



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Induction Machines Handbook

Electric Power Engineering Series

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Induction Machines Handbook

Transients, Control Principles, Design and Testing

Third Edition

Ion Boldea



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A humble, late, tribute to:

Nikola Tesla

Galileo Ferraris

Dolivo-Dobrovolski

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Preface

MOTIVATION

The 2010–2020 decade has seen notable progress in induction machine (IM) technology such as

- Extension of analytical and finite element modelling (FEM) for better precision and performance
- Advanced FEM-assisted optimal design methodologies with multi-physics character
- Introduction of upgraded premium efficiency IM international standards
- Development and fabrication of copper cage rotor IMs drives for traction on electric vehicles
- Extension of wound rotor induction generators (WRIGs) or doubly fed induction generators (DFIGs) with partial ratings A.C.–D.C.–A.C. converters in wind energy conversion and to pump storage reversible power plants (up to 400 MVA/unit)
- Extension of cage-rotor induction generators with full-power PWM converters for wind energy conversion (up to 5 MVA/unit)
- Development of cage (or nested cage)-rotor dual stator winding induction generators/motors with partial ratings power electronics for wind energy and vehicular technologies (autonomous operation)
- Development of line-start premium efficiency IMs with cage rotor, provided with PMs and (or) magnetic saliency for self-synchronization and operation at synchronism (three phase and single phase), for residential applications, etc.
- Introduction of multiphase ($m > 3$) IMs for higher torque density and more fault-tolerant electric drives.

All the above, reflected in a strong increase of line-start IMs and variable-speed IM motor and generator drives markets, have prompted us to prepare a new (third) edition of this book.

SHORT DESCRIPTION

As a way to mediate between not discomfoting the readers/users of second edition, but still bring/ add, wherever thought, proper, recent/representative knowledge, the titles of chapters from the second edition in this volume have been kept almost the same but reordered/corrected/improved and enhanced also with recent “knowledge pills” as **new** sections such as

- Chapter 1/1.17 Multiphase induction machines models for transients
- Chapter 1/1.18 Doubly fed induction machine models for transients
- Chapter 1/1.19 Cage-rotor synchronized reluctance motors
- Chapter 1/1.20 Cage-rotor-PM synchronous motors
- Chapter 9/9.11 Two-speed PM split-phase capacitor induction/synchronous motor
- Chapter 10/10.9 Transient operation of SEIG
- Chapter 10/10.13 DFIG space-phasor modelling for transients and control
- Chapter 10/10.14 Reactive-active power capability of DFIG
- Chapter 10/10.15 Dual stator winding cage and nested cage-rotor induction generators
- Chapter 10/10.16 DFIGs with diode-rectified output
- Chapter 12/12.15 LIM control with dynamic longitudinal end effect
- Chapter 13/13.6 Recent trends in IM testing
- Chapter 13/13.7 Cage PM-rotor line start IM testing
- Chapter 13/13.8 Linear induction motor (LIM) testing

Although efforts have been made to make all chapters rather self-sufficient within volume II, there is still a notable quantity of knowledge – expressions of parameters especially – from volume I, to reduce the text length.

Finally, the large number of numerical examples and representative graphic results from literature, processed and quoted exhaustively, should offer the reader solid understanding of phenomena with quantitative back-up as well as inspiration for follow-up work.

CONTENTS

Chapter 1: “Three and Multiphase Induction Machine Transients”/70 pages

To investigate IM transients or their control system design, *rotor – position – independent* machine inductance models are required, to simplify the mathematics. This is how the d-q for two phase and three phase – orthogonal space phasor – models have been developed 100 years ago from the phase – coordinate models, whose stator – rotor coupling inductances do depend on rotor position (to secure non-zero average torque production).

A zero sequence component is added for full power and loss equivalence. For five, six, nine phases multiple orthogonal (d-q) models with multiple zero sequence components are required.

The chapter derives the d-q model of three phase IMs from the phase coordinate model by the so called Park transformation directly in space phasor (complex variable) form in general coordinates (the airgap is ideally constant) if the slot openings are neglected or considered globally (as an airgap increase, by the Carter coefficient $K_c > 1$).

The steady state equivalent circuit and space phasor diagram are derived first and illustrated via a numerical example.

Then a fairly general structural diagram with stator and rotor flux linkage and space phasors as variables, valid at given speed (slip) is derived and proven to yield analytical solutions for transients in the form of complex eigen values. Magnetic saturation was introduced also in the d-q (space phasor) model together with core loss via a numerical example of transients which shed more light on the first milliseconds after a transient initiation behaviour of IM, with further decoupling of d-q axis needed for field oriented control (FOC).

Finally, recent models for transients of multiphase IMs, IMs with dual stator windings and regular or nested cage rotor and for PM assisted cage rotor IMs (for premium efficiency), are given in **new** Sections 1.17–1.20 to hopefully inspire the diligent reader to further self-study and application.

Chapter 2: “Single-Phase Source-Fed IM Transients”/8 pages

The d-q model as applied to starting and load transients for the single phase source (split – phase capacitor) IM via a MATLAB program is unfolded here in a dedicated specialized small chapter to facilitate the interested reader in this subject a quick introduction orientation. Multiple reference (+, -) modelling for transients and the inclusion of space harmonics is added to serve in advanced studies.

In general the possible asymmetry of stator windings of split-phase capacitor IM with cage (or cage +PM) rotor leads to the use of stator coordinates in pertinent orthogonal models.

Chapter 3: “Super-High Frequency Models and Behaviour of IMs/21 pages

Fast electric (voltage) transients (in the microseconds range), typical to atmospheric discharges and to voltage steep repetitive pulses of PWM static power converters with long power cables in variable speed motor/generator drives, require totally different models to describe properly the response of IMs to them.

A 2–3 p.u. voltage amplification was measured in long cable PWM-converter fed IM variable speed drives; also, specific bearings stray currents and shaft voltages with detrimental effects have been met in variable speed drives. The chapter treats these aspects discerning line (Z_m), -differential -, neutral (Z_{on}) ground (common made, Z_{og}) impedances for high (converter switching) frequency range, via circuit models and frequency responses. The distributed equivalent circuit for high frequency is

described with FEM calculations. Finally, the bearing currents caused by PWM converter and ways to reduce them, are presented.

Chapter 4: “Motor Specifications and Design Principles”/24 pages

Typical specifications (in numbers) are basis for any design (dimensioning) of IM. Typical load speed/torque profiles are given also; derating due voltage time harmonics, voltage and frequency variations, specifications for constant voltage and frequency (V and f), matching the IM with the given variable speed load, design factors, design features and the output coefficient and rotor tangential stress design concepts are treated in detail.

Coefficients, with numerical examples, are introduced to create a solid basis for IM dimensioning.

Chapter 5: “IM Design below 100 kW and Constant V_1 & f_1 ”/28 pages

An analytical nonlinear circuit model – based IM dimensioning rather complete sequence, with included starting current, torque, peak torque (in p.u.) and rated power factor and efficiency with limited winding over temperature for given equivalent heat transmission (convection) coefficient defined for the outer area of stator laminations (stack) is followed step by step through a case study in order to grasp the fundamentals of electromagnetic IM design.

The design methodology introduced in this chapter may serve as preliminary design in industry and/or as an initial design in developing optimal design methodologies (codes).

Chapter 6: “Induction Motor Design above 100 kW and Constant V_1 & f_1 ”/38 pages

Induction machines above 100 kW are built at low (up to 690 V_{line} -RMS) and medium (up to or even more than 6 kV_{line} -RMS) voltage, with cage or wound rotor.

The present chapter develops an IM electromagnetic design sequence for 736 kW, $V_{in} = 4$ kV(s), $f_1 = 60$ Hz, $2p_1 = 4$ pole, $m = 3$ phases in quite a few rotor variants: deep bar rotor, dual cage rotor, and, respectively, wound rotor ($V_{21} = 644$ V, star connection), including performance calculation for the latter case yielding an efficiency of 0.946 for a rather slip $s_n = 1.57\%$. In general, when used for variable speed, the WRIG is expected to a much higher maximum slip.

So, if the maximum rotor voltage equals the stator rated voltage at S_{max} , then the rotor number of turns /coil and the conductor cross – section (and current) are modified accordingly for around 0.3 p.u. rotor power capability.

The presented methodology avoids iterations and thus may constitute a solid preliminary design tool that requires a small computation time, while being fully intuitional and thus very useful to the young reader/designer in the field.

Chapter 7: “Induction Machine Design Principles for Variable Speed”/26 pages

The complex process of designing a variable speed IM to meet performance/cost and torque/speed envelopes per given D.C. input voltage (current) to the inverter is treated in this chapter by introducing key principles for electromagnetic design such as: general drives, constant power speed range (CPSR), power and voltage derating (due to the PWM converter supply), reducing skin effect in windings – especially in the rotor bars, torque pulsation reduction methods, increasing efficiency and breakdown torque, voltage management for wide constant power range (CPSR) design for high and super – high speed (a 21 kW, 47 krpm, 94% efficiency IM sample design, for wide CPSR is included).

Chapter 8: “Optimization Design Issues”/15 pages

Again, the “optimization design”, **an art of itself**, has been “reduced” here to a solid introduction of key issues related to specifications, single multi-dimensional objective function and the constraint function, variable vector and its range for the case in point, the machine model and the mathematical search method for finding of the global optimum geometry of IM. Four essential optimization methods (algorithms) are selected for orientative presentation here: augmented Lagrangian multiplier method, sequential unconstraint minimization, modified Hooke – Jeeves method and genetic algorithms. For the last two methods and IM, dedicated chapters with MATLAB computer programs on-line are available in [24]; more information on FEM based optimization design of IMs may be found in [25–27].

Chapter 9: “Single-Phase IM Design”/36 pages

Given the single (split) phase capacitor IM peculiarities and low power applications, this chapter is dedicated to the young reader designer centred on this subject.

A general/preliminary electromagnetic design (dimensioning) of a split phase capacitor induction motor is offered via a case study at 186.5 W, 115 V, 60 Hz, $2p_1 = 4$ poles, dealing with: sizing the stator and rotor magnetic circuits, sizing of stator windings (for quasi – sinusoidal mmf), resistances and leakage reactances, steady state performance around rated power, optimization design issues via a case study.

Finally, the design of a PM assisted cage rotor split phase capacitor IM is presented in some detail as it is credited for premium efficiency but with conflicting starting performance requirements (**new**).

Chapter 10: “Three-Phase Induction Generators”/49 pages

This chapter concentrates mainly on self (capacitor) excited cage rotor induction generator credited with good performance in small and medium power applications.

Issues such as IG classification, self-excited IG (SEIG) modelling, SEIG steady state performance, second order slip equation model for SEIG with numerical example, SEIG performance for constant speed and capacitors, unbalanced steady state operation of SEIG, SEIG transients with IM motor load, parallel operation of SEIGs are all treated in this chapter.

Also, the WRIG (or DFIG) with slip rings and brushes to connect a partial (s_{max}) p.u. PWM D.C.–D.C.–A.C. converter to the rotor is characterized.

Finally, in notable length **new** Sections 10.13–10.15 dual stator winding cage and nested – cage rotor IGs are treated together with DFIG with variable stator frequency and diode rectified output, credited with high potential in wind energy (with H(M)VDC interfacing) and for D.C. power bus vehicular (aircraft, marine vessel etc.) power systems.

Chapter 11: “Single-Phase Induction Generators”/10 pages

This short chapter, intended for the selective reader, deals with:

Single phase SEIGs topologies, steady state modelling performance, the d-q model via a case study and a battery fed inverter excited single phase IG with parallel output A.C. capacitor for variable speed bidirectional inverter power flow, to increase the power – speed range and reduce voltage regulation.

The rather complete model for steady state, with asymmetric stator orthogonal windings, emphasizing the role of magnetic saturation and the V-I characteristics with capacitors in both windings offer solid ground for application oriented industrial designs.

Chapter 12: “Linear Induction Motors (LIMs)”/51 pages

This is an extended review on steady state and transients modelling and performance of three phase LIMs with flat and tubular topologies, considering the transverse edge and dynamic longitudinal effects (including their coefficients in control: in a **new** Section 12.15 with numerical examples and data all over the place, to enforce quick assimilation of knowledge (more on LIMs, for transportation, especially).

Chapter 13: “Testing of Three Phase IMs”/56 pages

This rather comprehensive chapter follows in good part the international standards in use and adds nuances to virtual load testing and free acceleration – deceleration testing to offer a rounded view on a dynamic technology in full swing.

Loss segregation tests, stray load losses from no load overvoltage test and from reverse rotation test, no load and stall rotor tests with PWM converter (variable frequency and voltage) supply, calorimetric loss measurement, efficiency measurement, standard IEEE 1/2 – 1996, IEC – standard 34-2, efficiency test comparisons, motor/generator slip efficiency method, mixed frequency artificial (virtual) load testing with PWM converter supply, the slow free acceleration and deceleration testing – for parameters and efficiency measurement in medium/high power IMs, parameter (R_r , L_r) estimation from no-load and standstill, two frequency, catalogue data, standstill frequency testing and by general regression methods, noise measurements, are all treated in this rather comprehensive chapter.

To bring some very recent knowledge, new Sections 13.6 and 13.7 refer to very recent innovative IM testing sequences.

Chapter 14: “Single-Phase IM Testing”/12 pages

This short chapter deals with single phase IM testing as it is quite different from the three phase IM testing and has been given far less attention by Academia and Industry. The main issue treated hereby is the loss segregation tests of single phase IM based on a method developed by the now legendary C. G. Veinott in 1935 and unsurpassed until today (in our opinion). Reference is made to [5] where the method is extended /adopted for the cage – PM rotor IM of premium efficiency.

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He was a general chair of ten biannual IEEE-sponsored OPTIM International Conferences (www.info-optim.ro) and is the founding and current chief editor, since 2000, of the Internet-only *Journal of Electrical Engineering*, “www.jee.ro”.

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1 Induction Machine Transients

1.1 INTRODUCTION

Induction machines (IMs) undergo transients when voltage, current, and/or speed undergo changes. Turning on or off the power grid leads to transients in induction motors.

Reconnecting an IM after a short-lived power fault (zero current) is yet another transient. Bus switching for high-power IMs feeding urgent loads also qualifies as large deviation transients.

Sudden short circuits, at the terminals of large induction motors, lead to very large peak transient currents and torques. On the other hand, more and more induction motors are used in variable speed drives with fast electromagnetic and mechanical transients.

So, modelling transients is required for power-grid-fed (constant voltage and frequency) and pulse width modulation (PWM) converter-fed IM drives control.

Modelling the transients of IMs may be carried out through circuit models or through coupled field/circuit models (through finite element modelling or FEM). We will deal first with phase-coordinate abc model with inductance matrix exhibiting terms dependent on rotor position.

Subsequently, the space-phasor (d-q) model is derived. Both single- and double-rotor circuit models are dealt with. Saturation is included also in the space-phasor (d-q) model. The abc-d-q model is then derived and applied, as it is adequate for nonsymmetrical voltage supplies and for PWM converter-fed IMs.

Reduced-order d-q models are used to simplify the study of transients for low- and high-power motors, respectively.

Modelling transients with the computation of cage bar and end-ring currents is required when cage and/or end-ring faults occur. Finally, the FEM-coupled field circuit approach is dealt with.

Autonomous generator transients are left out as they are treated in Chapter 10 dedicated to induction generators (IGs) (in Volume 2).

1.2 THE PHASE-COORDINATE MODEL

The IM may be viewed as a system of electric and magnetic circuits which are coupled magnetically and/or electrically.

An assembly of resistances, self-inductances and mutual inductances is thus obtained. Let us first deal with the inductance matrix.

A symmetrical (healthy) cage may be replaced by a wound three-phase rotor [2]. Consequently, the IM is represented by six circuits (phases) (Figure 1.1). Each of them is characterized by a self-inductance and five mutual inductances.

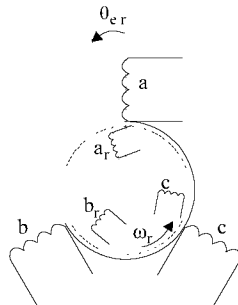


FIGURE 1.1 Three-phase IM with equivalent wound rotor.

The stator- and rotor-phase self-inductances do not depend on rotor position if slot openings are neglected. Also, mutual inductances between stator phases and rotor phases do not depend on rotor position. A sinusoidal distribution of windings is assumed. Finally, stator/rotor-phase mutual inductances depend on rotor position ($\theta_{er} = p_i \theta_r$).

The induction matrix, $L_{abca_r b_r c_r}(\theta_{er})$ is

$$\left[L_{abca_r b_r c_r}(\theta_{er}) \right] = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aa_r} & L_{ab_r} & L_{ac_r} \\ L_{ab} & L_{bb} & L_{bc} & L_{ba_r} & L_{bb_r} & L_{bc_r} \\ L_{ac} & L_{bc} & L_{cc} & L_{ca_r} & L_{cb_r} & L_{cc_r} \\ L_{aa_r} & L_{ba_r} & L_{ca_r} & L_{a_r a_r} & L_{a_r b_r} & L_{a_r c_r} \\ L_{ab_r} & L_{bb_r} & L_{cb_r} & L_{a_r b_r} & L_{b_r b_r} & L_{b_r c_r} \\ L_{ac_r} & L_{bc_r} & L_{cc_r} & L_{a_r c_r} & L_{b_r c_r} & L_{c_r c_r} \end{bmatrix} \quad (1.1)$$

with

$$\begin{aligned} L_{aa} &= L_{bb} = L_{cc} = L_{ls} + L_{ms}; \quad L_{ab} = L_{ac} = L_{bc} = -L_{sm}/2; \\ L_{aa_r} &= L_{bb_r} = L_{cc_r} = L_{srm} \cos \theta_{er}; \quad L_{a_r a_r} = L_{b_r b_r} = L_{c_r c_r} = L_{lr}^r + L_{rm}^r; \\ L_{c_r a} &= L_{a_r b} = L_{b_r c} = L_{srm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right); \quad L_{a_r b_r} = L_{a_r c_r} = L_{b_r c_r} = -L_{rm}^r/2; \\ L_{c_r b} &= L_{b_r a} = L_{a_r c} = L_{srm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) \end{aligned} \quad (1.2)$$

Assuming a sinusoidal distribution of windings, it may be easily shown that

$$L_{srm} = \sqrt{L_{sm} \cdot L_{rm}^r} \quad (1.3)$$

Reducing the rotor to stator is useful especially for cage-rotor IMs, as no access to rotor variables is available.

In this case, the mutual inductance becomes equal to self-inductance $L_{srm} \rightarrow L_{sm}$ and the rotor self-inductance equal to the stator self-inductance $L_{rm}^r \rightarrow L_{sm}$.

To conserve the fluxes and losses, with stator-reduced variables,

$$\frac{i_{ar}}{i_{ar}^r} = \frac{i_{br}}{i_{br}^r} = \frac{i_{cr}}{i_{cr}^r} = \frac{L_{srm}}{L_{sm}} = K_{rs} \quad (1.4)$$

$$\frac{V_{ar}}{V_{ar}^r} = \frac{V_{br}}{V_{br}^r} = \frac{V_{cr}}{V_{cr}^r} = \frac{i_{ar}^r}{i_{ar}} = \frac{i_{br}^r}{i_{br}} = \frac{i_{cr}^r}{i_{cr}} = \frac{1}{K_{rs}} \quad (1.5)$$

$$\frac{R_r}{R_r^r} = \frac{L_{lr}}{L_{lr}^r} = \frac{1}{K_{rs}^2} \quad (1.6)$$

The expressions of rotor resistance R_r and leakage inductance L_{lr} both reduced to the stator for both cage and wound rotors, are given in Chapter 6, Vol. 1.

The same is true for R_s and L_{ls} . The magnetization self-inductance L_{sm} has been calculated in Chapter 5, Vol. 1.

Now, the matrix form of phase-coordinate (variable) model is

$$[V] = [R][i] + \frac{d}{dt} [\Psi] \quad (1.7)$$

$$[V] = [V_a, V_b, V_c, V_{ar}, V_{br}, V_{cr}]^T$$

$$[i] = [i_a, i_b, i_c, i_{ar}, i_{br}, i_{cr}]^T$$

$$[R] = \text{Diag}[R_s, R_s, R_s, R_r, R_r, R_r]$$

$$[\Psi] = [L_{abca_r b_r c_r}(\theta_{er})][i] \quad (1.8)$$

$$\begin{bmatrix} L_{ls} + L_{sm} & -L_{sm}/2 & -L_{sm}/2 & L_{sm} \cos \theta_{er} & L_{srm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{srm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) \\ -L_{sm}/2 & L_{ls} + L_{sm} & -L_{sm}/2 & L_{srm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & L_{srm} \cos \theta_{er} & L_{srm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) \\ -L_{sm}/2 & -L_{sm}/2 & L_{ls} + L_{sm} & L_{srm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{srm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & L_{srm} \cos \theta_{er} \\ L_{sm} \cos \theta_{er} & L_{srm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & L_{srm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{ls} + L_{sm} & -L_{sm}/2 & -L_{sm}/2 \\ L_{srm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{sm} \cos \theta_{er} & L_{srm} \cos \theta_{er} & -L_{sm}/2 & L_{ls} + L_{sm} & -L_{sm}/2 \\ L_{srm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & L_{srm} \cos \left(\theta_{er} + \frac{2\pi}{3} \right) & L_{srm} \cos \left(\theta_{er} - \frac{2\pi}{3} \right) & L_{sm} \cos \theta_{er} & -L_{sm}/2 & L_{ls} + L_{sm} \end{bmatrix} \quad (1.9)$$

with (1.8), (1.7) becomes

$$[V] = [R][i] + \left([L] + \left[\frac{\partial L}{\partial i} \right] [i] \right) \frac{d[i]}{dt} + \frac{d[L]}{d\theta_{er}} [i] \frac{d\theta_{er}}{dt} \quad (1.10)$$

Multiplying (1.10) by $[i]^T$, we get

$$[i]^T [V] = [i]^T R[i] + \frac{d}{dt} \left(\frac{1}{2} [L][i][i]^T \right) + \frac{1}{2} [i]^T \frac{d}{d\theta_{er}} [L][i] \omega_r \quad (1.11)$$

where the first term represents the winding losses, the second, the stored magnetic energy variation, and the third, the electromagnetic power, P_e .

$$P_e = T_e \frac{\omega_r}{p_l} = \frac{1}{2} [i]^T \frac{d[L]}{d\theta_{er}} [i] \omega_r \quad (1.12)$$

The electromagnetic torque T_e is

$$T_e = \frac{1}{2} p_l [i]^T \frac{d[L]}{d\theta_{er}} [i] \quad (1.13)$$

The motion equation is

$$\frac{J}{p_l} \frac{d\omega_r}{dt} = T_e - T_{load}; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (1.14)$$

An eight-order nonlinear model with time-variable coefficients (inductances) has been obtained, even with core loss neglected.

Numerical methods are required to solve it, but the computation time is prohibitive. Consequently, the phase-coordinate model is to be used only for special cases as the inductance and resistance matrices may be assigned any amplitude and rotor position dependencies.

The complex or space vector variable model is now introduced to get rid of rotor position dependence of parameters.

1.3 THE COMPLEX VARIABLE MODEL

Let us use the following notations:

$$a = e^{j\frac{2\pi}{3}}; \quad \cos \frac{2\pi}{3} = \text{Re}[a]; \quad \cos \frac{4\pi}{3} = \text{Re}[a^2] \quad (1.15)$$

$$\cos \left(\theta_{er} + \frac{2\pi}{3} \right) = \text{Re}[ae^{j\theta_{er}}]; \quad \cos \left(\theta_{er} + \frac{4\pi}{3} \right) = \text{Re}[a^2 e^{j\theta_{er}}]$$

Based on the inductance matrix, expression (1.9), the stator phase a and rotor-phase a_r flux linkages, Ψ_a and Ψ_{a_r} , are

$$\Psi_a = L_{ls} i_a + L_{sm} \text{Re}[i_a + ai_b + a^2 i_c] + L_{sm} \text{Re}[(i_{a_r} + ai_{b_r} + a^2 i_{c_r}) e^{j\theta_{er}}] \quad (1.16)$$

$$\Psi_{a_r} = L_{lr} i_{a_r} + L_{sm} \text{Re}[i_{a_r} + ai_{b_r} + a^2 i_{c_r}] + L_{sm} \text{Re}[(i_a + ai_b + a^2 i_c) e^{-j\theta_{er}}] \quad (1.17)$$

We may now introduce the following complex variables as space phasors [1]:

$$\tilde{\mathbf{i}}_s^s = \frac{2}{3}(\mathbf{i}_a + a\mathbf{i}_b + a^2\mathbf{i}_c) \quad (1.18)$$

$$\tilde{\mathbf{i}}_r^r = \frac{2}{3}(\mathbf{i}_{ar} + a\mathbf{i}_{br} + a^2\mathbf{i}_{cr}) \quad (1.19)$$

Also,

$$\text{Re}(\tilde{\mathbf{i}}_s^s) = \mathbf{i}_a - \frac{1}{3}(\mathbf{i}_a + \mathbf{i}_b + \mathbf{i}_c) \quad (1.20)$$

$$\text{Re}(\tilde{\mathbf{i}}_r^r) = \mathbf{i}_{ar} - \frac{1}{3}(\mathbf{i}_{ar} + \mathbf{i}_{br} + \mathbf{i}_{cr}) \quad (1.21)$$

In symmetric steady-state and transient regimes (or for star connection of phases),

$$\mathbf{i}_a + \mathbf{i}_b + \mathbf{i}_c = \mathbf{i}_{ar} + \mathbf{i}_{br} + \mathbf{i}_{cr} = 0 \quad (1.22)$$

With the above definitions, Ψ_a and Ψ_{ar} become

$$\Psi_a = L_{ls} \text{Re}(\tilde{\mathbf{i}}_s^s) + L_m \text{Re}(\tilde{\mathbf{i}}_s^s + \tilde{\mathbf{i}}_r^r e^{j\theta_{er}}); \quad L_m = \frac{3}{2}L_{sm} \quad (1.23)$$

$$\Psi_{ar} = L_{lr} \text{Re}(\tilde{\mathbf{i}}_r^r) + L_m \text{Re}(\tilde{\mathbf{i}}_r^r + \tilde{\mathbf{i}}_s^s e^{-j\theta_{er}}) \quad (1.24)$$

Similar expressions may be derived for phases b_r and c_r . After adding them together, using the complex variable definitions (1.18) and (1.19) for flux linkages and voltages, we also obtain

$$\bar{\mathbf{V}}_s^s = \mathbf{R}_s \tilde{\mathbf{i}}_s^s + \frac{d\bar{\Psi}_s^s}{dt}; \quad \bar{\Psi}_s^s = L_s \tilde{\mathbf{i}}_s^s + L_m \tilde{\mathbf{i}}_r^r e^{j\theta_{er}} \quad (1.25)$$

$$\bar{\mathbf{V}}_r^r = \mathbf{R}_r \tilde{\mathbf{i}}_r^r + \frac{d\bar{\Psi}_r^r}{dt}; \quad \bar{\Psi}_r^r = L_r \tilde{\mathbf{i}}_r^r + L_m \tilde{\mathbf{i}}_s^s e^{-j\theta_{er}}$$

where

$$L_s = L_{sl} + L_m; \quad L_r = L_{rl} + L_m \quad (1.26)$$

$$\bar{\mathbf{V}}_s^s = \frac{2}{3}(\mathbf{V}_a + a\mathbf{V}_b + a^2\mathbf{V}_c); \quad \bar{\mathbf{V}}_r^r = \frac{2}{3}(\mathbf{V}_{ar} + a\mathbf{V}_{br} + a^2\mathbf{V}_{cr}) \quad (1.27)$$

In the above equations, stator variables are still given in stator coordinates and rotor variables in rotor coordinates.

Making use of a rotation of complex variables by the general angle θ_b in the stator and $\theta_b - \theta_{er}$ in the rotor, we obtain all variables in a unique reference rotating at electrical speed ω_b

$$\omega_b = \frac{d\theta_b}{dt} \quad (1.28)$$

$$\begin{aligned}\bar{\Psi}_s^s &= \bar{\Psi}_s^b e^{j\theta_b}; & \bar{i}_s^s &= \bar{i}_s^b e^{j\theta_b}; & \bar{V}_s^s &= \bar{V}_s^b e^{j\theta_b} \\ \bar{\Psi}_r^r &= \bar{\Psi}_r^b e^{j(\theta_b - \theta_{er})}; & \bar{i}_r^r &= \bar{i}_r^b e^{j(\theta_b - \theta_{er})}; & \bar{V}_r^r &= \bar{V}_r^b e^{j(\theta_b - \theta_{er})}\end{aligned}\quad (1.29)$$

With these new variables, Equations (1.25) become

$$\begin{aligned}\bar{V}_s &= R_s \bar{i}_s + \frac{d\bar{\Psi}_s}{dt} + j\omega_b \bar{\Psi}_s; & \bar{\Psi}_s &= L_s \bar{i}_s + L_m \bar{i}_r \\ \bar{V}_r &= R_r \bar{i}_r + \frac{d\bar{\Psi}_r}{dt} + j(\omega_b - \omega_r) \bar{\Psi}_r; & \bar{\Psi}_r &= L_r \bar{i}_r + L_m \bar{i}_s\end{aligned}\quad (1.30)$$

For convenience, the superscript b was dropped in (1.30). The electromagnetic torque is related to motion-induced voltage in (1.30).

$$T_e = \frac{3}{2} \cdot p_l \cdot \text{Re} \left(j \cdot \bar{\Psi}_s \cdot \bar{i}_s^* \right) = -\frac{3}{2} \cdot p_l \cdot \text{Re} \left(j \cdot \bar{\Psi}_r \cdot \bar{i}_r^* \right) \quad (1.31)$$

Adding the equations of motion, the complete complex variable (space-phasor) model of IM is obtained.

$$\frac{J}{p_l} \frac{d\omega_r}{dt} = T_e - T_{\text{load}}; \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (1.32)$$

The complex variables may be decomposed in a plane along two orthogonal d and q axes rotating at speed ω_b to obtain the d–q (Park) model [2].

$$\begin{aligned}\bar{V}_s &= V_d + j \cdot V_q; & \bar{i}_s &= i_d + j \cdot i_q; & \bar{\Psi}_s &= \Psi_d + j \cdot \Psi_q \\ \bar{V}_r &= V_{dr} + j \cdot V_{qr}; & \bar{i}_r &= i_{dr} + j \cdot i_{qr}; & \bar{\Psi}_r &= \Psi_{dr} + j \cdot \Psi_{qr}\end{aligned}\quad (1.33)$$

With (1.33), the voltage equations (1.30) become

$$\begin{aligned}\frac{d\Psi_d}{dt} &= V_d - R_s \cdot i_d + \omega_b \cdot \Psi_q \\ \frac{d\Psi_q}{dt} &= V_q - R_s \cdot i_q - \omega_b \cdot \Psi_d \\ \frac{d\Psi_{dr}}{dt} &= V_{dr} - R_r \cdot i_{dr} + (\omega_b - \omega_r) \cdot \Psi_{qr} \\ \frac{d\Psi_{qr}}{dt} &= V_{qr} - R_r \cdot i_{qr} - (\omega_b - \omega_r) \cdot \Psi_{dr} \\ T_e &= \frac{3}{2} P_l (\Psi_d i_q - \Psi_q i_d) = \frac{3}{2} P_l L_m (i_q i_{dr} - i_d i_{qr})\end{aligned}\quad (1.34)$$

Also from (1.27) with (1.19), the Park transformation for stator $P(\theta_b)$ is derived.

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = [P(\theta_b)] \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (1.35)$$

$$[P(\theta_b)] = \frac{2}{3} \cdot \begin{bmatrix} \cos(-\theta_b) & \cos\left(-\theta_b + \frac{2\pi}{3}\right) & \cos\left(-\theta_b - \frac{2\pi}{3}\right) \\ \sin(-\theta_b) & \sin\left(-\theta_b + \frac{2\pi}{3}\right) & \sin\left(-\theta_b - \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (1.36)$$

The inverse Park transformation is

$$[P(\theta_b)]^{-1} = \frac{3}{2} \cdot [P(\theta_b)]^T \quad (1.37)$$

A similar transformation is valid for the rotor but with $\theta_b - \theta_{er}$ instead of θ_b .

It may be easily proved that the homopolar (real) variables V_0 , i_0 , V_{0r} , i_{0r} , Ψ_0 and Ψ_{0r} do not interface in energy conversion:

$$\begin{aligned} \frac{d\Psi_0}{dt} &= V_0 - R_s \cdot i_0; \quad \Psi_0 \approx L_{0s} \cdot i_0 \\ \frac{d\Psi_{0r}}{dt} &= V_{0r} - R_r \cdot i_{0r}; \quad \Psi_{0r} \approx L_{0r} \cdot i_{0r} \end{aligned} \quad (1.38)$$

L_{0s} and L_{0r} are the homopolar inductances of stator and rotor, respectively. Their values are equal or lower (for chorded coil windings) to the respective leakage inductances L_{ls} and L_{lr} .

A few remarks on the complex variable (space-phaser) and d-q models are in order.

- Both models include, in the form presented here, only the space fundamental of mmfs and airgap flux distributions.
- Both models exhibit inductances independent of rotor position.
- The complex variable (space-phaser) model is credited with a reduction in the number of equations with respect to the d-q model, but it operates with complex variables.
- When solving the state-space equations, only the d-q model, with real variables, benefits from existing commercial software (Mathematica, MATLAB[®]–Simulink[®], Spice, etc.).
- Both models are very practical in treating the transients and control of symmetrical IMs fed from symmetrical voltage power grids or from PWM converters.
- Easy incorporation of magnetic saturation and rotor skin effect are yet two additional assets of complex variable and d-q models. The airgap flux density retains a sinusoidal distribution along the circumferential direction.
- Besides the widespread usage of complex variable, other models (variable transformations) that deal especially with asymmetric supply or asymmetric machine cases have also been introduced (for a summary, see Refs. [3,4]).

1.4 STEADY STATE BY THE COMPLEX VARIABLE MODEL

Constant speed and load are expressed by IM steady state. For a machine fed from a sinusoidal voltage symmetrical power grid, the phase voltages at IM terminals are

$$V_{a,b,c} = V\sqrt{2} \cdot \cos\left(\omega_1 t - (i-1) \cdot \frac{2\pi}{3}\right); \quad i = 1, 2, 3 \quad (1.39)$$

The voltage space-phasor $\bar{V}_s^b$ in random coordinates (from (1.27)) is

$$\bar{V}_s^b = \frac{2}{3} (V_a(t) + aV_b(t) + a^2V_c(t)) e^{-j\theta_b} \quad (1.40)$$

From (1.39) and (1.40),

$$\bar{V}_s^b = V\sqrt{2} [\cos(\omega_1 t - \theta_b) + j\sin(\omega_1 t - \theta_b)] \quad (1.41)$$

Only for steady state,

$$\theta_b = \omega_b t + \theta_0 \quad (1.42)$$

Consequently,

$$\bar{V}_s^b = V\sqrt{2} e^{j[(\omega_1 - \omega_b)t + \theta_0]} \quad (1.43)$$

For steady state, the current in the space-phasor model follows the voltage frequency: $(\omega_1 - \omega_b)$. Steady state in the state-space equations can be achieved by replacing d/dt with $j(\omega_1 - \omega_b)$.

Using this observation, Equations (1.30) become

$$\begin{aligned} \bar{V}_{s0} &= R_s \bar{i}_{s0} + j\omega_1 \bar{\Psi}_{s0}; \quad \bar{\Psi}_{s0} = L_{sl} \bar{i}_{s0} + \bar{\Psi}_{m0}; \quad \bar{\Psi}_{r0} = L_{rl} \bar{i}_{r0} + \bar{\Psi}_{m0}; \quad \bar{i}_{m0} = \bar{i}_{s0} + \bar{i}_{r0} \\ \bar{V}_{r0} &= R_r \bar{i}_{r0} + jS\omega_1 \bar{\Psi}_{r0}; \quad S = \frac{(\omega_1 - \omega_r)}{\omega_r}; \quad \bar{\Psi}_{m0} = L_m \bar{i}_{m0} \end{aligned} \quad (1.44)$$

So the form of space-phasor model voltage equations under the steady state is the same irrespective of the speed of the reference system ω_b .

When ω_b changes, only the frequency of voltages, currents, and flux linkages changes in the space-phasor model varies as $\omega_1 - \omega_b$.

No wonder this is so, as only Equations (1.44) exhibit the total emf, which should be independent of reference system speed ω_b . S is the slip, a well-known variable so far.

Notice that for $\omega_b = \omega_1$ (synchronous coordinates), for steady state, $d/dt = (\omega_1 - \omega_b) = 0$. Consequently, for synchronous coordinates the steady state means D.C. variables.

The space-phasor diagram of (1.44) is shown in Figure 1.2 for a cage-rotor IM.

From the stator space equations (1.44), the torque (1.31) becomes

$$T_e = \frac{3}{2} p_1 \left(j \bar{\Psi}_{r0} \bar{i}_{r0}^* \right) = \frac{3}{2} p_1 \Psi_{r0} i_{r0} \quad (1.45)$$

Also, from (1.44),

$$\bar{i}_{r0} = -jS\omega_1 \frac{\bar{\Psi}_{r0}}{R_r} \quad (1.46)$$

With (1.46), alternatively, the torque is

$$T_e = \frac{3}{2} p_1 \frac{\Psi_{r0}^2}{R_r} S\omega_1; \quad S\omega_1 = \omega_1 - \omega_r \quad (1.47)$$

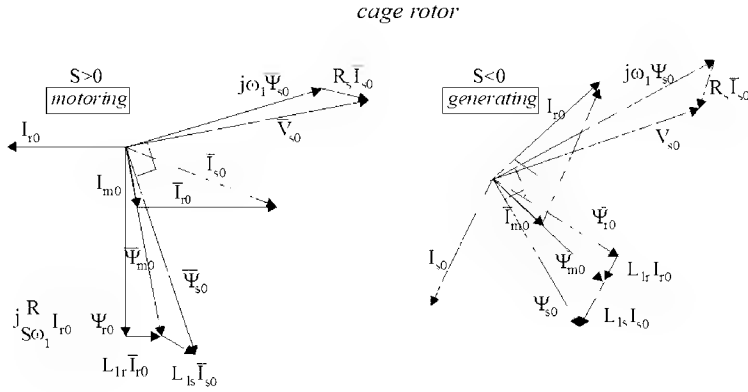


FIGURE 1.2 Space-phaser diagram for steady state.

Solving for Ψ_{r0} in Equations (1.44) leads to the standard torque formula:

$$T_e \approx \frac{3p_l}{\omega_l} \frac{V_s^2 \frac{R_r}{S}}{\left(R_s + C_l \frac{R_r}{S}\right)^2 + \omega_l^2 (L_{ls} + C_l L_{lr})^2}; \quad C_l = 1 + \frac{L_{ls}}{L_m} \quad (1.48)$$

Expression (1.47) shows that, for constant rotor flux space-phaser amplitude, the torque varies linearly with speed ω_r as it does in a separately excited D.C. motor. So all steady-state performance may be calculated using the space-phaser model as well.

1.5 EQUIVALENT CIRCUITS FOR DRIVES

Equations (1.30) lead to a general equivalent circuit good for transients, especially in variable speed drives (Figure 1.3).

$$\begin{aligned} \bar{V}_s &= R_s \bar{i}_s + (p + j\omega_b) L_{sl} \bar{i}_s + (p + j\omega_b) \bar{\Psi}_m \\ \bar{V}_r &= R_r \bar{i}_r + (p + j(\omega_b - \omega_r)) L_{rl} \bar{i}_r + (p + j(\omega_b - \omega_r)) \bar{\Psi}_m \end{aligned} \quad (1.49)$$

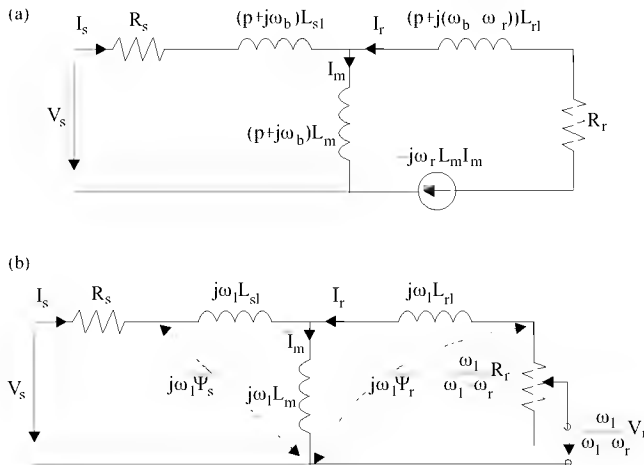


FIGURE 1.3 The general equivalent circuit with $V_s = 0$ (a) and for steady state, $V_r \neq 0$ (b).

The reference system speed ω_b may be random, but three particular values have met with rather wide acceptance.

- Stator coordinates: $\omega_b = 0$; for steady state: $p \rightarrow j\omega_l$
- Rotor coordinates: $\omega_b = \omega_r$; for steady state: $p \rightarrow jS\omega_l$
- Synchronous coordinates: $\omega_b = \omega_l$; for steady state: $p \rightarrow 0$.

Also, for steady state in variable speed drives, the steady-state circuit (the same for all values of ω_b) with $p \rightarrow j(\omega_l - \omega_b)$ is shown in Figure 1.3a and b.

Figure 1.3b shows, in fact, the standard T equivalent circuit of IM for steady state, but in space phasors and not in phase phasors.

A general method to “arrange” the leakage inductances L_{sl} and L_{rl} in various positions in the equivalent circuit consists of a change of variables.

$$\tilde{i}_r^a = \tilde{i}_r/a; \quad \bar{\Psi}_{ma} = aL_m(\tilde{i}_s + \tilde{i}_r^a) \quad (1.50)$$

Making use of this change of variables in (1.49) yields

$$\begin{aligned} \bar{V}_s &= R_s \tilde{i}_s + (p + j\omega_b)(L_s - aL_m) \tilde{i}_s + (p + j\omega_b) \bar{\Psi}_m^a \\ a\bar{V}_r &= a^2 R_r \tilde{i}_r^a + (p + j(\omega_b - \omega_r)) a(aL_{rl} - L_m) \tilde{i}_r^a + (p + j(\omega_b - \omega_r)) \bar{\Psi}_m^a \end{aligned} \quad (1.51)$$

An equivalent circuit may be developed based on (1.51), as shown in Figure 1.4.

The generalized equivalent circuit shown in Figure 1.4 warrants the following comments:

- For $a = 1$, the general equivalent circuit that is shown in Figure 1.3 is reobtained and $a\bar{\Psi}_m^a = \bar{\Psi}_m$: the main flux.
- For $a = L_m/L_r < 1$, the inductance term in the “rotor section” “disappears”, being moved to the primary section:

$$a\bar{\Psi}_m^a = \frac{L_m}{L_r} L_m \left(\tilde{i}_s + \tilde{i}_r \frac{L_r}{L_m} \right) = \frac{L_m}{L_r} \bar{\Psi}_r \quad (1.52)$$

- For $a = L_s/L_m > 1$, the leakage inductance term is lumped into the “rotor section”:

$$a\bar{\Psi}_m^a = \frac{L_s}{L_m} L_m \left(\tilde{i}_s + \tilde{i}_r \frac{L_m}{L_s} \right) = \bar{\Psi}_s \quad (1.53)$$

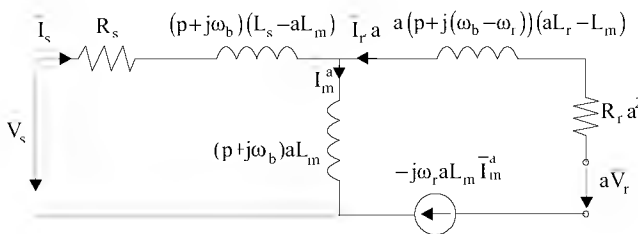


FIGURE 1.4 Generalized equivalent circuit.

If the rotor current is placed along real axis in the negative direction, the rotor flux magnetization current $\bar{I}_{r0}$ (Figure 1.6a) is

$$\bar{I}_{r0} = -j \frac{(-I_{r0}) \frac{R_r}{S} \frac{L_m}{L_r}}{\omega_l \frac{L_m^2}{L_r}} = -j \frac{(-I_{r0}) R_r}{S \omega_l L_m} = -j \frac{(+7.536) \cdot 1}{0.2 \cdot 2\pi 6 \cdot 0.2} = -j5 \text{ A}$$

The stator current is

$$\bar{I}_{s0} = -\bar{I}_{r0} \frac{L_r}{L_m} + \bar{I}_{0r0} = 7.536 \frac{0.205}{0.2} - j5 = 7.7244 - j5.0 \text{ A}$$

The stator flux $\bar{\Psi}_s$ is

$$\bar{\Psi}_{s0} = L_s \bar{I}_{s0} + L_m \bar{I}_{r0} = 0.205(7.7244 - j5.0) + 0.2(-7.536) = 0.076 - j1.025$$

The airgap flux $\bar{\Psi}_m$ is

$$\bar{\Psi}_{m0} = L_m (\bar{I}_{s0} + \bar{I}_{r0}) = 0.2(7.7244 - j5.0 - 7.536) = 0.03768 - j1.0$$

The rotor flux is

$$\bar{\Psi}_{r0} = \bar{\Psi}_{m0} + L_{rl} \bar{I}_{r0} = 0.03768 - j1.0 + 0.005(-7.536) = -j1.0 \text{ (as expected)}$$

The voltage V_{s0} is

$$V_{s0} = j\omega_l \bar{\Psi}_{s0} + R_s \bar{I}_{s0} = j2\pi 6(0.076 - j1.025) + 1(7.7244 - j5.0) = 46.346 - j2.136$$

The corresponding space-phasor diagram is shown in Figure 1.7.

1.6 ELECTRICAL TRANSIENTS WITH FLUX LINKAGES AS VARIABLES

Equations (1.30) may be transformed by changing the variables through the elimination of currents:

$$\begin{aligned} \bar{i}_s &= \sigma^{-1} \left[\frac{\bar{\Psi}_s}{L_s} - \bar{\Psi}_r \frac{L_m}{L_s L_r} \right]; \quad \sigma = 1 - \frac{L_m^2}{L_s L_r} \\ \bar{i}_r &= \sigma^{-1} \left[\frac{\bar{\Psi}_r}{L_r} - \bar{\Psi}_s \frac{L_m}{L_s L_r} \right] \end{aligned} \quad (1.54)$$

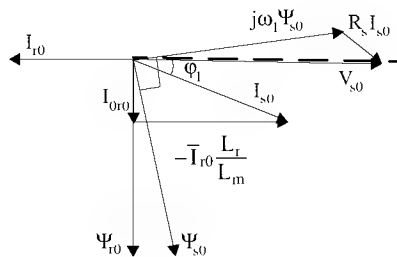


FIGURE 1.7 The space-phasor diagram.

$$\begin{aligned}\tau'_s \frac{d\bar{\Psi}_s}{dt} + (1 + j \cdot \omega_b \cdot \tau'_s) \cdot \bar{\Psi}_s &= \tau'_s \bar{V}_s + K_r \bar{\Psi}_r \\ \tau'_r \frac{d\bar{\Psi}_r}{dt} + (1 + j \cdot (\omega_b - \omega_r) \cdot \tau'_r) \cdot \bar{\Psi}_r &= \tau'_r \bar{V}_r + K_s \bar{\Psi}_s\end{aligned}\quad (1.55)$$

with

$$\begin{aligned}K_s &= \frac{L_m}{L_s}; \quad K_r = \frac{L_m}{L_r} \\ \tau'_s &= \tau_s \cdot \sigma; \quad \tau'_r = \tau_r \cdot \sigma \\ \tau_s &= \frac{L_s}{r_s}; \quad \tau_r = \frac{L_r}{r_r}\end{aligned}\quad (1.56)$$

By electrical transients, we mean constant speed transients. So both ω_b and ω_r are considered known. The inputs are the two voltage space phasors $\bar{V}_s$ and $\bar{V}_r$, and the outputs are the two flux linkage space phasors $\bar{\Psi}_s$ and $\bar{\Psi}_r$.

The structural diagram of Equations (1.55) is shown in Figure 1.8.

The transient behaviour of stator and rotor flux linkages as complex variables, at constant speeds ω_b and ω_r , for standard step or sinusoidal voltages $\bar{V}_s$ and $\bar{V}_r$ signals has analytical solutions. Finally, the torque transients have also analytical solutions as

$$T_e = \frac{3}{2} p_l \operatorname{Re}(j \Psi_s i_s^*) \quad (1.57)$$

The two complex eigenvalues of (1.55) are obtained from

$$\begin{vmatrix} \tau'_s p + 1 + j \omega_b \tau'_s & -K_r \\ -K_s & \tau'_r p + 1 + j(\omega_b - \omega_r) \tau'_r \end{vmatrix} = 0 \quad (1.58)$$

As expected, the eigenvalues $p_{1,2}$ depend on the speed of the motor and on the speed of the reference system ω_b .

Equation (1.58) may be put in the form

$$\begin{aligned}p^2 \tau'_s \tau'_r + p[(\tau'_s + \tau'_r) + j \tau'_s \tau'_r (2\omega_b - \omega_r)] - K_s K_r \\ + (1 + j \cdot \omega_b \cdot \tau'_s)(1 + j \cdot (\omega_b - \omega_r) \cdot \tau'_r) = 0\end{aligned}\quad (1.58')$$

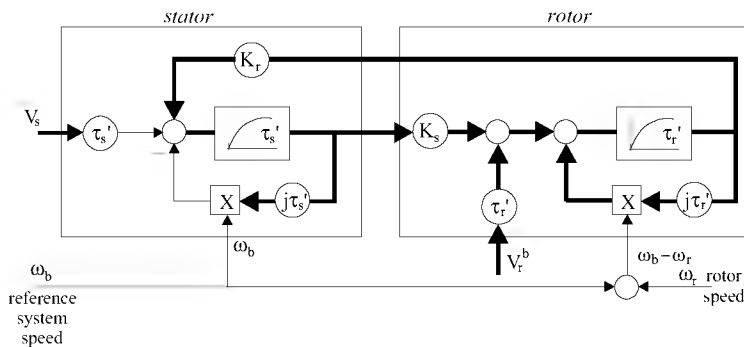


FIGURE 1.8 IM space-phasor diagram for constant speed.

In essence, in-rush current and torque (at zero speed), for example, have a rather straightforward solution through the knowledge of eigenvalues, with $\omega_r = 0 = \omega_b$,

$$p^2 \tau'_s \tau'_r + p(\tau'_s + \tau'_r) - K_s K_r + 1 = 0$$

$$(p_{1,2})_{\omega_r = \omega_b = 0} = \frac{-(\tau'_s + \tau'_r) \pm \sqrt{(\tau'_s + \tau'_r)^2 - 4\tau'_s \tau'_r (-K_s K_r + 1)}}{2\tau'_s \tau'_r} \quad (1.59)$$

The same capability of yielding analytical solutions for electrical transients is claimed by the spiral vector theory [5].

For constant amplitude stator or rotor flux conditions,

$$\left(\frac{d\bar{\Psi}_s}{dt} = j(\omega_1 - \omega_b) \bar{\Psi}_s \quad \text{or} \quad \frac{d\bar{\Psi}_r}{dt} = j(\omega_1 - \omega_b) \bar{\Psi}_r; \quad \omega_1 - \text{Stator frequency} \right)$$

Equations (1.55) are left with only one complex eigenvalue. Even a simpler analytical solution for electrical transients is feasible for constant stator or rotor flux conditions, so typical in fast response modern vector control drives.

Also, at least at zero speed, the eigenvalue with voltage type supply is about the same for stator or for rotor supply.

The same equations may be expressed with $\bar{i}_s$ and $\bar{\Psi}_r$ as variables, by simply putting $a = L_m/L_r$ and by eliminating $\bar{i}_r$ from (1.51) with (1.50).

1.7 INCLUDING MAGNETIC SATURATION IN THE SPACE-PHASOR MODEL

To incorporate magnetic saturation easily into the space-phasor model (1.49), we separate the leakage saturation from main flux path saturation with pertinent functions obtained *a priori* from tests or from field solutions (FEM).

$$L_{sl} = L_{sl}(|\bar{i}_s|), L_{lr} = L_{lr}(|\bar{i}_r|); \quad \bar{\Psi}_m = L_m(|\bar{i}_m|) \bar{i}_m \quad (1.60)$$

$$|\bar{i}_m| = |\bar{i}_s + \bar{i}_r| \quad (1.61)$$

Let us consider the reference system oriented along the main flux $\bar{\Psi}_m$, that is $\bar{\Psi}_m = \Psi_m$, and eliminate the rotor current, maintaining Ψ_m and $\bar{i}_s$ as variables.

$$\bar{V}_s = \left[R_s + (p + j\omega_b) L_{sl}(|\bar{i}_s|) \right] \cdot \bar{i}_s + (p + j\omega_b) \cdot \bar{\Psi}_m \quad (1.62)$$

$$\bar{V}_r = \left[R_r + (p + j(\omega_b - \omega_r)) L_{lr}(|\bar{i}_m - \bar{i}_s|) \right] \cdot (\bar{i}_m - \bar{i}_s) + (p + j(\omega_b - \omega_r)) \cdot \bar{\Psi}_m$$

$$i_m = \frac{\Psi_m}{L_m(i_m)} \quad (1.63)$$

We may add the equation of motion,

$$\frac{Jp\omega_r}{p_1} = \frac{3}{2} p_1 \operatorname{Re} \operatorname{al} \left(j \Psi_m \bar{i}_s^* \right) - T_{load} \quad (1.64)$$

Provided the magnetization curves $L_{sl}(i_s)$ and $L_{lr}(i_r)$ are known, Equations (1.62)–(1.64) may be solved only by numerical methods after splitting Equations (1.62) along d and q axes.

For steady state, however, it suffices that in the equivalent circuits, L_m is made a function of i_m , L_{sl} of i_{s0} and L_{lr} of (i_{r0}) (Figure 1.9). This is only in synchronous coordinates where steady state means D.C. variables.

In reality, in both the stator and the rotor, the magnetic fields are A.C. at frequency ω_l and $S\omega_l$. So, in fact, for transformer-induced voltages given in Equations (1.62), transient (A.C.) inductances should be used.

$$\begin{aligned} L_{ls}^t(i_s) &= L_{ls}(i_s) + \frac{\partial L_{ls}}{\partial i_s} i_s < L_{ls}(i_s) \\ L_{lr}^t(i_r) &= L_{lr}(i_r) + \frac{\partial L_{lr}}{\partial i_r} i_r < L_{lr}(i_r) \\ L_m^t(i_m) &= L_m(i_m) + \frac{\partial L_m}{\partial i_m} i_m < L_m(i_m) \end{aligned} \quad (1.65)$$

Typical curves are shown in Figure 1.10.

As the transient (A.C.) inductances are even smaller than the normal (D.C.) inductances, the machine behaviour at high currents is expected to show further increased currents.

Furthermore, as shown in Chapter 6, Vol. 1, the leakage flux circumferential flux lines at high currents influence the main (radial) flux and contribute to the resultant flux in the machine core. The saturation in the stator is given by the stator flux Ψ_s and in the rotor by the rotor flux for high levels of currents.

So, for large currents, it seems more appropriate to use the equivalent circuit with stator and rotor fluxes shown (Figure 1.11). However, two new variable inductances, L_{s1} and L_{r1} , are added, L_g refers to the airgap only. Finally, the stator and rotor leakage inductances are related only to end connections and slot volume: L_{ls}^e , L_{lr}^e [6].

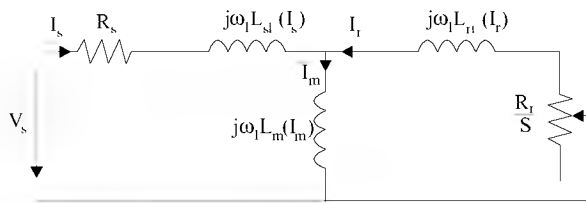


FIGURE 1.9 Standard equivalent circuit for steady-state leakage and main flux path saturation.

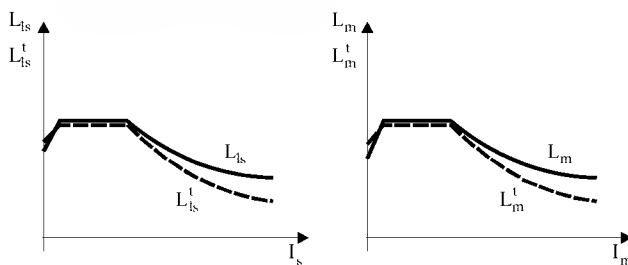


FIGURE 1.10 Leakage and main flux normal and transient inductances versus current.

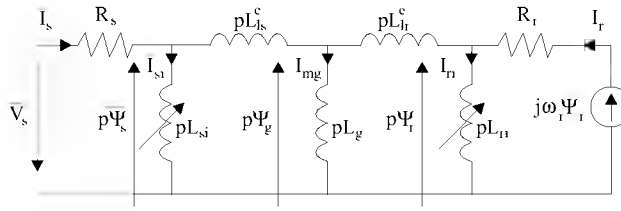


FIGURE 1.11 Space-phasor equivalent circuit with stator and rotor core saturation included (stator coordinates). (After Ref. [6].)

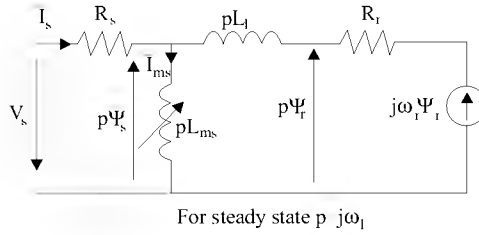


FIGURE 1.12 Space-phasor equivalent circuit with stator saturation included (stator coordinates).

As expected, the functions $\Psi_s(i_{si}) = L_{si}i_{si}$ and $\Psi_r(i_{ri}) = L_{ri}i_{ri}$ have to be known together with L_{ls}^e , L_{lr}^e , R_s , R_r and L_g which are hereby considered constant.

In the presence of skin effect, L_{lr}^e and R_r are functions of slip frequency $\omega_{sr} = \omega_l - \omega_r$. Furthermore, we should notice that it is not easy to measure all parameters in the equivalent circuit shown in Figure 1.11. It is, however, possible to calculate them either by sophisticated analytical or through FEM models. Depending on the machine design and load, the relative importance of the two variable inductances L_{si} and L_{ri} may be notable or negligible.

Consequently, the equivalent circuit may be simplified. For example, for heavy loads the rotor saturation may be ignored, and thus, only L_{si} remains. Then, L_{si} and L_g in parallel may be lumped into an equivalent variable inductance L_{ms} and L_{lr}^e and L_{ls}^e into the total constant leakage inductance of the machine L_l . Then, the equivalent circuit shown in Figure 1.10 degenerates into the one shown in Figure 1.12.

When high currents occur, during transients in electric drives, the equivalent circuit of Figure 1.11 indicates severe saturation while the conventional circuit (Figure 1.9) indicates moderate saturation of the main path flux and some saturation of the stator leakage path.

So when high current (torque) transients occur, the real machine, due to the L_{sm} reduction, produces torque performance quite different from the predictions by the conventional equivalent circuit (Figure 1.9). Up to 2–2.5 p.u. current, however, the differences between the two models are negligible.

In IMs designed for extreme saturation conditions (minimum weight), models like those shown in Figure 1.11 have to be used, unless FEM is applied.

1.8 SATURATION AND CORE LOSS INCLUSION INTO THE STATE-SPACE MODEL

To include the core loss into the space-phasor model of IMs, we assume here that the core losses occur, both in the stator and in the rotor, into equivalent orthogonal windings: $c_d - c_q$, $c_{dr} - c_{qr}$ (Figure 1.13) [7].

Alternatively, when rotor core loss is neglected (cage-rotor IMs), the $c_{dr} - c_{qr}$ windings account for the skin effect (the double-cage equivalence principle).

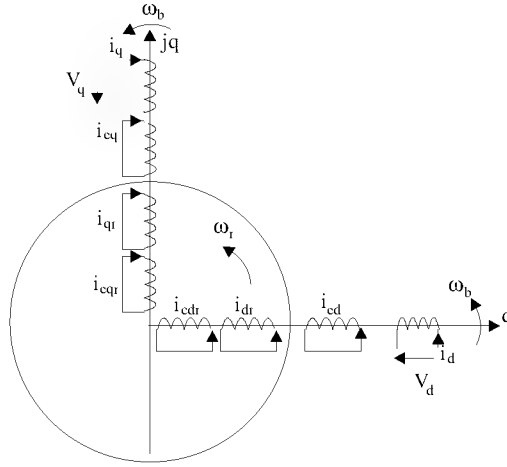


FIGURE 1.13 d–q model with stator and core loss windings.

Let us consider also that the core loss windings are coupled to the other windings only by the main flux path.

The space-phasor equations are thus straightforward (by addition).

$$\begin{aligned} R_s \bar{i}_s - \bar{V}_s &= -\frac{d\bar{\Psi}_s}{dt} - j\omega_b \bar{\Psi}_s; \quad \bar{\Psi}_s = \bar{\Psi}_m + L_{ls} \bar{i}_s \\ R_{cs} \bar{i}_{cs} &= -\frac{d\bar{\Psi}_{cs}}{dt} - j\omega_b \bar{\Psi}_{cs}; \quad \bar{\Psi}_{cs} = \bar{\Psi}_m + L_{lcs} \bar{i}_{cs} \end{aligned} \quad (1.66)$$

$$R_r \bar{i}_r - \bar{V}_r = -\frac{d\bar{\Psi}_r}{dt} - j(\omega_b - \omega_r) \bar{\Psi}_r; \quad \bar{\Psi}_r = \bar{\Psi}_m + L_{lr} \bar{i}_r$$

$$R_{cr} \bar{i}_{cr} = -\frac{d\bar{\Psi}_{cr}}{dt} - j(\omega_b - \omega_r) \bar{\Psi}_{cr}; \quad \bar{\Psi}_{cr} = \bar{\Psi}_m + L_{lcr} \bar{i}_{cr}$$

$$\bar{i}_m = (\bar{i}_s + \bar{i}_{cs} + \bar{i}_r + \bar{i}_{cr}); \quad \bar{\Psi}_m = L_m (\bar{i}_m) \bar{i}_m \quad (1.67)$$

The airgap torque now contains two components: one given by the stator current and the other one (braking) given by the stator core losses.

$$T_e = \frac{3}{2} p_l \text{Re} \left[j \left(\bar{\Psi}_s \bar{i}_s^* + \bar{\Psi}_{cs} \bar{i}_{cs}^* \right) \right] = \frac{3}{2} p_l \text{Re} \left[j \bar{\Psi}_m \left(\bar{i}_s^* + \bar{i}_{cs}^* \right) \right] \quad (1.68)$$

Equations (1.66) and (1.67) lead to a fairly general equivalent circuit when we introduce the transient magnetization inductance L_{mt} (1.65) and consider separately main flux and leakage paths saturation (Figure 1.14).

The apparently involved equivalent circuit shown in Figure 1.14 is fairly general and may be applied for many practical cases such as

- The reference system may be attached to the stator, $\omega_b = 0$ (for cage rotor and large transients), to the rotor (for the doubly fed IM) or to stator frequency $\omega_b = \omega_1$ for electric drives transients.

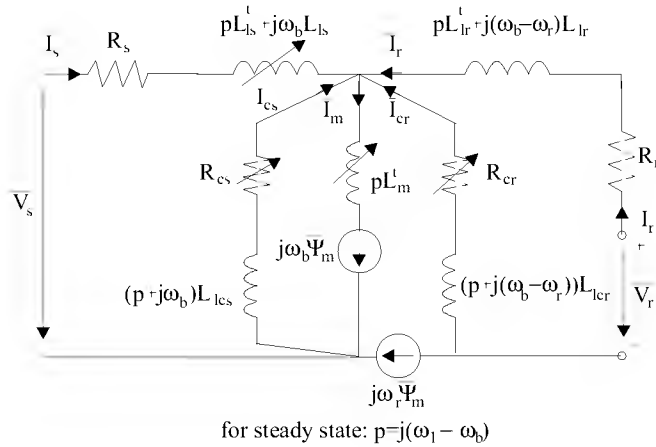


FIGURE 1.14 Space-phaser T equivalent circuit with saturation and rotor core loss (or rotor skin effect).

- To easily simplify Equations (1.66–1.67), the reference system may be attached to the main flux space phasor: $\bar{\Psi}_m = \Psi_m(i_m)$ and eventually using i_m , i_{cs} , i_{cr} , i_r , and ω_r as variables with $\bar{i}_s$ as a dummy variable [7.8]. As expected, d–q decomposition is required.
- For a cage rotor with skin effect (medium- and high-power motors fed from the power grid directly), we should simply make $V_r = 0$ and consider the rotor core loss winding (with its equations) as the second (fictitious) rotor cage.

Example 1.2 Saturation and Core Losses

Simulation results are presented in what follows. The motor constant parameters are $R_s = 3.41 \, \Omega$, $R_r = 1.89 \, \Omega$, $L_{sl} = 1.14 \cdot 10^{-2} \, \text{H}$, $L_{rl} = 0.9076 \cdot 10^{-2} \, \text{H}$, $J = 6.25 \cdot 10^{-3} \, \text{Kgm}^2$. The magnetization curve $\Psi_m(i_m)$ together with core loss in the stator at 50 Hz is shown in Figure 1.15.

The stator core loss resistance R_{cs} is

$$R_{cs} = \frac{3}{2} \frac{\omega_1^2 \Psi_m^2}{P_{\text{iron}}} \approx \frac{3}{2} \frac{(2\pi 50)^2 \cdot 2^2}{500} = 1183.15 \, \Omega$$

Considering that $L_{lcs} \cdot \omega_1 = R_{cs}$; $L_{lcs} = 1183.15 / (2\pi 50) = 3.768 \, \text{H}$.

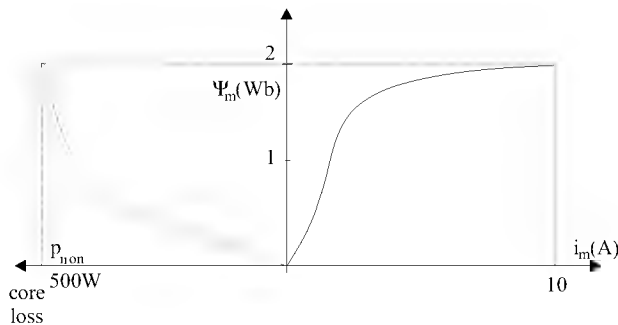


FIGURE 1.15 Magnetization curve and stator core loss.

For steady-state and synchronous coordinates ($d/dt = 0$), Equations (1.66) become

$$\begin{aligned} R_s \bar{i}_{s0} - \bar{V}_{s0} &= -j\omega_l \bar{\Psi}_{s0}; \quad V_{s0} = 380\sqrt{2}e^{j\delta_0} \\ R_{cs} \bar{i}_{cs0} &= -j\omega_l \bar{\Psi}_{cs0}; \quad \bar{i}_{m0} = \bar{i}_{s0} + \bar{i}_{cs0} + \bar{i}_{r0} \\ R_r \bar{i}_{r0} &= -jS\omega_l \bar{\Psi}_{r0} \end{aligned} \quad (1.69)$$

For given values of slip S , the values of stator current $\bar{i}_{s0}/\sqrt{2}$ (RMS/phase) and the electromagnetic torque are calculated for steady state using (1.69) and the flux–current relationship using (1.66) and (1.67) and Figure 1.15. The results are given in Figure 1.16a and b [7]. There are notable differences in stator current due to saturation. The differences in torque are rather small. Efficiency–power factor product and the magnetization current versus slip are shown in Figure 1.17a and b.

Again, saturation and core loss play an important role in reducing the efficiency–power factor although core loss itself tends to increase the power factor while it tends to reduce the efficiency. The reduction of magnetization current i_m with slip is small but still worth considering.

Two transients have been investigated by solving (1.66) and (1.67) and the motion equations through the Runge–Kutta–Gill method.

1. Sudden 40% reduction of supply voltage from 380 V per phase (RMS), steady-state constant load corresponding to $S = 0.03$.
2. Disconnection of the loaded motor at $S = 0.03$ for 10 ms and reconnection (at $\delta_0 = 0$, though any phasing could be used). The load is constant again.

The transients for transient 1 are shown in Figure 1.18a–c.

Some influence of saturation and core loss occurs in the early stages of the transients, but in general, the influence of them is moderate because at reduced voltage, saturation influence diminishes considerably.

The second transient, occurring at high voltage (and saturation level), causes more important influences of saturation (and core loss) as shown in Figure 1.19a–c.

The saturation and core loss lead to higher current peaks, but apparently for low but faster torque and speed transients.

In high-performance variable speed drives, high levels of saturation may be inflicted to increase the torque transient capabilities of IM. Vector control detuning occurs, and it has to be corrected. Also, in very precise torque control drives, such second-order phenomena are to be considered. Skin effect also influences the transients of IMs and the model shown in Figure 1.13 can handle it for cage-rotor IMs directly, as shown earlier in this paragraph.

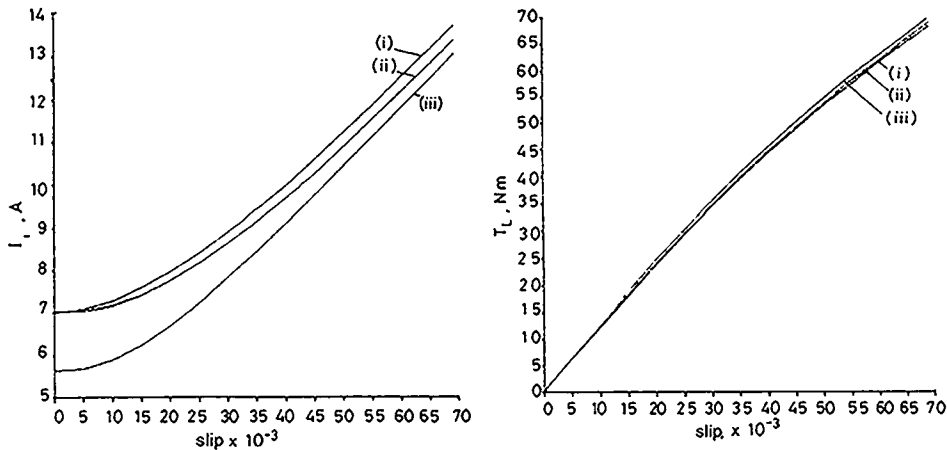


FIGURE 1.16 Steady-state phase current and torque versus slip (i) saturation and core loss considered, (ii) saturation considered, core loss neglected, and (iii) no saturation, no core loss. (After Ref. [7].)

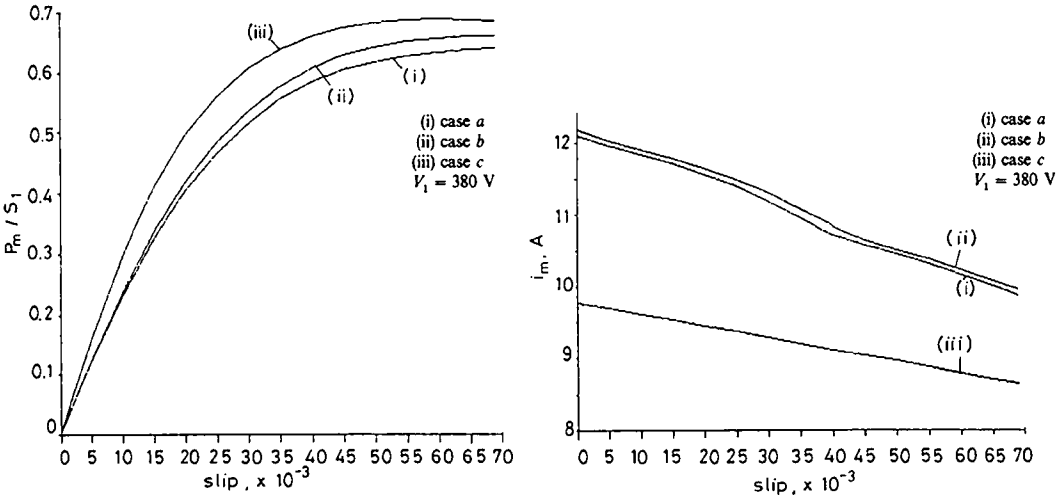


FIGURE 1.17 Efficiency–power factor product and magnetization current versus slip. (After Ref. [7].)

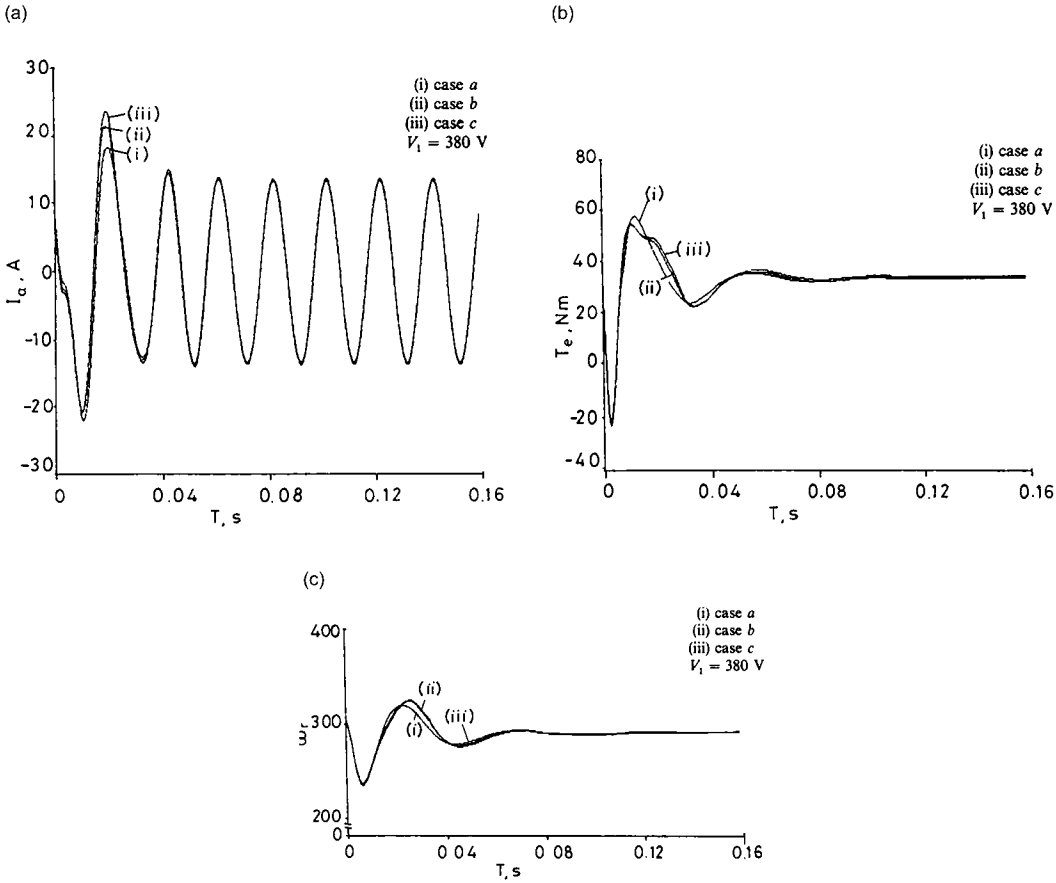


FIGURE 1.18 Sudden 40% voltage reduction at $S = 0.03$ constant load: (a) stator phase current and (b) torque (After Ref. [7]) and (c) speed.

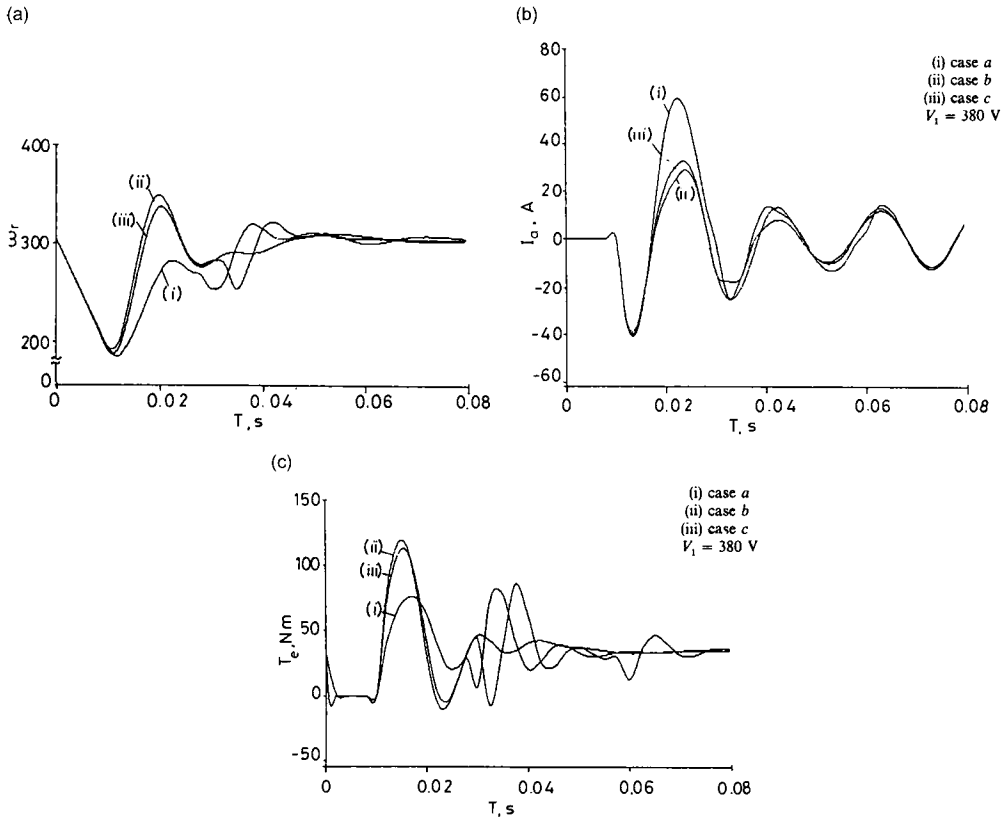


FIGURE 1.19 Disconnection–reconnection transients at high voltage: $V_{\text{phase}} = 380$ V (RMS): (a) speed, (b) phase current, (c) torque. (After Ref. [7].)

1.9 REDUCED-ORDER MODELS

The rather involved d–q (space-phaser) model with skin effect and saturation may be used directly to investigate the IM transients. More practical (simpler) solutions have been also proposed. They are called reduced-order models [9–11].

The complete model has, for a single cage, a fifth order in d–q writing and a third order in complex variable writing ($\bar{\Psi}_s, \bar{\Psi}_r, \omega_r$).

In general, the speed transients are slow, especially with high inertia or high loads, whereas stator flux $\bar{\Psi}_s$ and rotor flux $\bar{\Psi}_r$ transients are much faster for voltage source supplying.

The intuitive way to obtain reduced models is to ignore fast transients, of the stator, $\frac{d\bar{\Psi}_s}{dt} = 0$ (in synchronous coordinates) or stator and rotor transients $\left(\frac{d\bar{\Psi}_s}{dt} = \frac{d\bar{\Psi}_r}{dt} = 0 \right)$ (in synchronous coordinates) and update the speed by solving the motion equation by numerical methods.

This way, third-order $\left(\frac{d\bar{\Psi}_s}{dt} = 0 \right)$ or first-order $\left(\frac{d\bar{\Psi}_s}{dt} = \frac{d\bar{\Psi}_r}{dt} = 0 \right)$ models are obtained.

The problem is that the results of such order reductions obscure inevitably fast transients. Electric (supply frequency) torque transients (due to rotor flux transients) during starting may still be visible in such models, but for too a long time interval during starting in comparison with the reality.

So there are two questions with model order reduction:

- What kind of transients are to be investigated?
- What torque transients have to be preserved?

In addition to this, low- and high-power IMs seem to require different reduced-order models to produce practical results in simulating a group of IMs when the computation time is critical.

1.9.1 NEGLECTING STATOR TRANSIENTS

In this case, in synchronous coordinates, the stator flux derivative is considered zero (Equations (1.55)).

$$\begin{aligned}
 0 + (1 + j\omega_1\tau'_s)\bar{\Psi}_s &= \tau'_s\bar{V}_s + K_r\bar{\Psi}_r \\
 \tau'_r \frac{d\bar{\Psi}_r}{dt} + (1 + j(\omega_1 - \omega_r))\bar{\Psi}_r &= \tau'_r\bar{V}_r + K_s\bar{\Psi}_s \\
 T_e = -\frac{3}{2}p_l \text{Re}\left(j\bar{\Psi}_r \bar{i}_r^i\right); \quad \bar{i}_r^* &= \sigma^{-1} \left(\frac{\bar{\Psi}_r}{L_r} - \bar{\Psi}_s \frac{L_m}{L_s L_r} \right) \\
 \frac{J}{p_l} \frac{d\omega_r}{dt} &= T_e - T_{\text{load}}
 \end{aligned} \tag{1.70}$$

With two algebraic equations, there are three differential ones (in real d–q variables).

For cage-rotor IMs,

$$\bar{\Psi}_s = \frac{\tau'_s\bar{V}_s + K_r\bar{\Psi}_r}{1 + j\omega_1\tau'_s} \tag{1.71}$$

$$\frac{d\bar{\Psi}_r}{dt} = \left[- (1 + j(\omega_1 - \omega_r)) + \frac{K_s K_r}{1 + j\omega_1\tau'_s} \right] \bar{\Psi}_r + \frac{\tau'_s \cdot \bar{V}_s \cdot K_s}{(1 + j\omega_1\tau'_s)\tau'_r} \tag{1.72}$$

$$T_e = \frac{3}{2}p_l \text{Im} \left[\sigma^{-1} \bar{\Psi}_r \bar{\Psi}_s^* \frac{L_m}{L_s L_r} \right] = \frac{3}{2}p_l \frac{\sigma^{-1} L_m}{L_s L_r} \text{Im} \left[\bar{\Psi}_r \frac{\tau'_s \bar{V}_s + K_r \bar{\Psi}_r^i}{1 - j\omega_1\tau'_s} \right] \tag{1.73}$$

$$\frac{d\omega_r}{dt} = \frac{p_l}{J} (T_e - T_{\text{load}})$$

Fast (at stator frequency) transient torque pulsations are absent in this third-order model (Figure 1.20b).

The complete model and third-order model for a motor with $L_s = 0.05$ H, $L_r = 0.05$ H, $L_m = 0.0474$ H, $R_s = 0.29$ Ω , $R_r = 0.38$ Ω , $\omega_1 = 100\pi$ rad/s, $V_s = 220\sqrt{2}$ V, $p_l = 2$, $J = 0.5$ Kg m^2 yields, for no-load, starting transient results as shown in Figure 1.20 [11].

It is to be noted that steady-state torque at high slips (Figure 1.20a) falls into the middle of torque pulsations, which explains why calculating no-load starting time with steady-state torque produces good results.

A second-order system may be obtained by considering only the amplitude transients of $\bar{\Psi}_r$ in (1.73) [11], but the results are not good enough in the sense that the torque fast (grid frequency) transients are present during start-up at higher speed than for the full model.

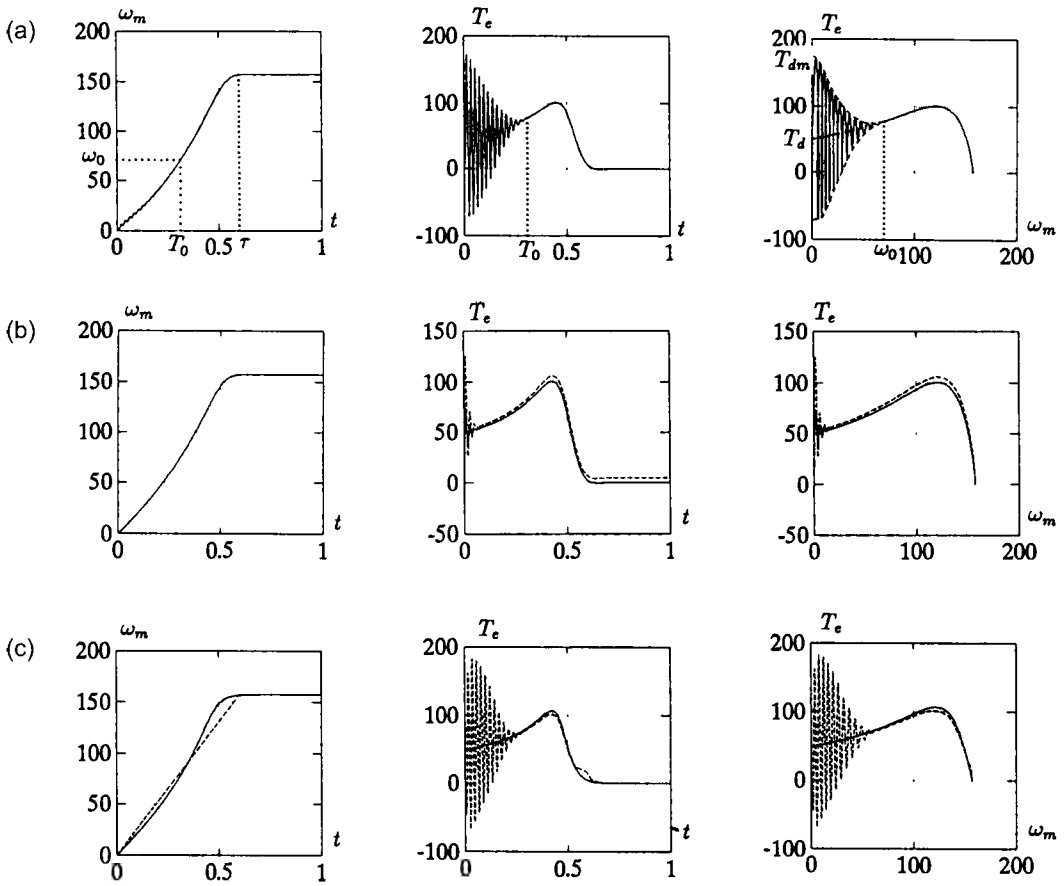


FIGURE 1.20 Starting transients: (a) fifth-order model, (b) third-order model, and (c) modified second-order model. (After Ref. [11].)

Modified second-order models have been proposed to improve the precision of torque results [11] (Figure 1.20), but the results are highly dependent on motor parameters and load during starting. So, in fact, for starting transients when the torque pulsations (peaks) are required with precision, model reduction may be exercised with extreme care.

The presence of leakage saturation makes the above results mostly of academic interest.

1.9.2 CONSIDERING LEAKAGE SATURATION

As shown in Chapter 9, Vol. 1, the leakage inductance ($L_{sc} = L_{s1} + L_{r1} = L_l$) decreases with current. This reduction is accentuated when the rotor has closed slots and more moderate when both, stator and rotor, have semiclosed slots (Figure 1.21).

The calculated (or measured) curves of Figure 1.21 may be fitted with analytical expressions of RMS rotor current such as [12]

$$X_1 = \omega_l L_l = K_1 + K_2 \left(\frac{I_r}{\sqrt{2}} \right)^{-K} \quad \text{for curve 1} \quad (1.74)$$

$$X_1 = \omega_l L_l = K_3 - K_4 \tanh(K_3 I_r^3 - K_6) + K_7 \cos(K_8 I_r) \quad \text{for curve 2} \quad (1.75)$$

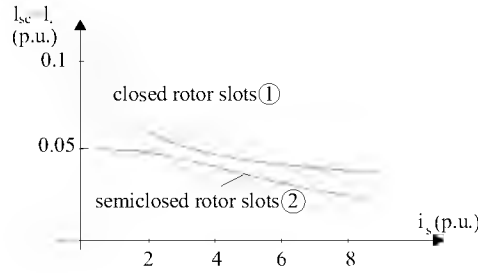


FIGURE 1.21 Typical leakage inductance versus stator current for semiclosed and closed stator slots.

During starting on no-load, the stator current envelope $I_{sp}(t)$ varies approximately as [12]

$$I_{sp} \approx \frac{Q_1 I_{rk}}{\sqrt{3 \left(1 + \sqrt{\frac{t}{t_{k0}}} + \left(\frac{t}{t_{k0}} \right)^2 \right)}} \quad (1.76)$$

Q_1 reflects the effect of point on wave connection when nonsymmetrical switching of supply is considered to reduce torque transients (peaks) [13]

$$Q_1 \approx \sqrt{2 + \left(\frac{X_l(S_k)}{R_s + R_r} \right) \cdot (\cos^2 \alpha + \sin^2(\alpha + \beta))} \quad (1.77)$$

α – point on wave connection of first phase-to-phase (b-c) voltage.

β – delay angle in connecting the third phase.

It has been shown that minimum current (and torque) transients are obtained for $\alpha = \pi/2$ and $\beta = \pi/2$ [14].

As the steady-state torque gives approximately the same starting time as the transient one, we may use it in the motion equation (for no-load).

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = \frac{3 \cdot V_{ph}^2 \cdot R_r \cdot p_1}{\omega_1} \cdot \frac{1}{\left(R_s + \frac{R_r}{S} \right)^2 S + \omega_1^2 L_1^2 S} = T_e \quad (1.78)$$

We may analytically integrate this equation with respect to time up to the breakdown point: S_K slip,

noting that $\frac{d\omega_r}{dt} = -\omega_1 \frac{dS}{dt}$.

$$t_{k0} = \frac{J\omega_1^2}{3V_{ph}^2 R_r p_1} \left[\frac{1}{2} \left(R_s^2 + (\omega_1 L_1 (S_{K0}))^2 \right) (1 - S_{K0}^2) + 2R_s R_r (1 - S_{K0}) + R_r^2 \ln \left(\frac{1}{S_{K0}} \right) \right] \quad (1.79)$$

$$\text{with } S_{K0} = \frac{R_r}{\sqrt{R_s^2 + (\omega_1 L_1 (S_{K0}))^2}} \quad (1.80)$$

On the other hand, the base current I_{rk} is calculated at standstill.

$$I_{rk} = \frac{V_{ph}}{\sqrt{(R_s + R_r)^2 + (\omega L_l(1))^2}} \quad (1.81)$$

Introducing L_l as function of I_r from (1.74) or (1.75) into (1.81), we may solve it numerically to find I_{rk} .

Now, we may calculate the time variation of I_{sp} (the current envelope) versus time during no-load starting. The current envelope compares favourably with complete model results and with test results [12].

As the current envelope varies with time, so does the leakage inductance $L_l(I_{sp})$, and finally, the torque value in (1.78) is calculated as a function of time through its peak values, since from Equation (1.78) we may calculate slip (S) versus time.

Sample results shown in Figure 1.22 [12] show that the approximation above is really practical and that neglecting the leakage saturation leads to an underestimation of torque peaks by more than 40%!

1.9.3 LARGE MACHINES: TORSIONAL TORQUE

Starting of large IMs causes not only heavy starting currents but also high torsional torque due to the large masses of the motor and load which may exhibit a natural torsional frequency in the range of electric torque oscillatory components.

Sudden voltage switching (through a transformer or star/delta switch), low leakage time constants τ'_s and τ'_r or a long starting interval causes further problems in starting large inertia loads in large IMs. The layout of such a system is shown in Figure 1.23.

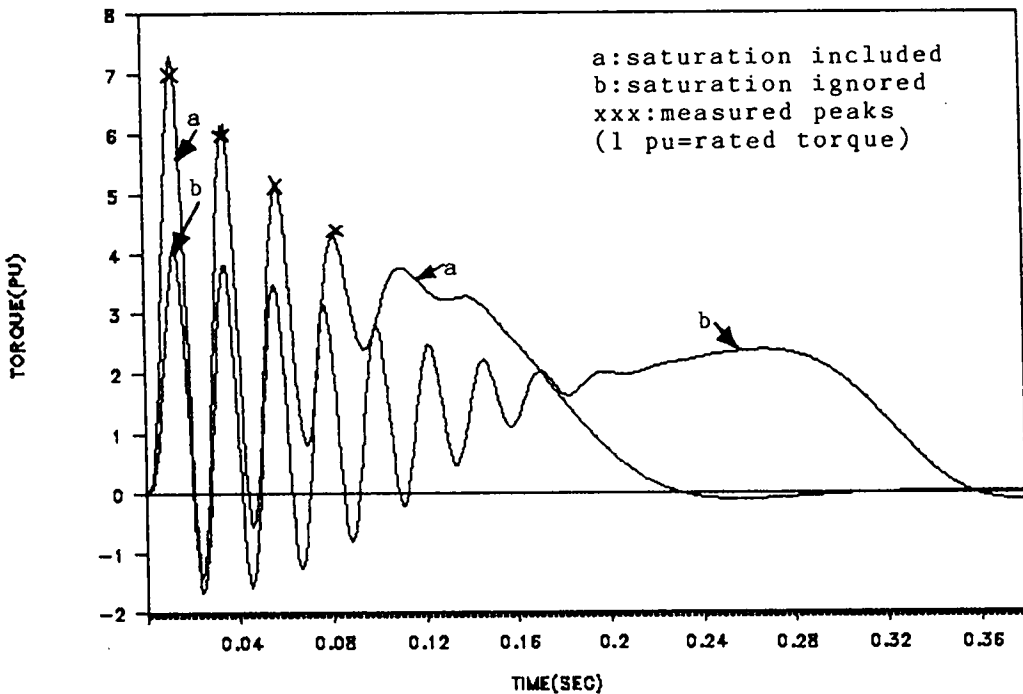


FIGURE 1.22 Torque transients during no-load starting. (After [12].)

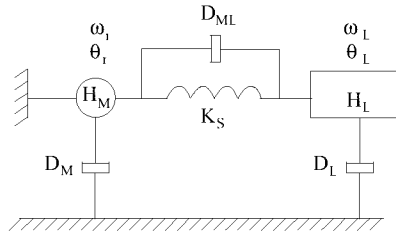


FIGURE 1.23 Large IM with inertia load.

The mechanical equations of the rotating masses in Figure 1.23, with elastic couplings, are straightforward [15].

$$\begin{bmatrix} \dot{\omega}_m \\ \dot{\theta}_m \\ \dot{\omega}_L \\ \dot{\theta}_L \end{bmatrix} = \begin{bmatrix} -\frac{(D_M+D_{ML})}{2H_M} & -\frac{K_S\omega_0}{2H_M} & \frac{D_{ML}}{2H_M} & \frac{K_S\omega_0}{2H_M} \\ 1 & 0 & 0 & 0 \\ -\frac{D_{ML}}{2H_M} & \frac{K_S\omega_0}{2H_L} & -\frac{(D_L+D_{HL})}{2H_L} & -\frac{K_S\omega_0}{2H_L} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_m \\ \theta_m \\ \omega_L \\ \theta_L \end{bmatrix} \quad (1.82)$$

ω_0 – base speed, H_M, H_L – inertia in seconds, D_{HL}, D_{ML}, D_M in p.u./rad/s, K_S in p.u./rad.

The rotor d–q model equation (1.34), with single rotor cage, in p.u. (relative units) is, after a few manipulations,

$$\begin{vmatrix} x_s & 0 & x_m & 0 \\ 0 & x_s & 0 & x_m \\ x_m & 0 & x_r & 0 \\ 0 & x_m & 0 & x_r \end{vmatrix} \cdot \frac{1}{\omega_0} \frac{d}{dt} \begin{vmatrix} i_q \\ i_d \\ i_{qr} \\ i_{dr} \end{vmatrix} = \begin{vmatrix} -r_s & -\frac{\omega_m x_s}{\omega_0} & 0 & -\frac{\omega_m x_m}{\omega_0} \\ \frac{\omega_m x_s}{\omega_0} & -r_s & \frac{\omega_m x_m}{\omega_0} & 0 \\ 0 & -Sx_m & -r_r & -Sx_r \\ Sx_m & 0 & Sx_r & -r_r \end{vmatrix} \begin{vmatrix} i_q \\ i_d \\ i_{qr} \\ i_{dr} \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} V_q \\ V_d \\ 0 \\ 0 \end{vmatrix} \quad (1.83)$$

where r_s, r_r, x_s, x_r , and x_m are the p.u. values of stator (rotor) resistances and stator, rotor and magnetization reactances

$$r_s = \frac{R_s}{X_n}; \quad x_s = \frac{\omega_0 L_s}{X_n}; \quad X_n = \frac{V_{nph}}{I_{nph}} \quad (1.84)$$

$$H = \frac{J \left(\frac{\omega_0}{p_l} \right)^2}{S_n} \quad (\text{seconds}) \quad (1.85)$$

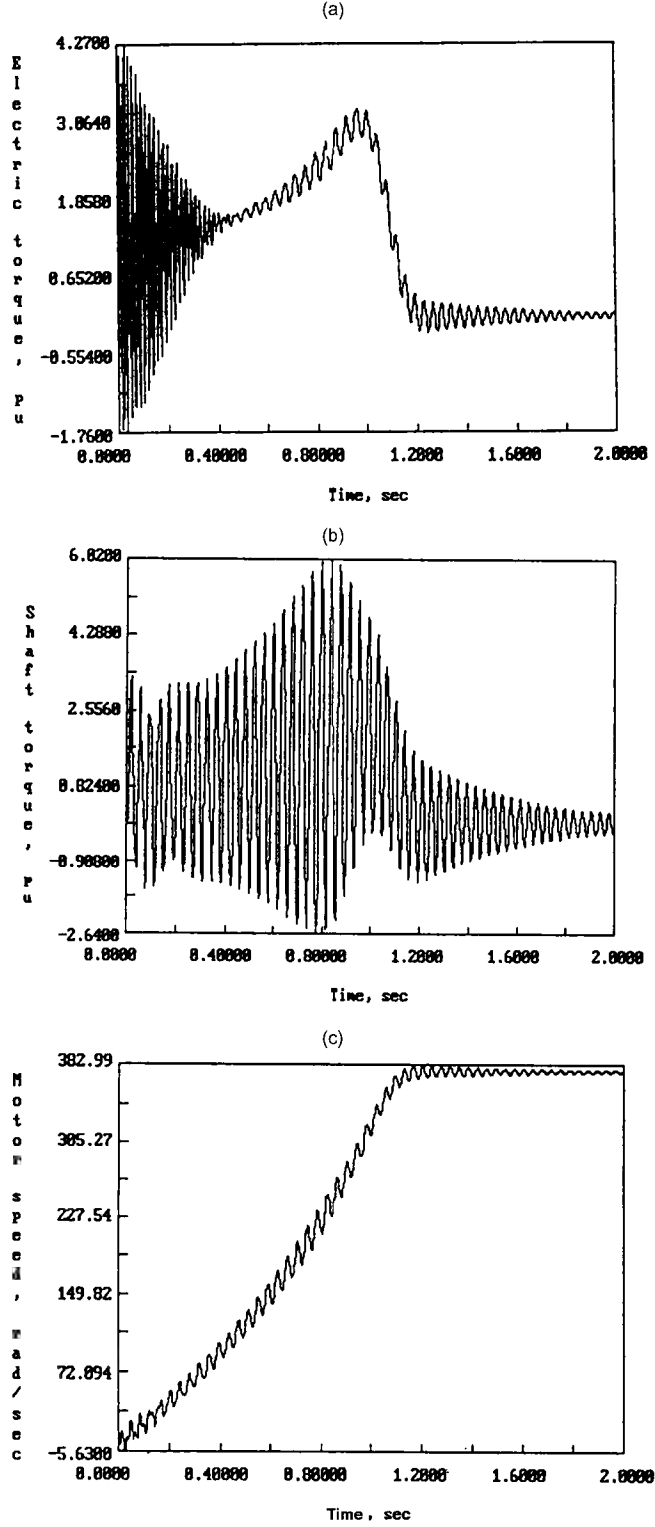


FIGURE 1.24 Starting transients of a large motor with elastic coupling of inertial load (After Ref. [15]).
(a) Electromagnetic torque, (b) shaft torque, and (c) motor speed.

$$V_d + jV_q = \frac{2}{3} \left(V_a + V_b e^{j\frac{2\pi}{3}} + V_c e^{-j\frac{2\pi}{3}} \right) e^{-j\omega_0 t} \quad (1.86)$$

$$\text{The torque is } t_e = x_m (\dot{i}_{dr} \dot{i}_q - \dot{i}_{qr} \dot{i}_d) \quad (1.87)$$

Equations (1.82) and (1.83) may be solved by numerical methods such as the Runge–Kutta–Gill method. For the data [15] $r_s = 0.0453$ p.u., $x_s = 2.1195$ p.u., $r_r = 0.0272$ p.u., $x_r = 2.0742$ p.u., $x_m = 2.042$ p.u., $H_M = 0.3$ s, $H_L = 0.74$ s, $D_L = D_M = 0$, $D_{ML} = 0.002$ p.u./(rad/s), $K_S = 30$ p.u./rad, $T_L = 0.0$, $\omega_0 = 377$ rad/s, the electromagnetic torque (t_e), the shaft torque (t_{sh}) and the speed ω_r are shown in Figure 1.24 [15].

The current transients include a stator frequency current component in the rotor, a slip frequency current in the rotor, and D.C. decaying components both in the stator and in the rotor. As expected, their interaction produces four torque components: unidirectional torque (by the rotating field components), supply frequency ω_0 , slip frequency $S\omega_0$, and speed frequency ω_m .

The three A.C. components may interact with the mechanical part whose natural frequency f_m is [15]:

$$f_m = \frac{1}{2\pi} \sqrt{\frac{K_S (H_M + H_L)}{2H_M H_L}} = 26 \text{ Hz} \quad (1.88)$$

The torsional (shaft) torque shown in Figure 1.24b (much higher than the electromagnetic torque) may be attributed to the interaction of the slip frequency component of electromagnetic torque, which starts at 60 Hz ($S = 1$) and reaches 26 Hz as the speed increases ($S \cdot 60 \text{ Hz} = 26 \text{ Hz}$, $\omega_m = (1 - S) \omega_0 = 215$ rad/s). The speed frequency component (ω_m) of electromagnetic torque is active when $\omega_m = 162$ rad/s ($\omega_m = 2\pi f_m$), but it turns out to be small.

A good start would imply the avoidance of torsional torques to prevent shaft damage.

For example, in the star/delta connection starting, the switching from star to delta may be delayed until the speed reaches 80% of rated speed to avoid the torsional torques occurring at 215 rad/s and reducing the current and torque peaks below it.

The total starting time with star/delta is larger than for direct (full voltage) starting, but shaft damage is prevented.

Using an autotransformer with 40%, 60%, 75%, and 100% voltage steps reduces further the shaft torque but at the price of even larger starting time.

Also, care must be exercised to avoid switching voltage steps near $\omega_m = 215$ rad/s (in our case).

1.10 THE SUDDEN SHORT CIRCUIT AT TERMINALS

Sudden short circuit at the terminals of large induction motors produces severe problems in the corresponding local power grid [16].

Large IMs with cage rotors are skin-effect-influenced, so the double-cage representation is required. Substituting $\bar{V}_r = 0$ in (1.66), the space-phasor model for double-cage IMs is obtained, and eliminating the stator core loss and the rotor core loss, the second rotor cage equation can be obtained. In rotor coordinates,

$$\begin{aligned} \bar{V}_s &= R_s \bar{i}_s + s \bar{\Psi}_s + j\omega_r \bar{\Psi}_s; \quad \bar{\Psi}_s = \bar{\Psi}_m + L_{sl} \cdot \bar{i}_s; \quad \bar{\Psi}_m = L_m \cdot \bar{i}_m \\ 0 &= R_{r1} \cdot \bar{i}_{r1} + s \bar{\Psi}_{r1}; \quad \bar{i}_m = \bar{i}_s + \bar{i}_{r1} + \bar{i}_{r2}; \quad \bar{\Psi}_{r1} = \bar{\Psi}_m + L_{r11} \cdot \bar{i}_{r1} \\ 0 &= R_{r2} \cdot \bar{i}_{r2} + s \bar{\Psi}_{r2}; \quad \bar{\Psi}_{r2} = \bar{\Psi}_m + L_{r12} \cdot \bar{i}_{r2} \end{aligned} \quad (1.89)$$

To simplify the solution, we may consider that the speed remains constant (eventually with initial slip $S_0 = 0$).

Eliminating $\bar{i}_{r1}$ and $\bar{i}_{r2}$ from (1.89) yields

$$\bar{\Psi}_s(p) = L(p)\bar{i}_s(p) \quad (1.90)$$

The operational inductance $L(p)$ is

$$L(p) = L_s \frac{(1+T'p)(1+T''p)}{(1+T'_0p)(1+T''_0p)} \quad (1.91)$$

The transient and subtransient inductances L'_s and L''_s for synchronous machines are given as follows:

$$L_s = \lim_{p \rightarrow 0} L(p)$$

$$L'_s = \lim_{p \rightarrow \infty} (L(p))_{T''-T''_0=0} = L_s \frac{T'}{T'_0} < L_s \quad (1.92)$$

$$L''_s = \lim_{s \rightarrow \infty} L(s) = L_s \frac{T'T''}{T'_0T''_0} \ll L_s \quad (1.93)$$

Using Laplace transformation (for $\omega_r = \omega_1 = ct$) makes the solution much easier to obtain. First, the initial (D.C.) stator current at no-load in rotor (synchronous) coordinates ($\omega_r = \omega_1$), $i_s^0(t)$, is

$$i_s^0 = \frac{V_s e^{j\gamma}}{j\omega_1 L_s}; \quad V_s = V\sqrt{2}; R_s \approx 0 \quad (1.94)$$

$$V_{a,b,c} = V\sqrt{2} \cos\left(\omega_1 t + \gamma - (i-1)\frac{2\pi}{3}\right) \quad (1.95)$$

To short-circuit the machine model, $-V_s e^{j\gamma}$ must be applied to the terminals, and thus, substituting Equation (1.89) in (1.90) yield (in Laplace form)

$$-\frac{V_s e^{j\gamma}}{p} = [R_s + (p + j\omega_1)L(p)]i_s(p) \quad (1.96)$$

$$\text{Denoting } \alpha = \frac{R_s}{L(p)} \approx \frac{R_s}{L''_s} = ct \quad (1.97)$$

the current $i_s(p)$ becomes

$$i_s(s) = \frac{-V_s e^{j\gamma}}{pL(p)(p + j\omega_1 + \alpha)} \quad (1.98)$$

with $L(p)$ of (1.91) written in the form

$$\frac{1}{L(p)} = \frac{1}{L_s} + \frac{\left(\frac{1}{L'_s} - \frac{1}{L_s}\right)p}{\frac{1}{T'} + p} + \frac{\left(\frac{1}{L''_s} - \frac{1}{L'_s}\right)p}{\frac{1}{T''} + p} \quad (1.99)$$

$i_s(p)$ may be put into an easy form to solve.

Finally, $i_s(t)$ is

$$\begin{aligned} \bar{i}_s(t) = & -\frac{V_s}{j\omega_1} e^{j\gamma} \left[\frac{1}{L_s} \left(1 - e^{-(\alpha + j\omega_1)t} \right) + \left(\frac{1}{L'_s} - \frac{1}{L_s} \right) \left(e^{-\frac{t}{T'}} - e^{-(\alpha + j\omega_1)t} \right) \right] \\ & + \left(\frac{1}{L''_s} - \frac{1}{L'_s} \right) \left(e^{-\frac{t}{T''}} - e^{-(\alpha + j\omega_1)t} \right) \end{aligned} \quad (1.100)$$

Now, the current in phase a, $i_a(t)$, is

$$\begin{aligned} i_a(t) &= \text{Re} \left[\left(\bar{i}_s(t) + \bar{i}_s^0(t) \right) e^{j\omega_1 t} \right] \\ &= V\sqrt{2} \left[\frac{1}{\omega_1 L''_s} e^{-\alpha t} \sin\gamma - \left[\left(\frac{1}{\omega_1 L'_s} - \frac{1}{\omega_1 L_s} \right) e^{-\frac{t}{T'}} + \left(\frac{1}{\omega_1 L''_s} - \frac{1}{\omega_1 L'_s} \right) e^{-\frac{t}{T''}} \right] \sin(\omega_1 t + \gamma) \right] \end{aligned} \quad (1.101)$$

A few remarks are in order.

- The subtransient component, related to α , T'' , L''_s reflects the decay of the leakage flux towards the rotor surface (upper cage).
- The transient component, related to T' and L'_s , reflects the decay of leakage flux pertaining to the lower cage.
- The final value of short-circuit current is zero.
- The torque expression requires first the stator flux solution.

$$T_e = \frac{3}{2} P_1 \left(\bar{\Psi}_s \bar{i}_s^* \right) \quad (1.102)$$

The initial value of stator flux ($R_s \approx 0$) is

$$\bar{\Psi}_s^0 = \frac{V_s e^{j\gamma}}{j\omega_1} = L_s \bar{i}_s^0 \quad (1.103)$$

with $\bar{i}_s^0$ from (1.94).

After short circuit,

$$\bar{\Psi}_s(p) = \frac{-V_s e^{j\gamma}}{p[p + j\omega_1 + \alpha]} = L(p) \bar{i}_s(p) \quad (1.104)$$

with $\bar{i}_s(p)$ from (1.98).

$$\text{So, } \bar{\Psi}_s(t) = \frac{-V_s e^{j\gamma}}{\alpha + j\omega_1} \left(1 - e^{-(\alpha + j\omega_1)t} \right) \quad (1.105)$$

We neglect α and add $\bar{\Psi}_s(t)$ with $\bar{\Psi}_s^0$ to obtain $\bar{\Psi}_{st}(t)$.

$$\bar{\Psi}_{st}(t) = \frac{V_s e^{j\gamma}}{j\omega_1} e^{-(\alpha + j\omega_1)t} \quad (1.106)$$

Now, from (1.100), (1.105) and (1.102), the torque is

$$T_e = 3V^2 \frac{P_1}{\omega_1} \left[\left(\frac{1}{\omega_1 L'_s} - \frac{1}{\omega_1 L_s} \right) e^{-\frac{t}{T'}} + \left(\frac{1}{\omega_1 L''_s} - \frac{1}{\omega_1 L'_s} \right) e^{-\frac{t}{T''}} \right] \sin(\omega_1 t) \quad (1.107)$$

Neglecting the damping components in (1.107), the peak torque is

$$T_{\text{peak}} \approx -\frac{3P_1 V^2}{\omega_1} \frac{1}{\omega_1 L_s''} \quad (1.108)$$

This result is analogous to that of peak torque at start-up.

Typical parameters for a 30 kW motors are [17,18]

$T' = 50.70$ ms, $T'' = 3.222$ ms, $T'_0 = 590$ ms, $T''_0 = 5.244$ ms, $T_0'' = 5.244$ ms, $1/\alpha = 20.35$ ms, $L_m = 30.65$ mH, $L_{sl} = 1$ mH ($L_s = 31.65$ mH), $R_s = 0.0868$ Ω .

Analysing, after acquisition, the sudden short-circuit current, the time constants (T'_0 , T''_0 , T' , T'') may be determined by curve-fitting methods.

Using power electronics, the IM may be separated very quickly from the power grid and then short-circuited at various initial voltages and frequency [17]. Such tests may be an alternative to standstill frequency response tests in determining the machine parameters.

Finally, we should note that the torque and current peaks at sudden short circuit are not (in general) larger than those for direct connection to the grid at any speed. It should be stressed that when direct connection to the grid is applied at various initial speeds, the torque and current peaks are about the same.

Reconnection, after a short turn-off period before the speed and the rotor current have decreased, produces larger transients if the residual voltage at stator terminals is in phase opposition with the supply voltage at the reconnection instant.

The question arises: which is the most severe transient? So far apparently there is no definite answer to this question, but the most severe transient up to now is apparently reported in [18].

1.11 MOST SEVERE TRANSIENTS (SO FAR)

We inferred above that reconnection, after a short supply interruption, could produce current and torque peaks which are higher than those for direct starting or sudden short circuit.

Whatever cause would prolong the existence of D.C. decaying currents in the stator (rotor) at high levels would also produce severe current and torque oscillations due to interaction between the A.C. source frequency current component caused by supply reconnection and the large D.C. decaying currents already in existence before reconnection.

One more reason for severe torque peaks would be the phasing between the residual voltages at machine terminals at the moment of supply reconnection. If the two add up, higher transients occur.

The case may be [13] when an IM, supplied through a power transformer whose primary is on turned off after starting the motor, at time t_d with the secondary connected to the motor, is left for an interval Δt , before supply reconnection takes place (Figure 1.25).

During the transformer primary turn-off, the total (no-load) impedance of the transformer (r_{ts} , x_{tos}), as seen from the secondary side, is connected in series with the IM stator resistance r_s and leakage reactance x_{ls} , given in relative (or absolute) units.

To investigate the above transients, we simply use the d-q model in p.u. in (1.83) with the simple motion equation:

$$H \frac{d\omega_r}{dt} = t_e - t_L \quad (1.109)$$

During the Δt interval, the model is changed only in the sense of replacing r_s with $r_s + r_{ts}$ and x_{ls} with $x_{ls} + x_{tos}$. When the transformer primary is reconnected, r_{ts} and x_{tos} are eliminated.

Digital simulations have been run for a 7.5 HP and a 500 HP IM.

The parameters of the two motors are [18] shown in Table 1.1.

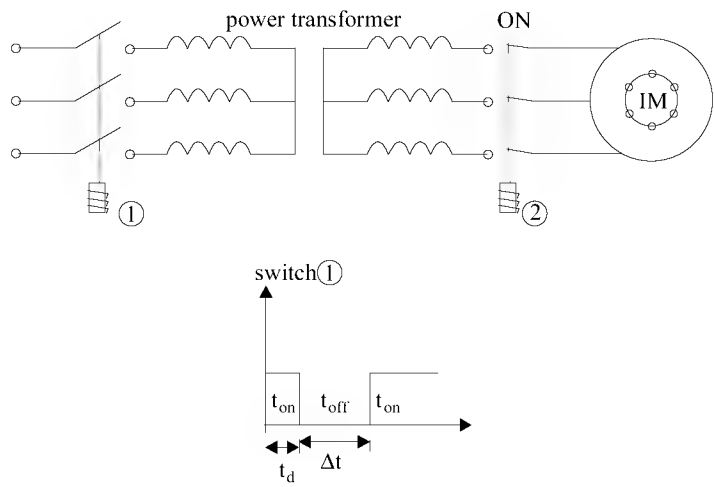


FIGURE 1.25 Arrangement for most severe transients.

TABLE 1.1

Machine Parameters

Size (HP)	V _L line	rpm	T _n (Nm)	I _n (A)	R _s (Ω)	X _{ls} (Ω)	X _{lm} (Ω)	R _r (Ω)	X _{lr} (Ω)	J (Kg·m ²)	f _i (Hz)
7.5	440	1440	37.2	9.53	0.974	2.463	68.7	1.213	2.463	0.042	50
500	2300	1773	1980	93.6	0.262	1.206	54.04	0.187	1.206	11.06	60

In Figure 1.26a,b,c, results shown have been obtained for the 7.5 HP machine. Figure 1.27 shows only 12 p.u. peak torque for the 500 HP machine, whereas 26 p.u. peak torque is noticeable for the 7.5 HP machine.

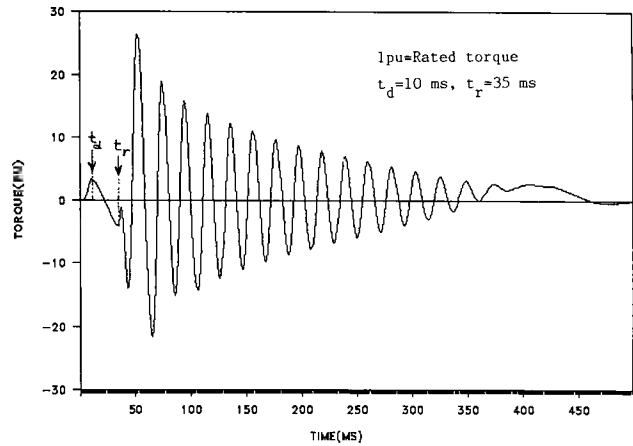
The influence of turn-off moment t_d (after starting) and of the transformer primary turn-off period Δt on peak torque is shown in Figure 1.28 for the 7.5 HP machines.

Figure 1.28 shows very high torque peaks. They depend on the turn-off time after starting t_d . There is a period difference between t_d values for curves a and c, and their behaviour is similar. This shows that the residual voltage phasing at the reconnection instant is important. If it adds to the supply voltage, the torque peaks increase. When it subtracts, the peak torques decrease. The periodic behaviour with Δt is a proof of this aspect.

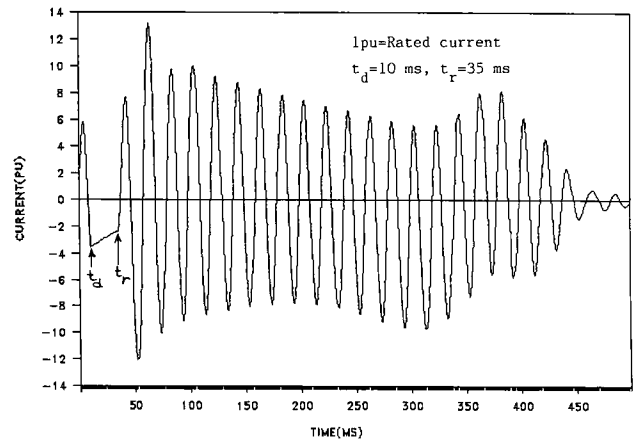
Also, up to a point, when the perturbation time Δt increases, the D.C. (nonperiodic) current components are high as the combined stator and secondary transformer (subtransient) and IM time constant is large.

This explains the larger value of peak torque with Δt increasing. Finally, the speed shows oscillations (even to negative values, for low inertia (power)) and the machine total starting time is notably increased.

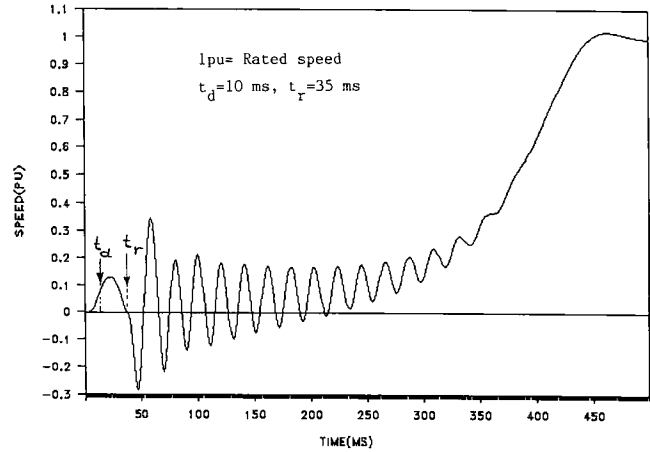
Unusually, high peak torques and currents are expected with this kind of transients for a rather long time, especially for low-power machines, whereas, even for high-power IMs, they are at least 20% higher than those for sudden short circuit.



(a) Torque-time pattern-7.5 HP machine



(b) Current-time pattern-7.5 HP machine



(c) Speed - time pattern - 7.5HP machine

FIGURE 1.26 Transformer primary turn-off and reconnection transients (7.5 HP): (a) torque, (b) current, (c) speed, versus time. (After Ref. [13].)

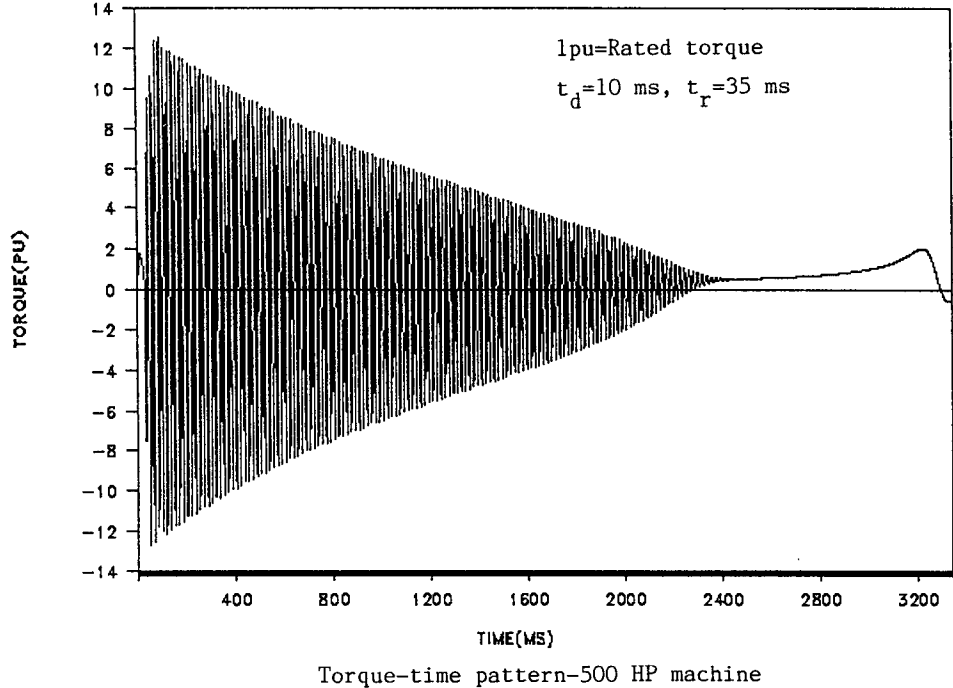


FIGURE 1.27 Transformer primary turn-off and reconnection transients (500 HP). (After Ref. [13].)

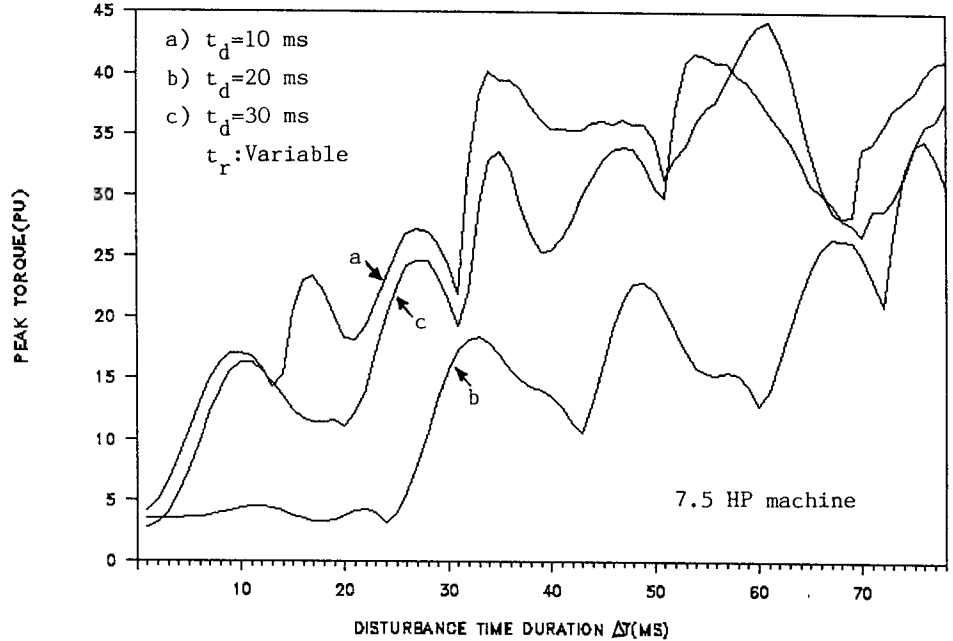


FIGURE 1.28 Peak torque versus perturbation duration (7.5 HP machine). (After Ref. [13].)

1.12 THE abc–d-q MODEL FOR PWM INVERTER-FED IMS

Time-domain modelling of PWM inverter-fed IMs, for both steady state and transients, symmetrical and fault conditions, may be approached through the so-called abc d-q model (Figure 1.29).

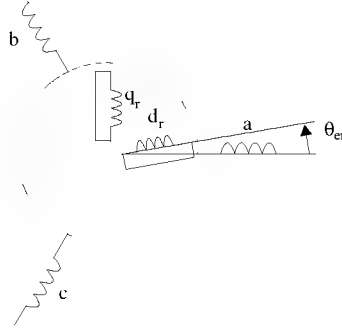


FIGURE 1.29 The abc–d–q model.

The derivation of the abc–d–q model in stator coordinates may be obtained starting again from, abc–a_rb_rc_r model with Park transformation of only rotor variables:

$$|P_{abcdq0}| = \begin{bmatrix} [1] & [0] \\ [0] & [P_{3 \times 3}(\theta_b - \theta_{er})] \end{bmatrix} \quad (1.110)$$

For stator coordinates, $\theta_b = 0$.

Instead of phase voltages, the line voltages may be used:

V_{ab} , V_{bc} , V_{ca} instead of V_a , V_b , V_c ,

$$V_{ab} = V_a - V_b; \quad V_{bc} = V_b - V_c; \quad V_{ca} = V_c - V_a \quad (1.111)$$

Using the 6×6 inductance matrix $|L_{abca_r b_r c_r}(\theta_{er})|$ from (1.1), the abc–d–q model equations become

$$[V] = [R][I] + \omega_r G[I] + L \frac{d}{dt}[I] \quad (1.112)$$

$$[R] = \text{Diag}[R_s, R_s, R_s, R_r, R_r, R_r]$$

$$[V] = [V_{ab}, V_{bc}, V_{ca}, 0, 0] \quad (1.113)$$

$$[I] = [i_a, i_b, i_c, i_{dr}, i_{qr}]$$

$$[G] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{L_m}{\sqrt{2}} & -\frac{L_m}{\sqrt{2}} & 0 & L_r \\ -\sqrt{\frac{2}{3}}L_m & \frac{L_m}{\sqrt{6}} & \frac{L_m}{\sqrt{6}} & -L_r & 0 \end{bmatrix} \quad (1.114)$$

$$[L] = \begin{bmatrix} L_s & -\frac{L_m}{3} & -\frac{L_m}{3} & L_m\sqrt{\frac{2}{3}} & 0 \\ -\frac{L_m}{3} & L_s & -\frac{L_m}{3} & -\frac{L_m}{\sqrt{6}} & \frac{L_m}{\sqrt{2}} \\ -\frac{L_m}{3} & -\frac{L_m}{3} & L_s & -\frac{L_m}{\sqrt{6}} & \frac{L_m}{\sqrt{2}} \\ \sqrt{\frac{2}{3}}L_m & -\frac{L_m}{\sqrt{6}} & -\frac{L_m}{\sqrt{6}} & L_r & 0 \\ 0 & \frac{L_m}{\sqrt{2}} & -\frac{L_m}{\sqrt{2}} & 0 & L_r \end{bmatrix} \quad (1.115)$$

The parameters L_s , L_r and L_m refer to phase inductances:

$$L_s = L_m + L_{ls}; \quad L_r = L_m + L_{lr} \quad (1.116)$$

The torque equation becomes

$$T_e = p_l [I]^T [G] [I] \quad (1.117)$$

Finally, the motion equation is

$$\frac{J}{P_l} \frac{d\omega_r}{dt} = T_e - T_{load} - B\omega_r \quad (1.118)$$

For PWM excitation and mechanical steady state ($\omega_r = \text{const.}$) based on $V_{ab}(t)$, $V_{bc}(t)$ and $V_{ca}(t)$ functions, as imposed by the corresponding PWM strategy, the currents may be found from (1.112) under the form:

$$\dot{[I]} = -[L]^{-1} [[R] + \omega_r [G]] [I] + [L]^{-1} [V] \quad (1.119)$$

Initial values of currents are to be given for steady-state sinusoidal supply and the same fundamental.

Although a computer program is required, the computation time is rather low as the coefficients in (1.119) are constant.

Results obtained for an IM with $R_s = 0.2 \Omega$, $R_r = 0.3 \Omega$, $X_m = 16 \Omega$, $X_r = X_s = 16.55 \Omega$, fed with a fundamental frequency $f_n = 60 \text{ Hz}$, at a switching frequency $f_c = 1260 \text{ Hz}$ ($F_{nc} = 21$) and sinusoidal PWM, are shown in Figure 1.30 [19].

When the speed varies, the PWM switching patterns may change and thus produce current and torque transients. A smooth transition is to be performed for good performance. The abc–d–q model, including the motion equation, also serves this purpose (e.g. switching from harmonic elimination PWM ($N = 5$) to square wave, as shown in Figure 1.31 [19]).

The current waveform changes to the classic six-pulse voltage response after notable current and torque transients.

1.12.1 FAULT CONDITIONS

Open line conditions or single phasing due to loss of gating signals of a pair of inverter switches are typical faults. Let us consider delta connection when line a is opened (Figure 1.32a). The relationships between the system currents $[i_{ab}, i_{bc}, i_{ca}, i_d, i_q]$ and the “fault” currents $i_{bac}, i_{bc}, i_d, i_q$ are (Figure 1.32a)

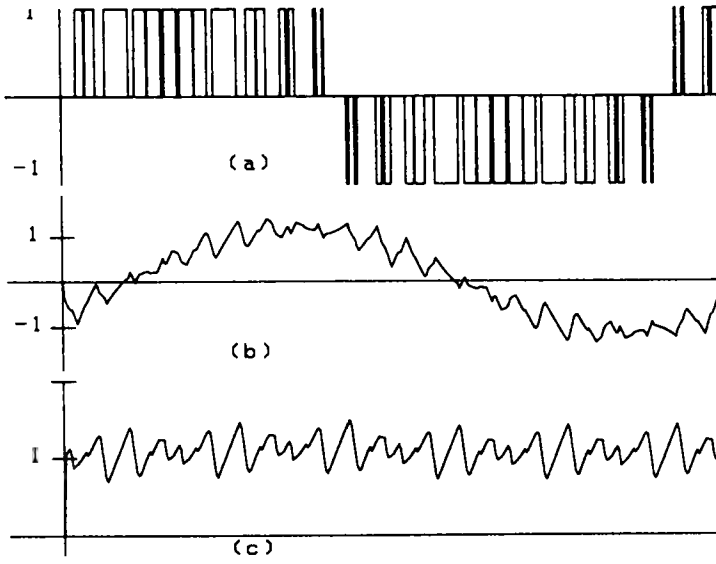


FIGURE 1.30 Computed waveforms for sinusoidal PWM scheme ($F_{nc} = 21$), $f = 60\text{Hz}$: (a) line-to-line voltage V_{ab} , (b) line current i_a , and (c) developed torque. (After Ref. [19].)

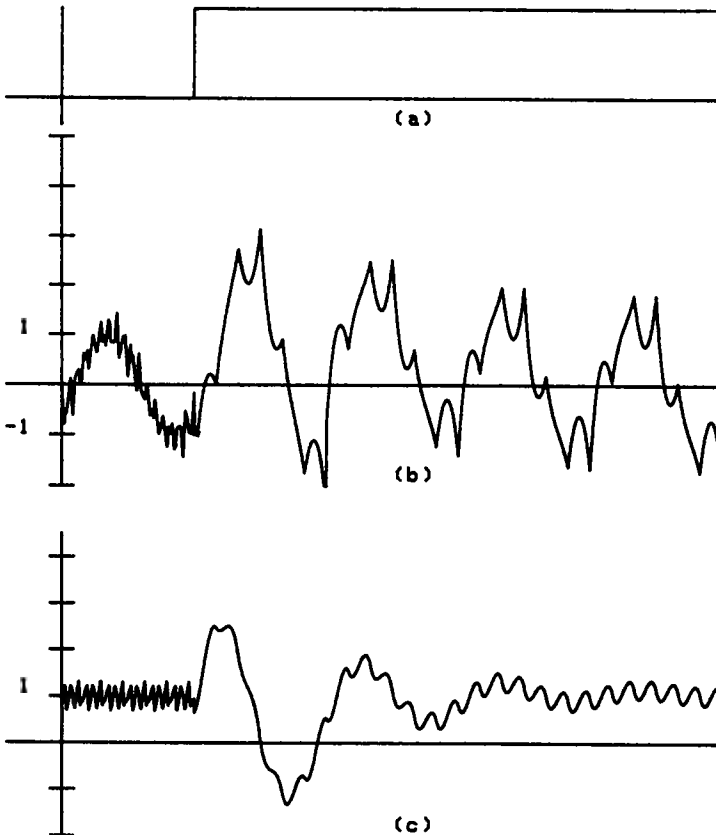


FIGURE 1.31 Computed transient response for a change from harmonic ($N = 5$) elimination PWM to square wave excitation at 60Hz : (a) voltage, (b) current, and (c) torque. (After Ref. [19].)

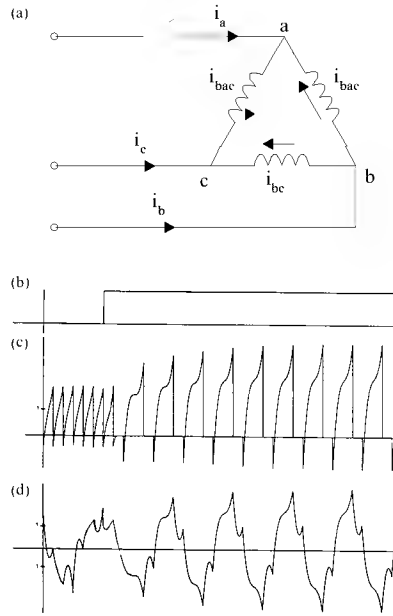


FIGURE 1.32 Computed transient response for a single phasing fault on line “a” at 60Hz (a) open line condition, (b) instant when single phasing occurs, (c) D.C. link current, and (d) line current i_c . (After Ref. [19].)

$$\begin{bmatrix} i_{ab} \\ i_{bc} \\ i_{ca} \\ i_d \\ i_q \end{bmatrix} = [C] \begin{bmatrix} i_{bac} \\ i_{bc} \\ i_d \\ i_q \end{bmatrix}; \quad [I_n] = [C][i_0] \quad (1.120)$$

$$[C] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.121)$$

The voltage relationship is straightforward as power has to be conserved:

$$\begin{bmatrix} V_{bac} \\ V_{bc} \\ V_d \\ V_q \end{bmatrix} = [C]^T \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \\ V_d \\ V_q \end{bmatrix}; \quad [V_n] = [C]^T [V_0] \quad (1.122)$$

Now, we may replace the voltage matrix in (1.112) to obtain

$$[V_n] = [C]^T [R][C][I_n] + [C]^T \omega_r [G][C][I_n] + [C]^T [L][C] \frac{d[I_n]}{dt} \quad (1.123)$$

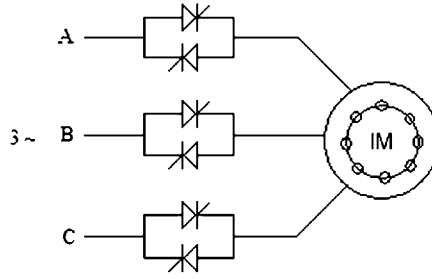


FIGURE 1.33 The soft-starter-fed IM.

All we need, to be able to solve Equation (1.123), is the initial conditions.

In reality, (Figure 1.32a) at time $t = 0$, the currents i_{ba} and i_{ca} are forced to be equal to I_{bac} when open line occurs. To avoid a discontinuity in the stored energy, the flux conservation law is applied to (1.112).

We impose $i_{ab} = i_{ca} = -i_{bac}$ and the conservation of flux will give the variation of rotor current to comply with flux conservation. Typical results for such a transient [19] are shown in Figure 1.32b–d. Severe D.C. line pulsations and line b current transients are visible.

Other fault conditions may be treated in a similar way.

The soft-starter-fed IM (Figure 1.33) may be investigated with the same ab , bc , ca , d_r , and q_r models.

The soft starter modifies the voltage amplitude while keeping the frequency constant. At small frequency (below 33%) by asymmetric control, a kind of frequency change can be accomplished and thus the starting torque is increased below 33% of rated speed. Also, by adequate current shape control, the torque pulsations during IM starting are reduced [20].

Typical results are shown in Figure 1.34.

It is clear that the torque pulsations may be reduced during starting by adequate control.

1.13 FIRST-ORDER MODELS OF IMS FOR STEADY-STATE STABILITY IN POWER SYSTEMS

The quasi-static (first order) or slip model of IMs with neglected stator and rotor transients may be

obtained from the general model (1.70) with $\frac{d\bar{\Psi}_s}{dt} = \frac{d\bar{\Psi}_r}{dt} = 0$ in synchronous coordinates.

$$\begin{aligned}
 (1 + j\omega_1 \tau'_s) \bar{\Psi}_s &= \tau'_s V_s + K_r \bar{\Psi}_r \\
 (1 + j(\omega_1 - \omega_r)) \bar{\Psi}_r &= K_s \bar{\Psi}_s \\
 T_e &= -\frac{3}{2} P_l \operatorname{Re} \left(j \bar{\Psi}_r \bar{i}_r^* \right); \quad \bar{i}_r = \sigma^{-1} \left(\frac{\bar{\Psi}_r}{L_r} - \bar{\Psi}_s \frac{L_m}{L_s L_r} \right) \\
 \frac{J}{P_l} \frac{d\omega_r}{dt} &= T_e - T_{\text{load}}
 \end{aligned} \tag{1.124}$$

It has been noted that this overly simplified speed model becomes erroneous for large IMs.

In near-rated conditions, the structural dynamics of low- and high-power IMs differ. The dominant behaviour of low-power IMs is a first-order speed model, whereas the high-power IMs is characterized by a first-order voltage model [21,22].

Starting again with the third-order model of (1.70), we may write the rotor flux space phasor as

$$\bar{\Psi}_r = \Psi_r e^{j\delta} \tag{1.125}$$

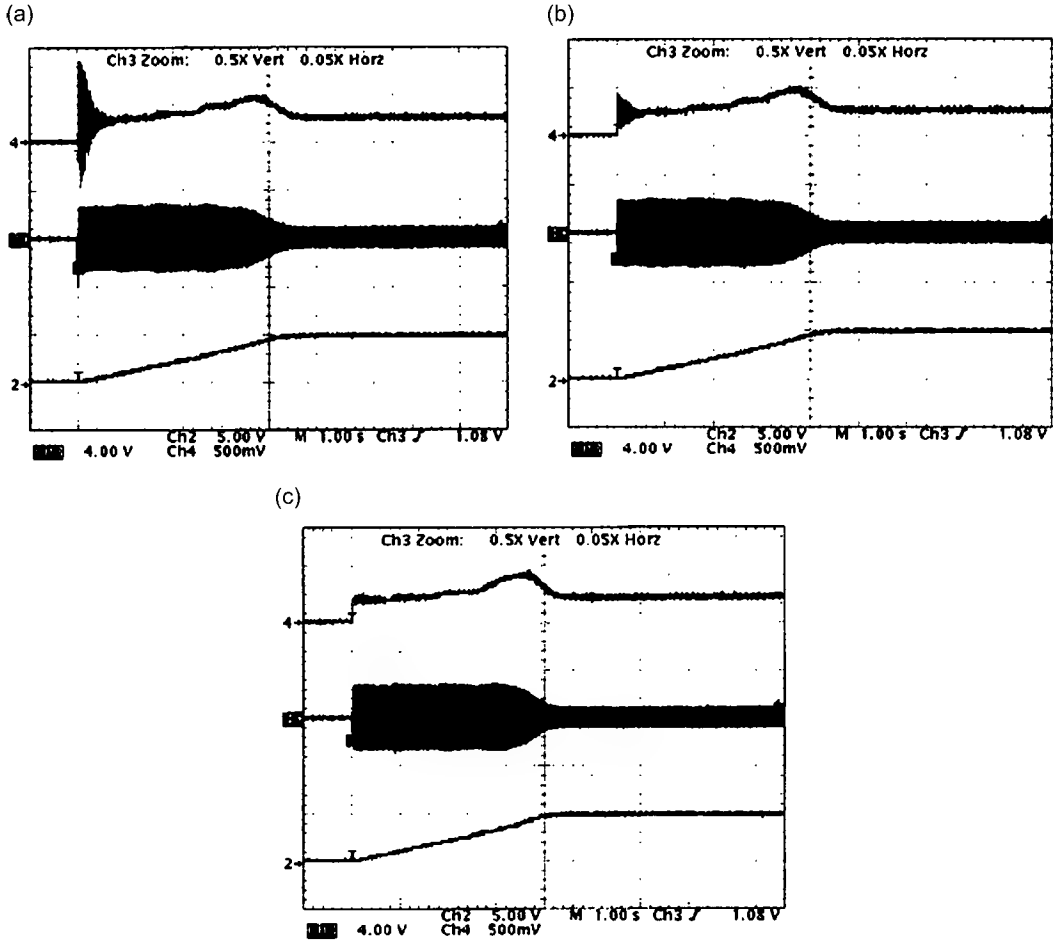


FIGURE 1.34 Starting performance (after Ref. [20]) (Ch. 4 – torque, Ch. 3 – current, Ch. 2 – speed). (a) Simultaneous switching, (b) all phases switched at $\alpha = 72^\circ$, and (c) with current shape control.

This way, we may separate the real and imaginary part of rotor equation of (1.70) as

$$\begin{aligned} \frac{\tau_r'}{K_r} \frac{d\Psi_r}{dt} K_r &= \frac{x}{x'} \Psi_r K_r + \frac{x-x'}{x'} V \cos \delta; \quad K_r \approx K_s = \frac{L_m}{L_s} = \frac{L_m}{L_r} \\ \frac{d\delta}{dt} &= \omega_r - \omega_1 - \frac{x-x'}{x'} K_r \frac{V \sin \delta}{\tau_r' \Psi_r K_r} \\ H' \frac{d\omega_r}{dt} &= -V \frac{\Psi_r}{x'} K_r \sin \delta - t_{load} \end{aligned} \quad (1.125')$$

where V – voltage, Ψ_r – rotor flux amplitude, x – no-load reactance, x' – short-circuit (transient) reactance in relative units (p.u.), and τ_r' , ω_1 and ω_r in absolute units and H' in seconds. The stator resistance is neglected.

In relative units, $\Psi_r K_r$ may be replaced by the concept of the voltage behind the transient reactance x' , E' .

$$\begin{aligned}\frac{\tau_r'}{K_r} \frac{dE'}{dt} &= \frac{x}{x'} E' + \frac{x-x'}{x'} V \cos \delta \\ \frac{d\delta}{dt} &= \omega_r - \omega_1 - \frac{x-x'}{x'} K_r \frac{V \sin \delta}{\tau_r' E'} \\ H' \frac{d\omega_r}{dt} &= -V \frac{E'}{x'} \sin \delta - t_{\text{load}}\end{aligned}\quad (1.126)$$

- The concept of E' is derived from the synchronous motor similar concept.
- The angle δ is the rotor flux vector angle with respect to the synchronous reference system.
- The steady-state torque is obtained from (1.126) with $d/dt = 0$.

$$E' = \frac{-x + x'}{x'} V \cos \delta \quad (1.127)$$

$$\omega_r - \omega_1 - \frac{x-x'}{x'} K_r \frac{V \sin \delta}{\tau_r' E'} < 0 \quad \text{for motoring} \quad (1.128)$$

$$t_e = -V \frac{E'}{x'} \sin \delta = -\left(\frac{1}{x'} - \frac{1}{x}\right) \frac{V^2}{2} \sin 2\delta > 0 \quad \text{for motoring} \quad (1.129)$$

for positive torque. $\delta < 0$ and for peak torque. $\delta_K = \pi/4$ as in a fictitious reluctance synchronous motor with $x_d = x$ (no-load reactance) and $x_q = x'$ (transient reactance).

For low-power IMs, E' and δ are the fast variables and $\omega_r - \omega_1$, the slow variable.

Consequently, we may replace δ and E' from the steady-state Equations (1.127) and (1.128) in the motion Equation (1.126).

$$x' H' \frac{d(\omega_r - \omega_1)}{dt} = -V^2 \frac{x-x'}{x'} \frac{(\omega_r - \omega_1) T'}{1 + T'^2 (\omega_r - \omega_1)^2} - x' t_{\text{load}}; \quad T' = \frac{\tau_r'}{K_r} \quad (1.130)$$

Also, from (1.127) and (1.128),

$$E' = \frac{(x-x') V}{x \sqrt{1 + T'^2 (\omega_r - \omega_1)^2}}; \quad \delta \approx \tan^{-1} T' (\omega_r - \omega_1) \quad (1.131)$$

In contrast, for high-power IMs, the slow variable is E' and the fast variables are δ and $\omega_r - \omega_1$. With $d\omega_r/dt = 0$ and $d\delta/dt = 0$ in (1.126),

$$\delta = \sin^{-1} \left(\frac{-x' t_{\text{load}}}{V E'} \right); \quad (\omega_r - \omega_1) = 0 \quad (1.132)$$

It is evident that $\omega_r = \omega_1 \neq 0$.

A nontrivial first-order approximation of $\omega_r - \omega_1$ is [22]

$$\omega_r - \omega_1 \approx \frac{x' t_{\text{load}}}{T' \sqrt{(V E')^2 - (x' t_{\text{load}})^2}} \quad (1.133)$$

Substituting δ from (1.132) in (1.126) yields

$$T' \frac{dE'}{dt} = -\frac{x}{x'} E' + \frac{x - x'}{x'} V \sqrt{1 - \left(\frac{x' t_{\text{load}}}{VE'} \right)^2} \quad (1.134)$$

Equations (1.132–1.134) represent the modified first-order model for high-power IMs. Good results with these first-order models are reported in [21] for 50 and 500 HP, respectively, in comparison with the third-order model (Figure 1.35).

Still better results for large IMs are claimed in [22] with a heuristic first-order voltage model based on sensitivity studies.

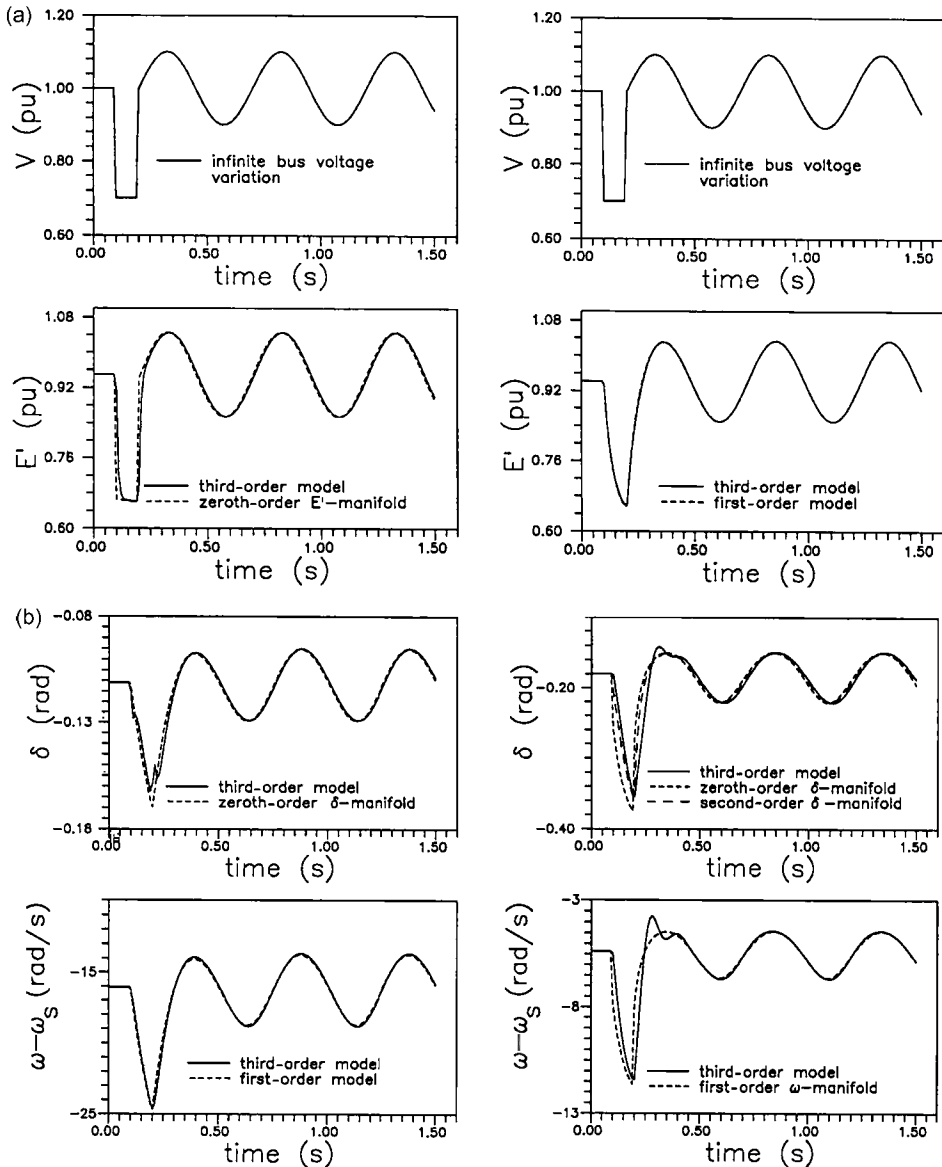


FIGURE 1.35 Third- and first-order model transients: (a) 50 HP IM and (b) 500 HP IM. (After Ref. [21].)

1.14 MULTIMACHINE TRANSIENTS

A good part of industrial loads is represented by induction motors. The power range of induction motors in industry spans from a few kW to more than 10 MW per unit. A local (industrial) transformer feeds through pertinent switchgears a group of induction motors which may undergo randomly load perturbations, direct turn-on starting or turn-off. Alternatively, bus transfer schemes are providing for energy-critical duty loads in processes such as thermal nuclear power plants.

During the time interval between turn-off from one power grid until the turn-on to the emergency power grid, the group of induction motors, with a series reactance common feeder or with a parallel capacitor bank (used for power factor correction), exhibits a residual voltage, which slowly dies down until reconnection takes place (Figure 1.36). During this period, the residual rotor currents in the various IMs of the group are producing stator emfs, and, depending on their relative phasing and amplitude (speed or inertia), some of them will act as generators and some as motors until their mechanical and magnetic energy dry out.

The obvious choice to deal with multimachine transients is to use the complete space-phasor model with flux linkages $\bar{\Psi}_s$ and $\bar{\Psi}_r$ and ω_r as variables (1.54–1.56 and 1.64).

Adding up to the fifth-order model of all n IMs ($5n$ equations), the equations related to the terminal capacitor (and the initial conditions before the transient is initiated) seem the natural way to solve any multimachine transient. A unique synchronous reference system may be chosen to speed up the solution.

Furthermore, to set the initial conditions, say, when the group is turned off, we may write that total stator current $\bar{i}_{st}(0+) = 0$. With a capacitor at the terminal, the total current is zero.

$$\sum_{j=1}^n \bar{i}_{sj}(0+) + \bar{i}_c = 0; \quad \frac{d\bar{V}_s}{dt} = \frac{\bar{i}_c}{C} \quad (1.135)$$

with $\bar{i}_s$ from (1.54),

$$\sum_{j=1}^n \bar{i}_{sj} = \sum_{j=1}^n \sigma_j^{-1} \left(\frac{\bar{\Psi}_{sj}}{L_{sj}} - \frac{\bar{\Psi}_{rj} L_{mj}}{L_{mj} L_{rj}} \right) \quad (1.136)$$

Differentiating Equations (1.135) with (1.136) with respect to time will produce an equation containing a relationship between the stator and rotor flux time derivatives. This way, the flux conservation law is met.

We take the expressions of these derivatives from the space-phasor model of each machine and find the only unknown, $(\bar{V}_s)_{t=0+}$, as the values of all variables $\bar{\Psi}_{sj}$, $\bar{\Psi}_{rj}$ and ω_{rj} , before the transient is initiated, are known.

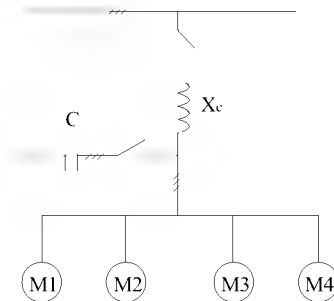


FIGURE 1.36 A group of induction motors.

Now, if we neglect the stator transients and make use of the third-order model, we may use the stator equations of any machine in the group to calculate the residual voltage $\bar{V}_s(t)$ as long as (1.135) with (1.136) is fulfilled.

This way no iteration is required and only solving the third model of each motor is, in fact, done [23,24]. Unfortunately, although the third-order model forecasts the residual voltage $V_s(t)$ almost as well as the full (fifth-order) model, the test results show a sudden drop in this voltage in time. This sharply contrasts with the experimental results.

A possible explanation for this is the influence of magnetic saturation which maintains the “self-excitation” process of some of the machines with larger inertia for some time, after which a fast deexcitation of them by the others, which act as motors, occurs.

Apparently magnetic saturation has to be accounted for to get any meaningful results related to residual voltage transient with or without terminal capacitors.

Simpler first-order models have been introduced earlier in this chapter. They have been used to study load variation transients once the flux in the machine is close to its rated value [25,26].

Furthermore, multimachine transients (such as limited-time bus fault) with unbalanced power grid voltages have also been treated, based on the d-q (space-phasor) model – complete or reduced. However, in all these cases, no strong experimental validation has been brought up so far.

So we feel that model reduction, or even saturation and skin effect modelling, has to be pursued with extreme care when various multimachine transients are investigated.

1.15 SUBSYNCHRONOUS RESONANCE (SSR)

Subsynchronous series resonance [27,28] may occur when the induction motors are fed from a power source which shows a series capacitance (Figure 1.37). It may lead to severe oscillations in speed, torque, and current. It appears, in general, during machine starting or after a power supply fault.

In Figure 1.37, f_b is the base frequency, f_l is the current frequency, and n is the rotor speed. The reactances are written at base frequency.

It is self-evident that the space-phasor model may be used directly to solve this problem completely, provided a new variable (capacitor voltage V_c) is introduced.

$$\frac{d\bar{V}_c}{dt} = \frac{1}{C} \bar{i}_s \quad (1.137)$$

The total solution contains a forced solution and a free solution. The complete mathematical model is complex when magnetic saturation is considered. It simplifies greatly if the magnetization reactance

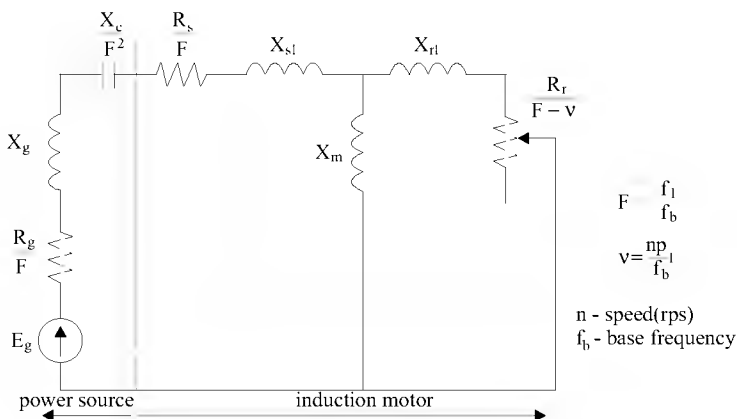


FIGURE 1.37 Equivalent circuit with power source parameters, R_g , X_g , and X_c in series.

X_m is considered constant (though at a saturated value). In contrast to the induction capacitor-excited generator, the free solution here produces conditions that lead to machine heavy saturation.

In such conditions, we may treat separately the resonance conditions for the free solution.

To do so, we lump the power source impedance R_g and X_g into stator parameters to get

$$R_e = R_s + R_g; \quad X_e = X_g + X_{sl} \quad (1.138)$$

and eliminate the generator (power source) emf E_g (Figure 1.38).

To simplify the analysis, let us use a graphical solution. The internal impedance locus is, as known, a circle with the centre 0 at $0\left(\frac{R_e}{F}, a\right)$ [29].

$$a \approx X_e + \frac{X_e^2}{2(X_e + X_m)} \quad (1.139)$$

and the radius r

$$r \approx \frac{X_e^2}{2(X_e + X_m)} \quad (1.140)$$

On the other hand, the external (the capacitor, in fact) impedance is represented by the imaginary axis (Figure 1.39). The intersection of the circle with the imaginary axis, for a given capacitor, takes place in two points F_1 and F_2 which correspond to reference conditions. F_1 corresponds to the lower slip and is the usual self-excitation point of IGs.

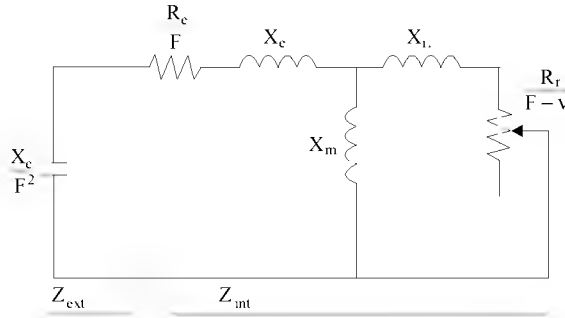


FIGURE 1.38 Equivalent circuit for free solution.

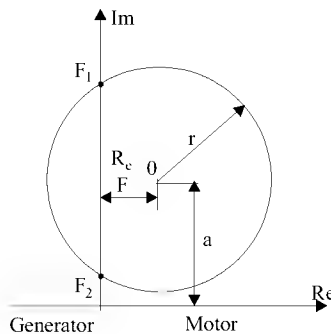


FIGURE 1.39 Impedance locus.

The p.u. values of frequencies F_1 and F_2 , for a given series capacitor X_c , resistor R_e , and reactance X_e , are

$$\begin{aligned} F_1 &\approx \sqrt{\frac{X_e + X_m}{X_c}} \\ F_2 &\approx \sqrt{\frac{X_e + \frac{X_m X_{rl}}{X_m + X_{rl}}}{X_c}} \end{aligned} \quad (1.141)$$

Alternatively, for a given resonance frequency and capacitor, we may obtain the resistance R_e to cause SSR. With $F = F_1$ given, we may use the impedance locus to obtain R_e .

$$\frac{X_c}{F_1^2} = a + \sqrt{r^2 - \left(\frac{R_e}{F_1}\right)^2} \quad (1.142)$$

Also, from the internal impedance at $R_e(\underline{Z}_{int}) = 0$ equal to X_c/F_1^2 , the value of speed (slip: $F - \nu$) may be obtained.

$$\text{slip} = F_1 - \nu \approx \pm \sqrt{\frac{R_r \left(X_m - X_e - \frac{X_c}{F_1^2} \right)}{(X_m + X_{rl}) \left[\left(\frac{X_c}{F_1^2} - X_e \right) (X_m + X_{rl}) - X_m X_{rl} \right]}} \quad (1.143)$$

In general, we may use Equation (1.143) with both $\pm$ to calculate the frequencies for which resonance occurs.

$$F^4 (a^2 - r^2) + F^2 (R_e^2 - 2aX_c) + X_c^2 = 0 \quad (1.144)$$

The maximum value of R_e , which still produces resonance, is obtained when the discriminant of (1.144) becomes zero.

$$R_{e\max} = \sqrt{2X_c \left(\sqrt{a^2 - r^2} + a \right)} = KX_c \quad (1.145)$$

The corresponding frequency is

$$F(R_{e\max}) = \sqrt{\frac{2X_c^2}{2aX_c - R_{e\max}^2}} \quad (1.146)$$

This way the range of possible source impedance that may produce SSR has been defined.

Preventing SSR may be done either by increasing R_e above $R_{e\max}$ (which means notable losses) or by connecting a resistor R_{ad} in parallel with the capacitor (Figure 1.40), which means less losses.

A graphical solution is at hand as $\underline{Z}_{int}$ locus is already known (the circle). The impedance locus of the $R_{ad}||X_c$ circuit is a half circle with the diameter R_{ad}/F and the centre along the horizontal negative axis (Figure 1.41).

The SSR is eliminated if the two circles, at same frequency, do not intersect each other. Analytically, it is possible to solve the equation:

$$\underline{Z}_{ext} + \underline{Z}_{int} = 0 \quad (1.146')$$

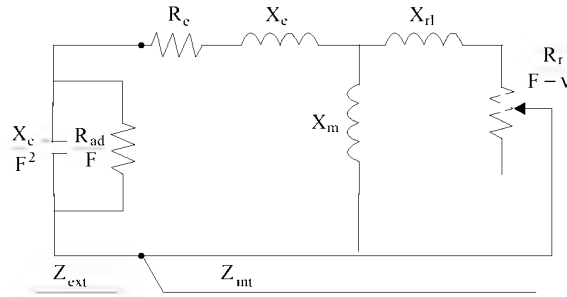


FIGURE 1.40 Additional resistance R_{ad} for preventing SSR.

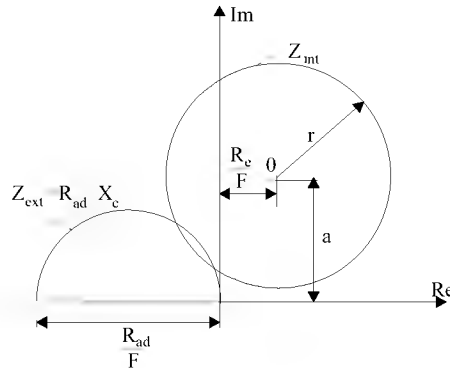


FIGURE 1.41 Impedances locus for SSR prevention.

again, with $R_{ad} \parallel X_c$ as Z_{ext} .

A sixth-order polynomial equation is obtained. The situation with a double real root and four complex roots corresponds to the two circles being tangent. Only one SSR frequency occurs. This case defines the critical value of R_{ad} for SSR.

In general, R_{ad} (critical) increases with X_c and so does the corresponding SSR frequency [28,29].

Using a series capacitor for starting medium power IMs to reduce temporary supply voltage sags and starting time is a typical situation when the prevention of SSR is necessary.

1.16 THE m/N_r ACTUAL WINDING MODELLING FOR TRANSIENTS

By m/N_r winding model, we mean the induction motor with m stator windings and N_r actual (rotor loop) windings [30].

In the general case, the stator windings may exhibit faults such as local short circuits, or some open coils, when the self and mutual inductances of stator windings have special expressions that may be defined using the winding function method [31–33].

While such a complex methodology may be justifiable in large machine preventive fault diagnostics, for a symmetrical stator the definition of inductances is simpler.

The rotor cage is modelled through all its N_r loops (bars). One more equation for the end ring is added. So the machine model is characterized by $(m + N_r + 1 + 1)$ equations (the last one is the motion equation). Rotor bar and end-ring faults, so common in IMs, may thus be modelled through the m/N_r actual winding model.

The trouble is that it is not easy to measure all inductances and resistances entering the model. In fact, relying on analysis (or field) computation is the preferred choice.

The stator phase equations (in stator coordinates) are

$$\begin{aligned}
 V_a &= R_s i_a + \frac{d\Psi_a}{dt} \\
 V_b &= R_s i_b + \frac{d\Psi_b}{dt} \\
 V_c &= R_s i_c + \frac{d\Psi_c}{dt}
 \end{aligned} \tag{1.147}$$

The rotor cage structure and unknowns are shown in Figure 1.42.

The K_r^{th} rotor loop equation (in rotor coordinates) is

$$0 = 2 \left(R_b + \frac{R_e}{N_r} \right) i_{kr} + \frac{d\Psi_{kr}}{dt} - R_b (i_{(k-1)r} + i_{(k+1)r}) \tag{1.148}$$

Also, for one of the end rings,

$$0 = R_e i_e + L_e \frac{di_e}{dt} - \sum_i^{N_r} \left(\frac{R_e}{N_r} i_{kr} + L_e \frac{di_{kr}}{dt} \right) \tag{1.149}$$

As expected, for a healthy cage, the end-ring loop cage current $i_e = 0$.

The relationships between the bar-loop and end-ring currents i_b and i_e are given by Kirchhoff's law:

$$i_{bk} = i_{kr} - i_{(k+1)r}; \quad i_{ek} = i_{kr} - i_e \tag{1.150}$$

This explains why only $N_r + 1$ -independent rotor current variables remain.

The self and mutual stator phase inductances are given through their standard expressions (see Chapter 5, Vol. 1):

$$\begin{aligned}
 L_{aa} &= L_{bb} = L_{cc} = L_{ls} = L_{sm} \\
 L_{ab} &= L_{bc} = L_{ca} = -\frac{1}{2} L_{sm} \\
 L_{sm} &= \frac{4\mu_0 (W_l K_{wl})^2 \tau L}{\pi^2 K_c p_l g (1 + K_s)}
 \end{aligned} \tag{1.151}$$

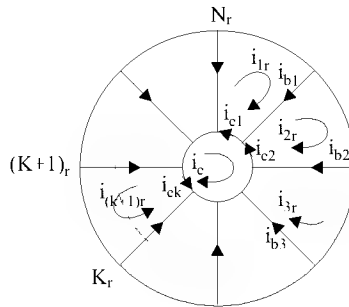


FIGURE 1.42 Rotor cage with rotor loop currents.

The saturation coefficient K_s is considered constant. L_{ls} – leakage inductance, τ – pole pitch, L – stack length, p_1 – pole pairs, g – airgap, W_l – turns/phase, and K_{w1} – winding factor.

The self-inductance of the rotor loop is calculated on the base of rotor loop area,

$$L_{K_r, K_r} = \frac{2\mu_0 (N_r - 1) p_1 \tau L}{N_r^2 K_c g (1 + K_s)} \quad (1.152)$$

The mutual inductance between rotor loops is

$$L_{K_r, (K+j)_r} = -\frac{2\mu_0 P_i \tau L}{N_r^2 K_c g (1 + K_s)} \quad (1.153)$$

This is a kind of average small value, as in reality, the coupling between various rotor loops varies notably with the distance between them.

The mutual inductances L_{aK_r} , L_{bK_r} and L_{cK_r} of stator–rotor loops are

$$\begin{aligned} L_{aK_r}(\theta_r) &= L_{sr} \cos[\theta_{er} + (K_r - 1)\alpha] \\ L_{bK_r}(\theta_r) &= L_{sr} \cos\left[\theta_{er} + (K_r - 1)\alpha - \frac{2\pi}{3}\right] \\ L_{cK_r}(\theta_r) &= L_{sr} \cos\left[\theta_{er} + (K_r - 1)\alpha + \frac{2\pi}{3}\right] \end{aligned} \quad (1.154)$$

$$\alpha = P_i \frac{2\pi}{N_r}; \quad \theta_{er} = p_1 \theta_r; \quad p_1 - \text{pole pairs} \quad (1.155)$$

$$L_{sr} = \frac{-(W_l K_{w1}) \mu_0 \tau L}{2g K_c (1 + K_s)} \sin \frac{\alpha}{2} \quad (1.156)$$

The $m + N_r + 1$ electrical equations may be written in matrix form as

$$[V] = [R][i] + [L'(\theta_{er})] \left[\frac{di}{dt} \right] + [G][i] \quad (1.157)$$

$$\begin{aligned} [V] &= [V_a, V_b, V_c, 0, 0, \dots, 0]^T \\ [I] &= [i_a, i_b, i_c, i_{1r}, i_{2r}, \dots, i_{N_r}, i_e]^T \end{aligned} \quad (1.158)$$

The motion equations are

$$\frac{d\omega_r}{dt} = \frac{P_l}{J} (T_e - T_{load}); \quad \frac{d\theta_{er}}{dt} = \omega_r \quad (1.162)$$

The torque T_e is

$$T_e = P_l [I]^T \frac{\partial [L(\theta_{er})]}{d\theta_{er}} [I] \quad (1.163)$$

Finally,

$$T_e = p_l L_{sr} \left[\left(i_a - \frac{1}{2} i_b - \frac{1}{2} i_c \right) \sum_1^{N_r} i_{Kr} \sin[\theta_{er} + (K_r - 1)\alpha] + \frac{\sqrt{3}}{2} (i_b - i_c) \sum_1^{N_r} i_{Kr} \sin[\theta_{er} + (K_r - 1)\alpha] \right] \quad (1.164)$$

A few remarks are in order:

- The $(3 + N_r + 2)$ -order system has quite a few coefficients (inductances) depending on rotor position θ_{er} .
- When solving such a system using the Runge–Kutta–Gill or the like method, a matrix inversion is required for each integration step.
- The case of healthy cage can be handled by a generalized space vector (phasor) approach when the coefficients' dependence on rotor position is eliminated.
- The case of faulty bars is handled by increasing the resistance of that (those) bar (R_b) to a few orders of magnitude.
- When an end-ring segment is broken, the corresponding R_e/N_r term in the resistance matrix is increased to a few orders in magnitude (both along the diagonal and along its vertical and horizontal direction in the last row and column, respectively).

In Ref. [34], a motor with the following data was investigated:

$R_s = 10 \, \Omega$, $R_b = R_c = 155 \, \mu\Omega$, $L_{ls} = 0.035 \, \text{mH}$, $L_{ms} = 378 \, \text{mH}$, $W_l = 340$ turns/phase, $K_{\alpha 1} = 1$, $N_r = 30$ rotor slots, $p_l = 2$ pole pairs, $L_e = L_b = 0.1 \, \mu\text{H}$, $L_{sr} = 0.873 \, \text{mH}$, $\tau = 62.8 \, \text{mm}$ (pole pitch), $L = 66 \, \text{mm}$ (stack length), $g = 0.375 \, \text{mm}$, $K_c = 1$, $K_s = 0$, $J = 5.4 \cdot 10^{-3} \, \text{Kg m}^2$, $f = 50 \, \text{Hz}$, $V_L = 380 \, \text{V}$, $P_n = 736 \, \text{W}$, $I_0 = 2.1 \, \text{A}$.

Numerical results for bar 2 broken ($R_{b2} = 200R_b$) with the machine under load $T_L = 3.5 \, \text{Nm}$ are shown in Figure 1.43.

For the ring segment 3 broken ($R_{e3} = 10^3 R_e/N_r$), the speed and torque are given in Figure 1.44 [34].

- Small pulsations in speed and torque are caused by a single bar or end-ring segment faults.
- A few broken bars would produce notable torque and speed pulsations. Also, a $2Sf_l$ frequency component will show up, as expected, in the stator phase currents.

The assumption that the interbar resistance is infinitely large (no interbar currents or insulated bars) is hardly true in reality. In presence of interbar currents, the effect of one to two broken bars tends to be diminished [35].

The transients of IM have also been approached by FEM, specifically by time stepping coupled FEM circuit (or state space) models [36,37]. The computation time is still large, but the results are getting better and better as new contributions are made. The main advantage of such method is the

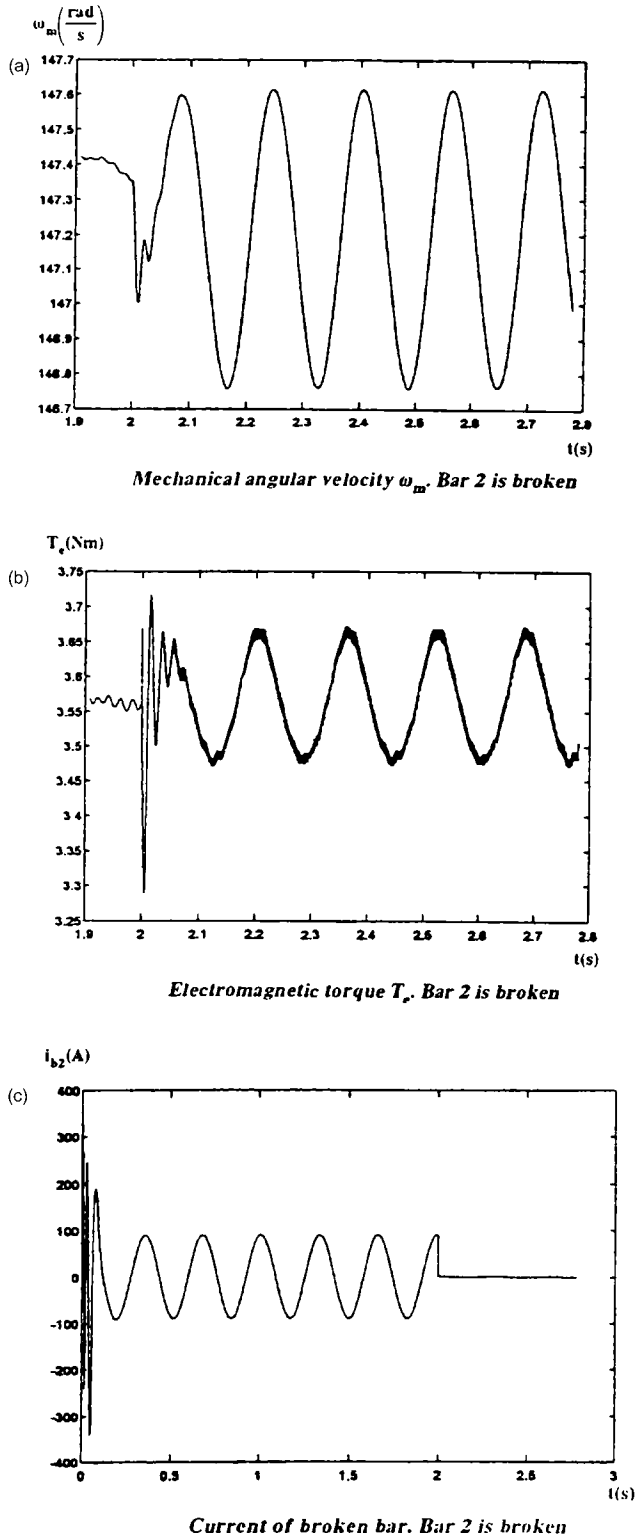
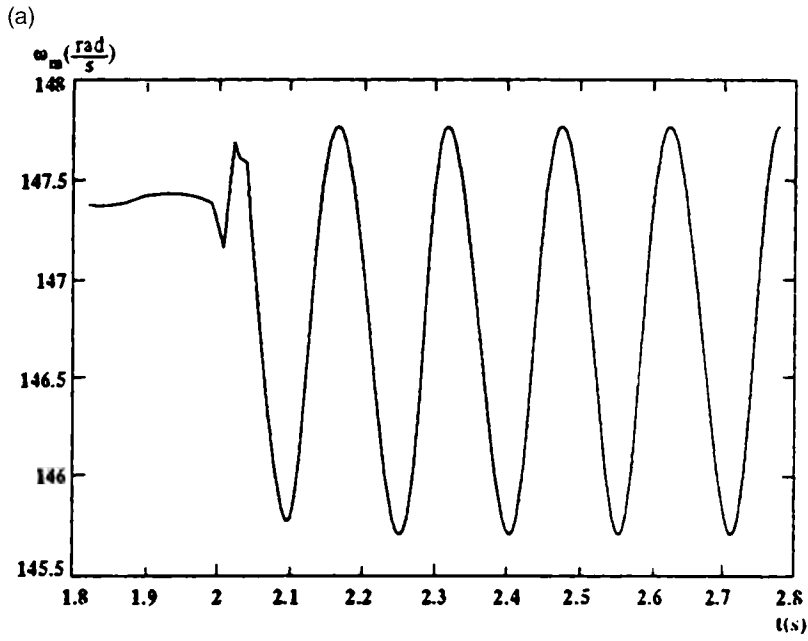
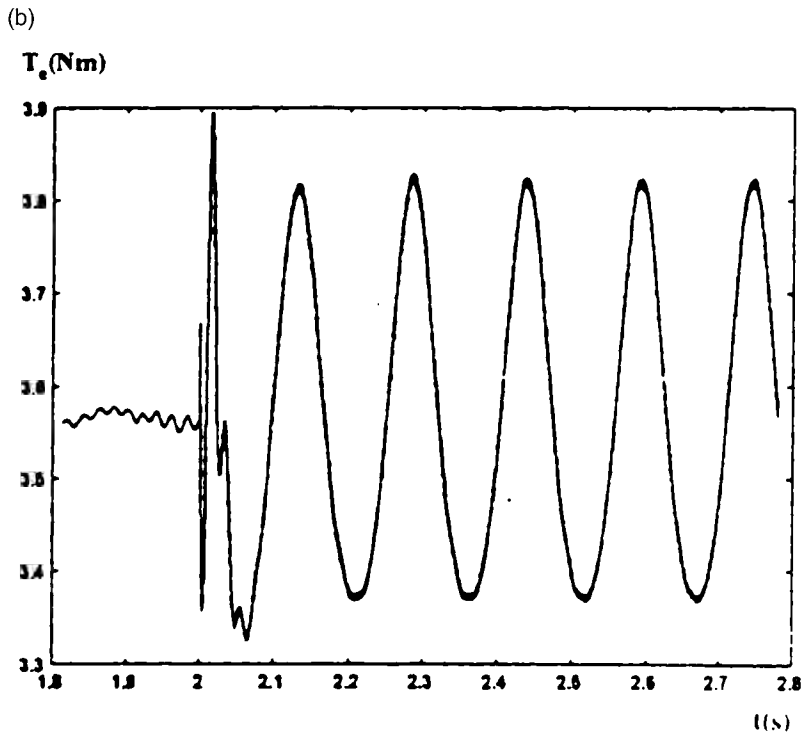


FIGURE 1.43 Bar 2 broken at steady state ($T_L = 3.5 \text{ Nm}$): (a) speed, (b) torque (after Ref. [34]), and (c) broken bar current (after Ref. [34]).



Mechanical angular velocity. Ring-section 3 is broken



Electromagnetic torque. Ring-section 3, is broken

FIGURE 1.44 End-ring 3 broken at steady state: (a) speed (after Ref. [34]) and (b) torque (after Ref. [34]).

possibility to account for most detailed topological aspects and for nonlinear material properties. Moreover, coupling the electromagnetic to thermal and mechanical modelling seems feasible in the near future.

Quite a few more analytical models with FEM backup to handle broken bar effects have been introduced recently [38–40].

1.17 MULTIPHASE INDUCTION MACHINES MODELS FOR TRANSIENTS

For multiphase IMs, i.e. IMs with more than three phases, five, six, and seven phases would be typical choices. They have recently been investigated in order to provide more freedom in inverter control for variable speed, more fault tolerance and more torque density. In terms of efficiency, however, they have not been proven yet are superior to typical three-phase distributed winding IMs.

The windings of an m -phase machine may be symmetrical or asymmetrical as the spatial electrical angle of adjoint phases is $\alpha_{es} = 2\pi/m$ or $\alpha'_{es} = \pi/m$.

Odd number of phase windings ($m = 5, 7, \dots$) may be only symmetric, whereas even number of phase windings may be both symmetric and asymmetric. For example, the six-phase IM may be built symmetric ($\alpha_{es} = 2\pi/6$) with one single neutral point or of two three-phase windings ($\alpha'_{es} = \pi/6$) when two independent neutral points are available but not mandatory.

In general for IM, sinusoidal current and rather sinusoidal mmf (distributed windings) contain additional space/time harmonics effects.

However, to increase torque density essentially by reducing the magnetic saturation in the machine, a third harmonics may be injected in the reference current.

1.17.1 THE SIX-PHASE MACHINE

Let us first consider a typical six-phase (dual three-phase) IM with a single neutral point (Figure 1.45).

Extending the orthogonal (α - β or d - q or space vector) model theory from three phase to $m > 3$ cases, the six-phase phase-coordinate model, whose mutual stator/rotor inductance depends on rotor position, is transformed for the case in point (Figure 1.45), for stator coordinates, into three orthogonal batches of variables: α - β , x - y , O_1 - O_2 . The transformation matrix is

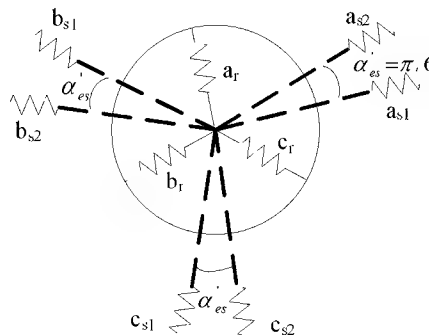


FIGURE 1.45 Dual three-phase (six-phase) IM.

$$\begin{bmatrix} V_{s\alpha} \\ V_{s\beta} \\ V_{sx} \\ V_{sy} \\ V_{so1} \\ V_{so2} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{2} & -1 \\ 1 & -\frac{1}{2} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{2} & -1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_{sa1} \\ V_{sa2} \\ V_{sb1} \\ V_{sb2} \\ V_{sc1} \\ V_{sc2} \end{bmatrix} \quad (1.165)$$

Now as magnetic saturation and space harmonics have been neglected, the three sets of orthogonal pairs of equations are (similar to three-phase IMs)

$$\begin{bmatrix} V_{s\alpha} \\ V_{s\beta} \\ V_{r\alpha} \\ V_{r\beta} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & M\dot{\theta}_{er} & R_r & L_r\dot{\theta}_{er} \\ -M\dot{\theta}_{er} & 0 & -L_r\dot{\theta}_{er} & R_r \end{bmatrix} \cdot \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix} + \begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix} \quad (1.166)$$

where L_s , L_r , R_s and R_r are stator and rotor cyclical-phase inductances and resistances, $\dot{\theta}_{er} = \omega_r$ is electrical rotor speed and M is the cyclical coupling inductance between stator and rotor (magnetization inductance).

Approximately, the xy and O_1O_2 component equations are related only to leakage flux and losses and are driven to zero by control:

$$\begin{aligned} V_{sx} &\approx R_s i_{sx} + L_{ls} \frac{d}{dt} i_{sx}; & V_{sy} &\approx R_s i_{sy} + L_{ls} \frac{d}{dt} i_{sy} \\ V_{so1} &\approx R_s i_{so1} + L_{ls} \frac{d}{dt} i_{so1}; & V_{so2} &\approx R_s i_{so2} + L_{ls} \frac{d}{dt} i_{so2} \\ V_{ro} &\approx R_r i_{ro} + L_{lr} \frac{d}{dt} i_{ro} \end{aligned} \quad (1.167)$$

with L_{ls} and L_{lr} as stator and rotor leakage inductances. Consequently, the torque is produced by the α - β sequence, which then may be transformed into d-q synchronous coordinates for convenient FOC:

$$T_e = p_l M (i_{s\beta} i_{r\alpha} - i_{s\alpha} i_{r\beta}) \quad (1.168)$$

As the three new sets of variables are well decoupled, the FOC is straightforward.

It is however feasible to connect the six-phase machine neutral to the inverter's D.C. bus neutral point (by a divided capacitor filter or an additional leg), and an additional zero sequence is ultimately generated to produce a third harmonic flux, which limits the peak flux in the machine and thus produces more torque [41,42].

1.17.2 THE FIVE-PHASE MACHINE

A typical five-phase IM may have 2 poles with 20 stator slots and 30 rotor slots (Figure 1.46) [43].

Here again the third harmonic injection principle is used to flatten the flux pattern and produce additional torque. The extended Park transformation $P_5(\theta_{er})$ now in d-q coordinates of synchronous speeds ω_{e1} and $3\omega_{e1}$ leads to

$$V_{d1s} = R_s i_{d1se} + p\Psi_{d1se} - \omega_{e1} \Psi_{q1se}; \quad 0 = R_{r1} i_{d1re} + p\Psi_{d1re} - (\omega_{e1} - \omega_r) \Psi_{q1re} \quad (1.169)$$

$$V_{q1s} = R_s i_{q1se} + p\Psi_{q1se} + \omega_{e1} \Psi_{d1se}; \quad 0 = R_{r1} i_{q1re} + p\Psi_{q1re} + (\omega_{e1} - \omega_r) \Psi_{d1re}$$

$$V_{d3se} = R_s i_{d3se} + p\Psi_{d3se} - 3\omega_{e3} \Psi_{q3se}$$

$$V_{q3se} = R_s i_{q3se} + p\Psi_{q3se} + 3\omega_{e3} \Psi_{d3se}$$

$$0 = R_{r3} i_{d3re} + p\Psi_{d3re} - 3(\omega_{e3} - \omega_r) \Psi_{q3re}$$

$$0 = R_{r3} i_{q3re} + p\Psi_{q3re} + 3(\omega_{e3} - \omega_r) \Psi_{d3re}$$

(1.170)

$$\text{with } \Psi_{d3se} + j\Psi_{q3se} = (L_{ls3} + L_{m3})(i_{d3se} + ji_{q3se}) + L_{m3}(i_{d3se} + ji_{q3se})$$

(1.171)

$$\Psi_{d3re} + j\Psi_{q3re} = (L_{lr3} + L_{m3})(i_{d3re} + ji_{q3re}) + L_{m3}(i_{d3se} + ji_{q3se})$$

$$T_e = p_l [L_{m1}(i_{q1se} i_{d1re} - i_{d1se} i_{q1re}) + 3L_{m3}(i_{q3se} i_{d3re} - i_{d3se} i_{q3re})]$$

(1.172)

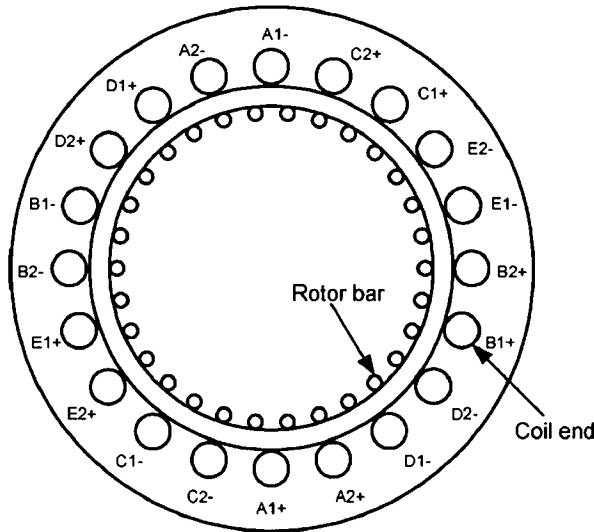


FIGURE 1.46 Five-phase IM. (After Ref. [43].)

$$\begin{bmatrix} V_{d1se} \\ V_{q1se} \\ V_{d3se} \\ V_{q3se} \\ V_{0se} \end{bmatrix} = \sqrt{\frac{2}{5}} \begin{bmatrix} \cos\theta_{er1} & \cos(\theta_{er1} - \alpha) & \cos(\theta_{er1} - 2\alpha) & \cos(\theta_{er1} - 3\alpha) & \cos(\theta_{er1} - 4\alpha) \\ -\sin\theta_{er1} & -\sin(\theta_{er1} - \alpha) & -\sin(\theta_{er1} - 2\alpha) & -\sin(\theta_{er1} - 3\alpha) & -\sin(\theta_{er1} - 4\alpha) \\ \cos\theta_{er3} & \cos(3\theta_{er3} - 3\alpha) & \cos(3\theta_{er3} - \alpha) & \cos(3\theta_{er3} - 4\alpha) & \cos(3\theta_{er3} - 3\alpha) \\ -\sin\theta_{er3} & -\sin(3\theta_{er3} - 3\alpha) & -\sin(3\theta_{er3} - \alpha) & -\sin(3\theta_{er3} - 4\alpha) & -\sin(3\theta_{er3} - 2\alpha) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \\ V_{ds} \\ V_{es} \end{bmatrix} \quad (1.173)$$

The machine may be run on the fundamental only, or on the third space harmonic only (the speed is three times smaller than that for the fundamental) or with both, to increase the torque density ($\alpha = 2\pi/5$).

We have to notice however that the machine parameters for the first and third space harmonics of the machine are quite different [44].

$$\begin{aligned} L_{m1} &= \frac{5}{2} \mu_0 l_r \pi N_{s1}^2; \quad L_{m3} = \frac{5}{2} \mu_0 l_r \pi N_{s3}^2; \quad N_{s1} = \frac{4}{\pi} N_s \cos \frac{\pi}{20} \\ R_{r1} &= \left(\frac{5\pi}{2} \right)^2 \frac{N_{s1}^2}{n_{bar} \sin^2(\alpha_{er}/2)} [R_b (1 - \cos \alpha_{er}) + R_{ering}]; \quad N_{s3} = \frac{4}{\pi} \frac{N_s}{3} \cos \frac{3\pi}{20} \\ L_{lr1} &= \left(\frac{5\pi}{2} \right)^2 \frac{N_{s1}^2}{n_{bar} \sin^2(\alpha_{er}/2)} [L_b (1 - \cos \alpha_{er}) + L_{ering}] \\ R_{r3} &= \frac{45\pi^2}{2} \cdot \frac{N_{s3}^2}{n_{bar} \sin^2(3\alpha_{er}/2)} [R_b (1 - \cos 3\alpha_{er}) + R_{ering}] \\ L_{lr3} &= \frac{45\pi^2}{2} \cdot \frac{N_{s3}^2}{n_{bar} \sin^2(3\alpha_{er}/2)} [L_b (1 - \cos 3\alpha_{er}) + L_{ering}] \end{aligned} \quad (1.174)$$

where l – stack length; r – rotor radius; N_s – conductor slot; $\alpha_{er} = 2\pi p/n_{bar}$; n_{bar} – bars in the rotor; R_b , L_b – rotor bar resistance and leakage inductance; and R_{ering} , L_{ering} – end ring sector resistance and inductance. Magnetic saturation is still neglected in the above expressions.

A method to estimate on line these machines parameters is available in [45].

Due to the reshaped (trapezoidal) flux pattern on account of third space harmonics component, the inverter pole voltage tends to be rectangular, which allows wider torque-speed envelope for given D.C. link voltage in the inverter. Also it has been demonstrated [43] that more torque for given RMS current is feasible with their improved flux pattern, at high load levels.

For a comprehensive up-to-date review of multiple-phase machine drives, see [46].

1.18 DOUBLY FED INDUCTION MACHINE MODELS FOR TRANSIENTS

Dual-stator winding induction machines (DSWIMs) with cage (or nested cage) rotor may be considered a peculiar case of multiphase IMs which have received strong R&D attention especially for stand-alone or grid-connected generators with D.C. or A.C. output-limited kVA ratings converter (feeding the control winding) for limited variable speed range mainly because they are brushless in contrast to standard doubly fed induction machines (DFIMs) with wound rotor which

require, in general, brushes to feed by partial kVA rating converter (RSC) connected to the rotor winding.

First, let us describe the model of the regular doubly fed induction generator (DFIG) in stand-alone mode with A.C. and D.C. outputs [47] (Figure 1.47).

The main problem is the nonlinearities, primarily the magnetic saturation in $L_m(i_m)$ which is required for self-excitation when the system is started.

As a mere example the space-phasor equations of the six-phase/three-phase dual-stator (p and c) winding IG (Figure 1.48) are straightforward in stator coordinates:

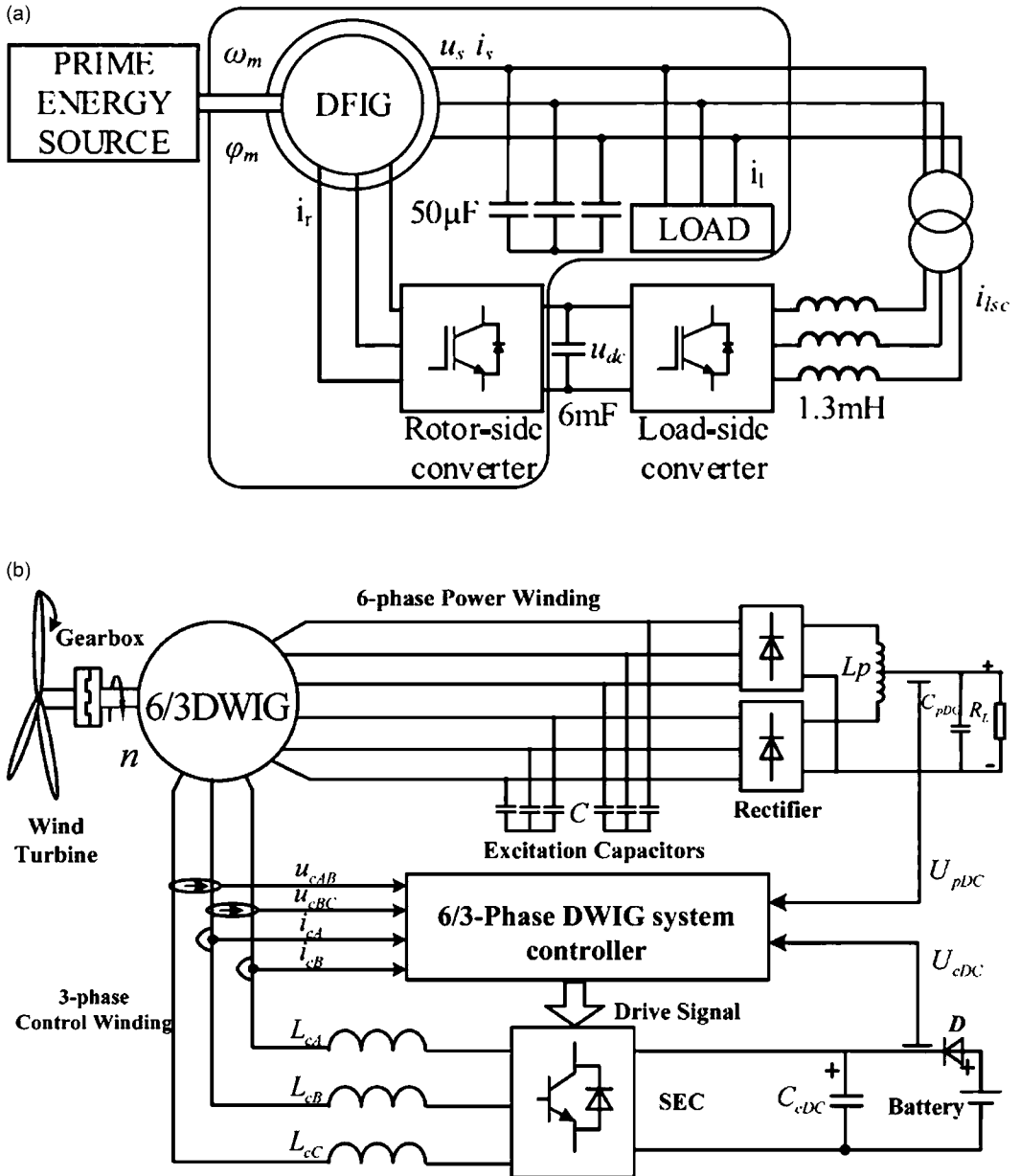


FIGURE 1.47 Stand-alone DFIG: (a) with nonlinear and unbalanced A.C. loads [47] and (b) BDFIG with D.C. loads [48].

$$\begin{aligned}
\bar{V}_{p1} &= R_{p1} \bar{i}_{p1} + \frac{d\bar{\Psi}_{p1}}{dt}; \quad \bar{\Psi}_{p1} = (L_{p11} + L_m) \bar{i}_{p1} + L_{lmp12} \bar{i}_{p2} + L_{lmcp1} \bar{i}_c + L_m \bar{i}_r \\
\bar{V}_{p2} &= R_{p2} \bar{i}_{p2} + \frac{d\bar{\Psi}_{p2}}{dt}; \quad \bar{\Psi}_{p2} = L_{lmp12} \bar{i}_{p1} + (L_{p21} + L_m) \bar{i}_{p2} + L_{lmcp2} \bar{i}_c + L_m \bar{i}_r \\
\bar{V}_c &= R_c \bar{i}_c + \frac{d\bar{\Psi}_c}{dt}; \quad \bar{\Psi}_c = L_{lmcp1} \bar{i}_{p1} + L_{lmcp2} \bar{i}_{p2} + (L_{cl} + L_m) \bar{i}_c + L_m \bar{i}_r \\
0 &= R_r \bar{i}_r - j\omega_r \bar{\Psi}_r + \frac{d\bar{\Psi}_r}{dt}; \quad \bar{\Psi}_r = L_m (\bar{i}_{p1} + \bar{i}_{p2} + \bar{i}_c + \bar{i}_r) + L_{re} \bar{i}_r
\end{aligned} \tag{1.175}$$

The power winding (p) contains 2 three-phase windings (1, 2) which are coupled only by the mutual leakage inductance L_{lmp12} . The 3 + 3-phase power winding is chosen to reduce the machine current harmonics in the presence of a diode rectifier (Figure 1.47b); c refers to the three-phase control stator winding connected through an active rectifier/inverter to a D.C. link which needs a low-voltage battery for safe starting self-excitation. The excitation capacitor equations have to be added together with load equations. The equivalent circuit of this machine resembles that of the standard cage-rotor three-phase IM, but the fact is that it shows three inputs (Figure 1.48).

Note that the control winding fed at variable frequency, shown in Figure 1.48, as a current source, may have a number of pole pairs different from the power winding (which may be three phase if A.C. loads only are accommodated). But then, it is usual to make the rotor with a bar-loop cage with $(p_p + p_c)$ – poles when the so-called brushless DFIG (BDFIG) is obtained [49,50]. In yet another version of BDFIG, the cascade BDFIG reintroduced [51] this time has been fed (controlled) by PWM converters in both stators (Figure 1.49) when a wide constant power speed range (CPSR) is obtained.

As shown in Figure 1.49, the windings of the two twin rotors are connected in counter series; also there are in fact two IMs here with wound rotors and p_1 and p_2 pole pairs fed at ω_1 and ω_2 stator frequencies. The model in stator coordinates for transients (in field-oriented or spiral vector-phase variables) looks the same [51]:

$$\begin{aligned}
V_1 &= \left(R_{s1} + s \left(L_{ls1} + \frac{3}{2} M_{s1r} \right) \right) i_1 + s \frac{3}{2} M_{s1r} i_r \\
-V_2^* &= (s - j(p_1 + p_2) \Omega_r) \frac{3}{2} M_{s2r} i_r - \left(R_{s2} + \left[s - j(p_1 + p_2) \Omega_r \left(L_{ls2} + \frac{3}{2} M_{s2r} \right) \right] \right) i_2^* \\
0 &= (s - jp_1 \Omega_r) \frac{3}{2} M_{s1r} i_1 - (s - jp_1 \Omega_r) \frac{3}{2} M_{s2r} i_2^* + \left[R_r + (s - jp_1 \Omega_r) \left(L_{lr} + \frac{3}{2} (M_{s1r} + M_{s2r}) \right) \right] i_r
\end{aligned} \tag{1.176}$$

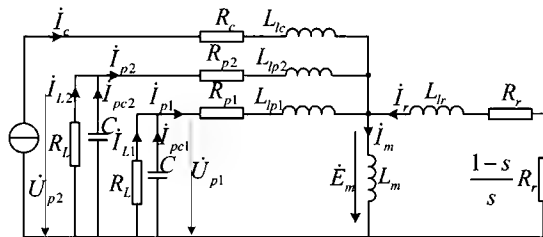


FIGURE 1.48 Six three-phase dual-stator winding IG space-phasor circuit in stator coordinates.

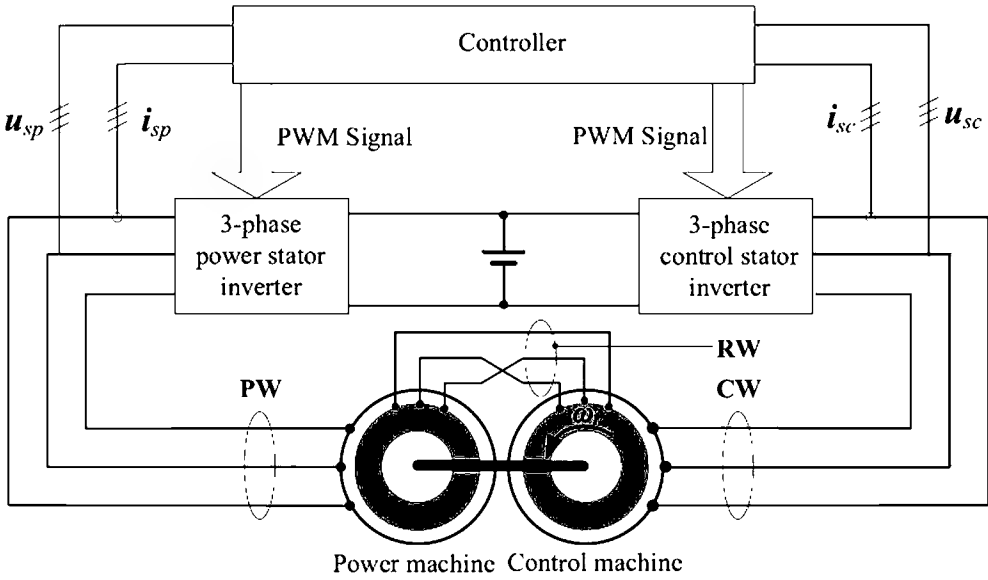


FIGURE 1.49 The cascaded BDFIM.

Ω_r – mechanical angular speed; s – Laplace operator (d/dt).

The electromagnetic torque T_e is

$$T_e = \frac{9}{4} \left(p_1 M_{s1r} \text{Imag}(i_1 i_r^*) + p_1 M_{s2r} \text{Imag}(-i_2^* \cdot (-i_r^*)) \right) \quad (1.177)$$

All parameters are referred to as the power winding (1) and are phase values (this is why $\frac{3}{2} N_{s1r}$ and $\frac{3}{2} N_{s2r}$ occur). In space-phaser form, $\bar{V}_1$, $\bar{V}_2$, $\bar{i}_1$, $\bar{i}_2$, and $\bar{i}_r$ as space vectors are to be used, whereas in spiral vector form, they are simply phase variables [52].

Finally, the space vector model of the brushless doubly fed induction machine (BDFIM) with p_p , p_c stator pole pair windings and $p_1 + p_2$ rotor pole loop cage [53] is straightforward:

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} \bar{i}_{s1} \\ \bar{i}_{s2} \\ \bar{i}_r \end{bmatrix} &= \begin{bmatrix} L_{s1} & 0 & M_{s1r} \\ 0 & L_2 & M_{s2r} \\ M_{s1r} & M_{s2r} & L_r \end{bmatrix}^{-1} \cdot \begin{bmatrix} R_{s1} & 0 & 0 \\ 0 & R_{s2} & 0 \\ 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} \bar{i}_{s1} \\ \bar{i}_{s2} \\ \bar{i}_r \end{bmatrix} \\ &+ \omega_r \begin{bmatrix} 0 & 0 & \frac{dM_{s1r}}{d\theta_r} \\ 0 & 0 & \frac{dM_{s2r}}{d\theta_r} \\ \frac{dM_{s1r}}{d\theta_r} & \frac{dM_{s2r}}{d\theta_r} & 0 \end{bmatrix} \begin{bmatrix} \bar{i}_{s1} \\ \bar{i}_{s2} \\ \bar{i}_r \end{bmatrix} + \begin{bmatrix} V_{s1} \\ V_{s2} \\ 0 \end{bmatrix} \times T_e = \begin{bmatrix} i_{s1}^T & i_{s1}^T \end{bmatrix} \begin{bmatrix} \frac{dM_{s1r}}{d\theta_r} \\ \frac{dM_{s2r}}{d\theta_r} \end{bmatrix} [i_r] \quad (1.178) \end{aligned}$$

It goes without saying that mutual inductances M_{s1r} and M_{s2r} between the stator windings and the rotor loop cage depend on rotor position in (1.178).

Also the synchronous speed Ω_r is

$$\omega_r = \frac{\omega_1 + \omega_2}{p_1 + p_2} \quad (1.179)$$

For starting as a motor, the control winding will be short-circuited (asynchronous starting) and then connected to its PWM converter for self-synchronization. Alternatively, the control winding, with main winding open, will be fed from PWM converter for slow starting (the converter kVAs are limited), and then, the stator is connected as for regular DFIGs.

1.19 CAGE-ROTOR SYNCHRONIZED RELUCTANCE MOTORS

In line-start applications, to secure good starting and better efficiency the cage rotor may be provided with magnetic saliency. A now historical such configuration is shown in Figure 1.50 [54].

More than one flux barrier may be stamped below uniform slots in the rotor to secure higher magnetic saliency. In a two-pole cage rotor with only shaft ends (Figure 1.51), the saliency may be improved to the point of providing 0.8+ power factor in a two-pole 15 kW induction motor stator.

In essence, the machine starts as an induction motor, and then self-synchronizes and operates as a reluctance synchronous machine.

If the saliency ratio (L_d/L_q) is high enough, the machine self-synchronizes under sizeable load torque with a rather large resistance rotor cage that provides good starting performance.

Arguably, the better efficiency in synchronous operation, even with a smaller power factor, justifies the practical utilization of such machines, especially when the speed should not vary with load.

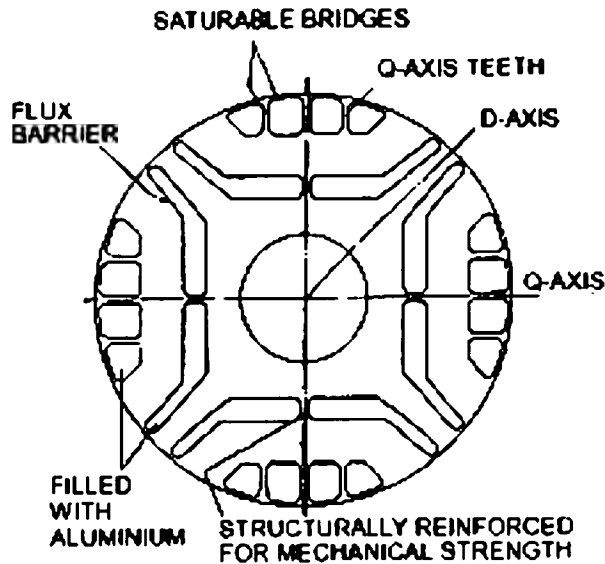


FIGURE 1.50 Single flux barrier four-pole cage rotor.

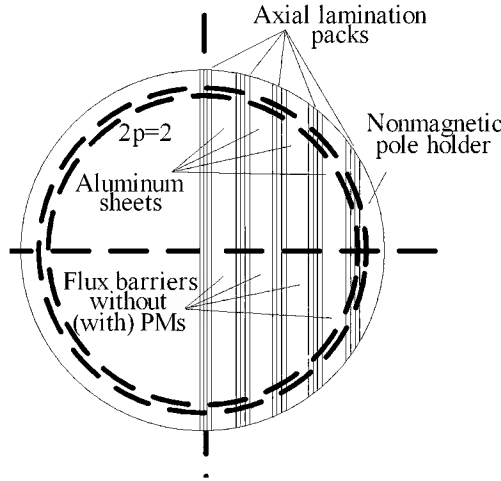


FIGURE 1.51 Two-pole “shaftless” cage variable reluctance rotor.

The d-q model of cage rotor Relsyn (reluctance synchronous) motor is shown in Figure 1.52, which leads to the following equations:

$$\begin{aligned} \frac{d\Psi_d}{dt} &= V_d - R_s i_d + \omega_r \Psi_q; \quad \Psi_d = L_{sl} i_d + L_{dm} (i_d + i_{dr}) \\ \frac{d\Psi_q}{dt} &= V_q - R_s i_q - \omega_r \Psi_d; \quad \Psi_q = L_{sl} i_q + L_{qm} (i_q + i_{qr}) \\ \frac{d\Psi_0}{dt} &= V_0 - R_s i_0; \quad \Psi_0 = L_{sl} i_0 \end{aligned} \quad (1.180)$$

$$\begin{aligned} \frac{d\Psi_{dr}}{dt} &= -R_{dr} i_{dr}; \quad \Psi_{dr} = L_{drl} i_{dr} + L_{dm} (i_d + i_{dr}) \\ \frac{d\Psi_{qr}}{dt} &= -R_{qr} i_{qr}; \quad \Psi_{qr} = L_{qrl} i_{qr} + L_{qm} (i_q + i_{qr}) \end{aligned} \quad (1.181)$$

$$\frac{d\omega_r}{dt} = \frac{p_l}{J} (T_e - T_{load}); \quad T_e = \frac{3}{2} p_l (\Psi_d i_q - \Psi_q i_d)$$

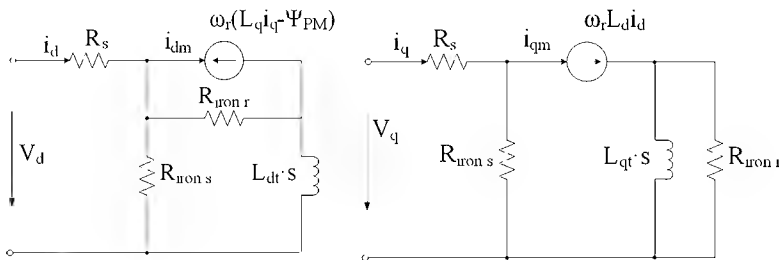


FIGURE 1.52 The d–q model of cage rotor Relsyn motor.

$$\frac{d\theta_{er}}{dt} = \omega_r; P(\theta_{er}) = \frac{2}{3} \begin{vmatrix} \cos(-\theta_{er}) & \cos\left(-\theta_{er} + \frac{2\pi}{3}\right) & \cos\left(-\theta_{er} - \frac{2\pi}{3}\right) \\ \sin(-\theta_{er}) & \sin\left(-\theta_{er} + \frac{2\pi}{3}\right) & \sin\left(-\theta_{er} - \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{vmatrix} \quad (1.182)$$

$$\begin{vmatrix} V_d \\ V_q \\ V_0 \end{vmatrix} = |P(\theta_{er})| \cdot \begin{vmatrix} V_a \\ V_b \\ V_c \end{vmatrix}; \begin{vmatrix} i_d \\ i_q \\ i_0 \end{vmatrix} = |P(\theta_{er})| \cdot \begin{vmatrix} i_a \\ i_b \\ i_c \end{vmatrix}; \quad \omega_r = p_1 \Omega_r \quad (1.183)$$

Note that if permanent magnets are added in axis d, in the rotor, then Ψ_d and Ψ_{dr} contain one more term: Ψ_{PMd} . In that case, however, in general (but not mandatory), $L_{dm} < L_{qm}$ (inversed saliency).

Defining $\alpha = L_q/L_d$, $\beta = R_s/\omega_1 L_d$ finally the torque (T_{e0}) expression, for steady state (in rotor coordinates: $d/dt = 0$),

$$T_{e0} = 3p_1 \frac{V^2}{\omega_1^2 L_d} \cdot \frac{(1-\alpha)}{(\alpha+\beta^2)^2} \cdot [(\alpha-\beta^2)\sin 2\delta - 2\beta(1-\alpha)\sin^2 \delta + 2\alpha\beta] \quad (1.184)$$

The space angle between the stator voltage vector $\bar{V}_{so}$ and the q axis is $\theta_0 = \frac{\pi}{2} + \delta$: δ is the power angle, and V is the phase voltage RMS value.

The max torque is obtained at the power angle δ_m :

$$\delta_m = \frac{1}{2} \tan^{-1} \left[\frac{\alpha - \beta^2}{\beta(1+\alpha)} \right] \leq \frac{\pi}{4} \quad (1.185)$$

In small machines, β may not be considered zero and thus $\delta_m < \pi/4$. For large machines, $\beta(R_s)$ may be neglected and $\delta_m \approx \pi/4$. The machine currents during steady state are

$$i_{d0} = \frac{V\sqrt{2}(\omega_1 L_q \cos \delta - R_s \sin \delta)}{R_s^2 + \omega_1 L_d L_q} \quad (1.186)$$

$$i_{q0} = \frac{V\sqrt{2}(\omega_1 L_d \sin \delta + R_s \cos \delta)}{R_s^2 + \omega_1 L_d L_q}; \quad i_{s0}^2 = i_{d0}^2 + i_{q0}^2$$

The iron losses may be considered as

$$P_{iron} = \frac{3(V\sqrt{2})^2}{2R_{iron}}; \quad V_{abc} = V\sqrt{2} \cos \left(\omega_r t - (i-1) \frac{2\pi}{3} \right) \quad (1.187)$$

So the synchronous operation efficiency (η) and power factor ($\cos \varphi$) may be calculated:

$$\eta = \frac{T_e \frac{\omega_1}{p_1} - p_{mec}}{T_e \frac{\omega_1}{p_1} + \frac{3}{2} R_s i_{s0}^2 + \frac{3}{2} \frac{(V\sqrt{2})^2}{R_{iron}}} \quad (1.188)$$

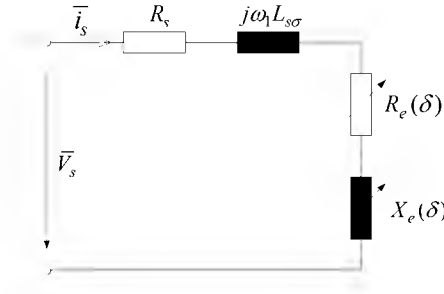


FIGURE 1.53 Equivalent circuit for synchronous operation.

Again, when neglecting the losses in the machine, the maximum/ideal power factor is

$$\cos \phi_1 = \frac{1 - \alpha}{1 + \alpha} \quad (1.189)$$

and is obtained for $\delta_i = \tan^{-1} \sqrt{\alpha}$; $i_{d0i}/i_{q0i} = \sqrt{\alpha}$.

The higher the saliency ($1/\alpha$), the higher the maximum power factor.

An equivalent steady-state (synchronous operation) circuit in space vector may be obtained by considering (Figure 1.53).

$$i_{d0} = \frac{\bar{i}_{s0} + \bar{i}_{s0}^*}{2}; \quad i_{q0} = \frac{\bar{i}_{s0} - \bar{i}_{s0}^*}{2j}; \quad \bar{i}_{s0}^* = \bar{i}_{s0} e^{2j(\delta - \phi_1)} \quad (1.190)$$

$$\bar{V}_{s0} = \bar{i}_{s0} \left(\text{Re}(\delta, \phi_1) + R_s + j(X_e(\delta, \phi_1) + X_{se}) \right); \quad X_{se} = \omega_1 L_{sl} \quad (1.191)$$

$$X_e(\delta, \phi_1) = \frac{\omega_1}{2} \left[(L_{dm} + L_{qm}) - (L_{dm} - L_{qm}) \cos 2(\phi_1 - \delta) \right] \quad (1.192)$$

$$\text{Re}(\delta, \phi_1) = \frac{1}{\omega_1} (L_{dm} - L_{qm} \sin 2(\phi_1 - \delta)); \quad \cos \phi_1 = \frac{R_e}{\sqrt{X_e^2 + R_e^2}}$$

$\text{Re}(\delta, \phi_1)$ should be negative for generating ($\delta < 0$, also).

For a standard but thorough self-synchronization and stability study when such a motor with low saliency ($L_d/L_q < 2.5$) is started on line, see Ref. [54].

When saliencies $L_d/L_q > 4$ –5 are reached, with multiple flux barriers behind the rotor slots (or with an axial lamination anisotropic rotor with aluminium sheets instead of insulation layers) the Relsyn motor with cage rotor offers higher efficiency at only slightly lower power factor than the corresponding IM with the same stator.

1.20 CAGE ROTOR PM SYNCHRONOUS MOTORS

In yet another effort to improve efficiency in line-start A.C. motors, PMs are added under the cage rotor. The PMs may be mounted on rotor surface (above the cage) when the equivalent airgap is large and IM power factor is low, but the PM utilization is very good; alternatively, the PMs may be planted in $1/2$ flux barriers per pole-hub below the cage (Figure 1.54 [55]).

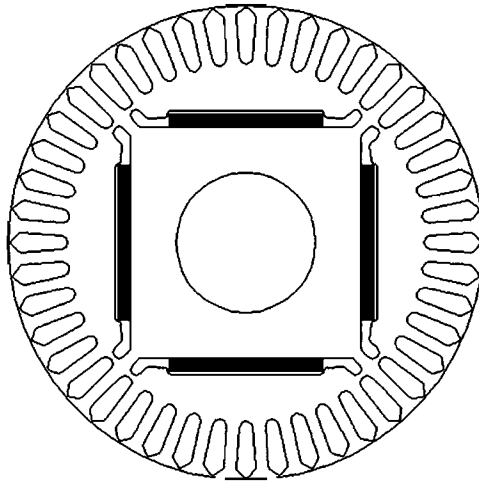


FIGURE 1.54 Line-start permanent magnet synchronous motor (PMSM) with single rotor flux barrier and PM per pole.

Quite a few winding configurations (Δ /parallel, star/parallel, Δ /series, star/ Δ , star/series) which provided adequate switches are available and have been proposed to yield good efficiency and power factor (even leading) for various input voltage levels and loads [48].

1.21 SUMMARY

- IMs undergo transients when the amplitude and frequency of electric variables and (or) the speed vary in time.
- Direct starting, after turn-off, sudden mechanical loading, sudden short circuit, reconnection after a short supply fault, behaviour during short intervals of supply voltage reduction, performance when PWM converter-fed are all typical examples of IM transients.
- The investigation of transients may be approached directly by circuit models or by coupled FEM circuit models of different degrees of complexity.
- The phase-coordinate model is the “natural” circuit model, but its stator–rotor mutual inductances depend on rotor position. The order of the model is 8 for a single three-phase winding in the rotor. The symmetrical rotor cage may be replaced by one-, two-, and, three-phase windings to cater for skin (frequency) effects.
- Solving such a model for transients requires a notable computation effort on a contemporary PC. Dedicated numerical software (such as MATLAB) has quite a few numerical methods to solve such systems of nonlinear equations.
- To eliminate the parameter (inductance) dependence on rotor position, the space-phasor (complex variable) or d–q model is used. The order of the system (in d–q, real, variables) is not reduced, but its solution is much easier to obtain through numerical methods.
- In complex variables, with zero homopolar components, only two electrical and one mechanical equations remain for a single cage rotor.
- With stator and rotor flux space phasors $\bar{\Psi}_s$ and $\bar{\Psi}_r$ as variables and constant speed, the machine model exhibits only two complex eigenvalues. Their expressions depend on the speed of the reference system.
- As expected, below breakdown torque–speed, the real part of the complex eigenvalues tends to be positive suggesting unstable operation.

- The model has two transient electrical time constants τ'_s and τ'_r : one of the stator and one of the rotor. So, for voltage supply, the stator and rotor flux transients are similar. Not so for current supply (only the rotor equation is used) when the rotor flux transients are slow (marked by the rotor time constant $\tau_r = L_r/R_r$).
- Including magnetic saturation in the complex variable model is easy for main flux path if the reference system is oriented to main flux space phasor $\bar{\Psi}_m = \Psi_m$, as $\Psi_m = L_m(i_m)i_m$.
- The leakage saturation may be accounted for separately by considering the leakage inductance dependence on the respective (stator or rotor) current.
- The standard equivalent circuit may be amended to reflect the leakage and main inductances, L_{sl} , L_{rl} , and L_m , dependence on respective currents, i_s , i_r , and i_m . However, as both the stator and rotor cores experience A.C. fields, the transient values of these inductances should be used to get better results.
- For large stator (and rotor) currents such as in high-performance drives, the main and leakage fluxes contribute to the saturation of teeth tops. In such cases, the airgap inductance L_g is separated as constant, and total core inductances L_{sl} and L_{rl} of one stator and one rotor, variables with respective currents, are responsible for the magnetic saturation influence in the machine.
- Further on, the core loss may be added to the complex variable model by orthogonal short-circuited windings in the stator (and rotor). The ones in the rotor may be alternatively used as a second rotor cage.
- Core losses influence slightly the efficiency torque and power factor during steady state. Only in the first few milliseconds, the core loss “leakage” time constant influences the behaviour of IM transients. The slight detuning in field orientation controlled drives, due to core loss, is to be considered mainly when precise (sub 1% error) torque control is required.
- Reduced-order models of IMs are used in the study of power system steady state or transients, as the number of motors is large.
- Neglecting the stator transients (stator leakage time constant τ'_r) is the obvious choice in obtaining a third-order model (with $\bar{\Psi}_r$ and ω_r as state variables). It has to be used only in synchronous coordinates (where steady-state means D.C.). However, as the supply frequency torque oscillations are eliminated, such a simplification is not to be used in calculating starting transients.
- The supply frequency oscillations in torque during starting are such that the average transient torque is close to the steady-state torque. This is why calculating the starting time of a motor via the steady-state circuit produces reasonable results.
- Considering leakage saturation in investigating starting transients is paramount in calculating correctly the peak torque and current values.
- Large IMs are coupled to plants through a kind of elastic couplings which, together with the inertia of motor and load, may lead to torsional frequencies which are equal to those of transient torque components. Large torsional torques occur at the load shaft in such cases. Avoiding such situations is the practical choice.
- The sudden short circuit at the terminals of high-power IMs represents an important liability for not-so-strong local power grids. The torque peaks are not, however, severely larger than those occurring during starting. The sudden short-circuit test may be used to determine the IM parameters in real saturation and frequency effect conditions.
- More severe transients than direct starting, reconnection on residual voltage or sudden short circuit have been found. The most severe case so far occurs apparently when the primary of transformer feeding an IM is turned off for a short interval (tens of milliseconds), very soon after direct starting. The secondary of the transformer (now with primary on no-load) introduces a large stator time constant which keeps alive the D.C. decaying stator

current components. This way, very large torque peaks occur. They vary from 26 to 40 p.u. in a 7.5 HP to 12 to 14 p.u. in a 500 HP machine.

- To treat unsymmetrical stator voltages (connections) so typical with PWM converter-fed drives, the abc–d–q model seems the right choice. Fault conditions may be treated rather easily as the line voltages are the inputs to the system (stator coordinates).
- For steady-state stability in power systems, first-order models are recommended. The obvious choice of neglecting both stator and rotor electrical transients in synchronous coordinates is not necessarily the best one. For low-power IMs, the first-order system thus obtained produces acceptably good results in predicting the mechanical (speed) transients. The subtransient voltage (rotor flux) E' and its angle δ represent the fast variables.
- For high-power motors, a modified first-order model is better. This new model reflects the subtransient voltage E' (rotor flux, in fact) transients, with speed ω_r and angle δ as the fast transients calculated from algebraic equations.
- In industry, a number of IMs of various power levels are connected to a local power grid through a power bus which contains a series reactance and a parallel capacitor (to increase power factor).
- Connection and disconnection transients are very important in designing the local power grid. Complete d–q models, with saturation included, proved to be necessary to simulate residual voltage (turn-off) of a few IMs when some of them act as motors and some as generators until the mechanical energy and magnetic energy in the system die down rather abruptly. The parallel capacitor delays the residual voltage attenuation, as expected.
- Series capacitors are used in some power grids to reduce the voltage sags during large motor starting. In such cases, SSR conditions might occur.
- In such a situation, high current and torque peaks occur at a certain speed. The maximum value of power bus plus stator resistance R_s for which SSR occurs shows how to avoid SSR. A better solution (in terms of losses) to avoid SSR is to put a resistance in parallel with the series capacitor. In general, the critical resistance R_{ad} (in parallel) for which SSR might occur increases with the series capacitance and so does the SSR frequency.
- Rotor bar and end-ring segment faults occur frequently. To investigate them, a detailed modelling of the rotor cage loops is required. Detailed circuit models to this end are available. In general, such fault introduces mild torque and speed pulsations and $(1-2S)f_1$ pulsations in the stator current. Information like this may be instrumental in IM diagnosis and monitoring. Interbar currents tend to attenuate the occurring asymmetry.
- Even more complete circuit models that handle both rotor bar currents and also the mixed eccentricity have been recently developed, based on the winding function method [56].
- Finite element-coupled circuit models have been developed in the last 10 years to deal with the IM transients. Still when skin and saturation effects, skewing and the rotor motion, and the IM structural details are all considered, the computation time is still prohibitive but feasible [57]. Worldwide aggressive R&D FEM efforts should render in the near future such complete (3D) models feasible (at least for prototype design refinements with thermal and mechanical models linked together).
- New circuit models for better converter/IM modelling software have recently been introduced [58].
- Multiphase IMs for variable speed drives for better fault tolerance have been proposed recently.
- Dual-stator winding IMs with cage rotor and a fractional kVA converter have been introduced for variable speed generators, and they are expected to be applied in both stand-alone and grid applications [59,60].
- PM-assisted cage-rotor IMs with no or notable magnetic rotor saliency are now promoted for superpremium efficiency line-start applications.

- The bar-to-bar circuit modelling of cage-rotor IM is useful to detect broken bars in “counter-current” braking tests [61].
- Very recent refinements on saturation and core loss modelling for transients in [7] are available in [62], as the subject is very important.
- A recent study treats analytically the broken bar and stator inter-turn faults together with promising results [61,63].

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2 Single-Phase IM Transients

2.1 INTRODUCTION

Single-phase induction motors undergo transients during starting, load perturbation or voltage sags, etc. When inverter fed, in variable speed drives, transients occur even for mechanical steady state, during commutation mode.

To investigate the transients, for orthogonal stator windings, the cross-field (or d-q) model in stator coordinates is traditionally used [1].

In the absence of magnetic saturation, the motor parameters are constant. Skin effect may be considered through a fictitious double cage on the rotor.

The presence of magnetic saturation may be included in the d-q model through saturation curves and flux linkages as variables. Even for sinusoidal input voltage, the currents may not be sinusoidal. The d-q model is capable of handling it. The magnetization curves may be obtained either through special flux decay standstill tests in the d-q (m.a) axes (one at a time) or from finite element modelling (FEM) – in D.C. with zero rotor currents. The same d-q model can handle nonsinusoidal input voltages such as those produced by a static power converter or by power grid polluted with harmonics by other nearby loads.

In the case of nonorthogonal windings on stator, a simplified equivalence with a d-q (orthogonal) winding system is worked out. Alternatively, a multiple-reference system + – model is used [2].

The d-q model uses stator coordinates, which imply A.C. steady-state quantities, whereas the multiple-reference model uses + – synchronous reference systems which imply D.C. steady-state quantities. Consequently, for the investigation of stability, the frequency response approach is typical to the d-q model, whereas small deviation linearization approach is typical to the multiple-reference + – model.

Finally, to consider the number of stator and rotor slots, that is space flux harmonics, the winding function approach is preferred [3]. This way, the torque/speed deep around 33% of no-load ideal speed, the effect of the relative numbers of stator and rotor slots, broken bars, and rotor skewing may be considered. Still saturation remains a problem as long as superposition is used.

A complete theory of single-phase induction machine (IM), valid for both steady state and transients, may be approached only by a FEM-circuit-coupled model, yet to be developed in an elegant computation-time competitive software. The abovementioned models will be described in detail next.

2.2 THE d-q MODEL PERFORMANCE IN STATOR COORDINATES

As in general the two windings are orthogonal but not identical, only stator coordinates may be used directly in the d-q (cross-field) model of the single-phase IMs (Figure 2.1).

It is the fact that when one phase is open, any coordinates will do it, but this works only in the case of the split-phase IM after starting [4].

First, all the d-q model variables are reduced to the main winding:

$$\begin{aligned}V_{ds} &= V_s(t) \\V_{qs} &= (V_s(t) - V_c(t))/a \\I_{ds} &= I_m \\I_{qs} &= I_a \cdot a\end{aligned}\tag{2.1}$$

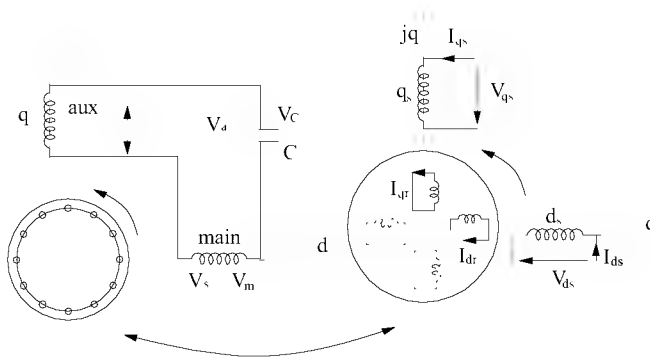


FIGURE 2.1 The d-q model.

where a is the auxiliary winding-to-main winding ratio of the equivalent turns.

Also

$$\frac{dV_c}{dt} = \frac{1}{C} \frac{I_{qs}}{a} \quad (2.2)$$

The d-q model equations, in stator coordinates, are now straightforward:

$$\begin{aligned} \frac{d\psi_{ds}}{dt} &= V_s(t) - R_{sm} I_{ds} \\ \frac{d\psi_{qs}}{dt} &= (V_s(t) - V_c(t))/a - \frac{R_{sa}}{a^2} I_{qs} \\ \frac{d\psi_{dr}}{dt} &= I_{dr} R_{rm} - \omega_r \psi_{qr} \\ \frac{d\psi_{qr}}{dt} &= I_{qr} R_{rm} + \omega_r \psi_{dr} \end{aligned} \quad (2.3)$$

The flux–current relationships are given by the following equations:

$$\begin{aligned} \Psi_{ds} &= L_{sm} I_{ds} + \Psi_{dm}; \quad \Psi_{dm} = L_m (I_{ds} + I_{dr}) = L_m I_{dm} \\ \Psi_{dr} &= L_{rm} I_{dr} + \Psi_{dm} \\ \Psi_{qs} &= \frac{L_{sa}}{a^2} I_{qs} + \Psi_{qm}; \quad \Psi_{qm} = L_m (I_{qs} + I_{qr}) = L_m I_{qm} \\ \Psi_{qr} &= L_{rm} I_{qr} + \Psi_{qm} \end{aligned} \quad (2.4)$$

The magnetization inductance L_m depends on both I_{dm} and I_{qm} when saturation is accounted for because of cross-coupling saturation effects. As the magnetic circuit is nonisotropic even with slotting neglected, the total flux vector Ψ_m is a function of total magnetization current I_m :

$$\begin{aligned} \Psi_m &= L_m(I_m) I_m \\ I_m &= \sqrt{I_{dm}^2 + I_{qm}^2}; \quad \Psi_m = \sqrt{\Psi_{dm}^2 + \Psi_{qm}^2} \end{aligned} \quad (2.5)$$

Once $L_m(I_m)$ is known, it may be used in the computation process with its previous computation step value, provided the sampling time is small enough.

We have to add the motion equation:

$$\begin{aligned} \frac{J}{p_l} \frac{d\omega_r}{dt} &= T_e - T_{load}(\theta_{er}, \omega_r) \\ \frac{d\theta_{er}}{dt} &= \omega_r / p_l \\ T_e &= (\Psi_{ds} I_{qs} - \Psi_{qs} I_{ds}) p_l \end{aligned} \quad (2.6)$$

Equations (2.2)–(2.6) constitute a nonlinear seventh-order system with V_c , I_{ds} , I_{qs} , I_{dr} , I_{qr} , ω_r , and θ_{er} as variables. The inputs are the source voltage $V_s(t)$ and the load torque T_{load} .

The load torque may vary with speed (for pumps, fans) and also with rotor position (for compressors).

In most applications, θ_{er} does not intervene as the load torque depends only on speed, so the order of the system becomes six.

When the two stator windings do not occupy the same number of slots, there may be slight differences between the magnetization curves along the two axes, but they are in many cases small enough to be neglected in the treatment of transients.

A good approximation of the magnetization curve may be obtained through FEM, at zero speed, with D.C. excitation along one axis and contribution from the second axis. Direct current decay tests at standstill in one axis, with D.C. injection in the second axis, should provide similar results. After many “data points” shown in Figure 2.2 are calculated, curve fitting may be used to yield a unique correspondence between Ψ_m and I_m .

As expected, if enough digital simulation time for transients is allowed for, steady-state behaviour may be reached.

For sinusoidal power source voltage, when the magnetic saturation is not considered, the line current is sinusoidal for no-load and under-load conditions, whereas when the magnetic saturation is considered, the line current is nonsinusoidal for no-load and load conditions (Figure 2.3) [5].

The influence of saturation is visible. It is even more visible in the main winding current RMS computation versus output power experiments (Figure 2.4) [5].

The differences are smaller in the auxiliary winding for the case in point: $P_n = 1.1$ kW, $U_n = 220$ V, $I_n = 6.8$ A, $C = 25$ μ F, $f_n = 50$ Hz, $a = 1.52$, $R_{sm} = 2.6$ Ω , $R_{sa} = 6.4$ Ω , $R_{rm} = 3.11$ Ω , $L_{sm} = L_{rm} = 7.5$ mH, $J = 1.1 \times 10^{-3}$ Kg m² and $L_{sa} = 17.3$ mH. The magnetization curve is shown in Figure 2.2. The motor power factor, calculated for the fundamental, at low loads, is much less than that predicted by the nonsaturated d-q model. On the other hand, it is similar to the saturated d-q model that produces results, for a wide range of loads, which are very close to the experimental ones.

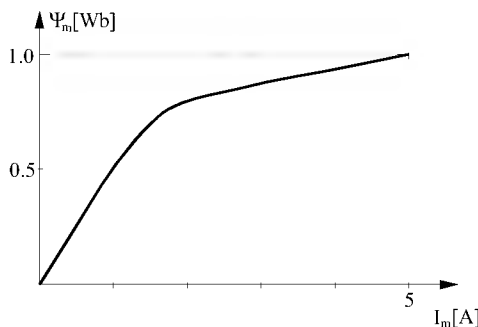


FIGURE 2.2 The magnetization curve.

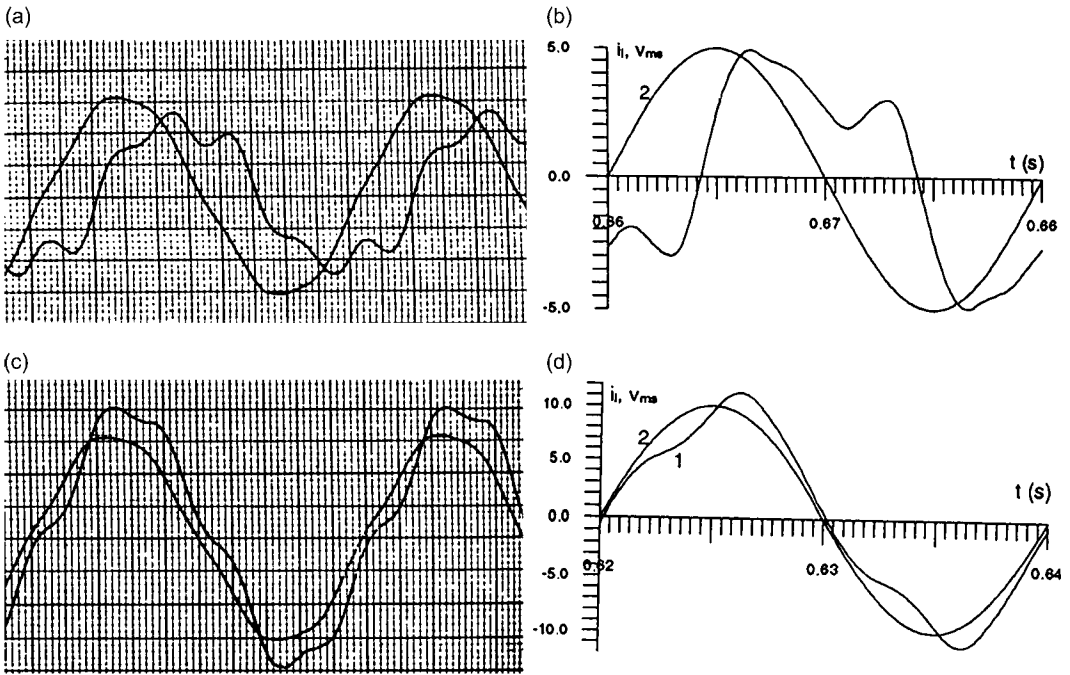


FIGURE 2.3 Steady state by the d-q model with saturation included. (a,b) No load, (c,d) on load, (a,c) test results, and (b,d) digital simulation.

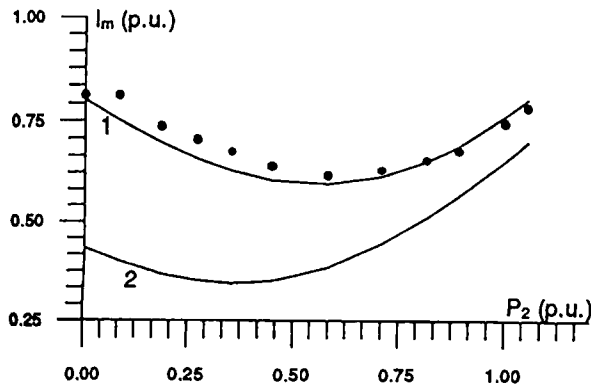


FIGURE 2.4 RMS of main winding current versus output power. Curve 1 – saturated model, curve 2 – unsaturated model, and . – test results.

This may be due to the fact that the saturation curve has been obtained through tests and thus the saturation level is tracked for each instantaneous value of magnetization current (flux).

The RMS current correct prediction by the saturated d-q model for steady state represents notable progress in assessing more correctly the losses in the machine.

Still, the core losses – fundamental and additional – and additional losses in the rotor cage (including the interbar currents) are not yet included in the model. Space harmonics though apparently somehow included in the $\Psi_m(I_m)$ curve are not thoroughly treated in the saturated d-q model.

2.3 STARTING TRANSIENTS

It is a known fact that during severe starting transients, the main flux path saturation does not play a crucial role. However, it is embedded in the model and may be used if needed.

For a single-phase IM with the data, $V_s = 220$ V, $f_{ln} = 50$ Hz, $R_{sm} = R_{sa} = 1 \Omega$, $a = 1$, $L_m = 1.9$ H, $L_{sm} = L_{sa} = 0.2$ H, $R_{rm} = 35 \Omega$, $L_{rm} = 0.1$ H, $J = 10^{-3}$ Kg m^2 , $C_a = 5 \mu$ F, $\zeta = 90^\circ$, the starting transients calculated via the d-q model are presented in Figure 2.5. The load torque is zero from start to $t = 0.4$ s, when a 0.4 Nm load torque is suddenly applied.

The average torque is negative because of the choice of signs.

The torque pulsations are large which are shown in Figure 2.5b, while the average torque is 0.4 Nm, equal to the load torque. The large torque pulsations reflect the departure from symmetry conditions. The capacitor voltage goes up to a peak value of 600 V, which indicates that a larger capacitor might be needed.

The hodograph of stator current vector is shown in Figure 2.6.

$$I_s = I_d + jI_q \quad (2.7)$$

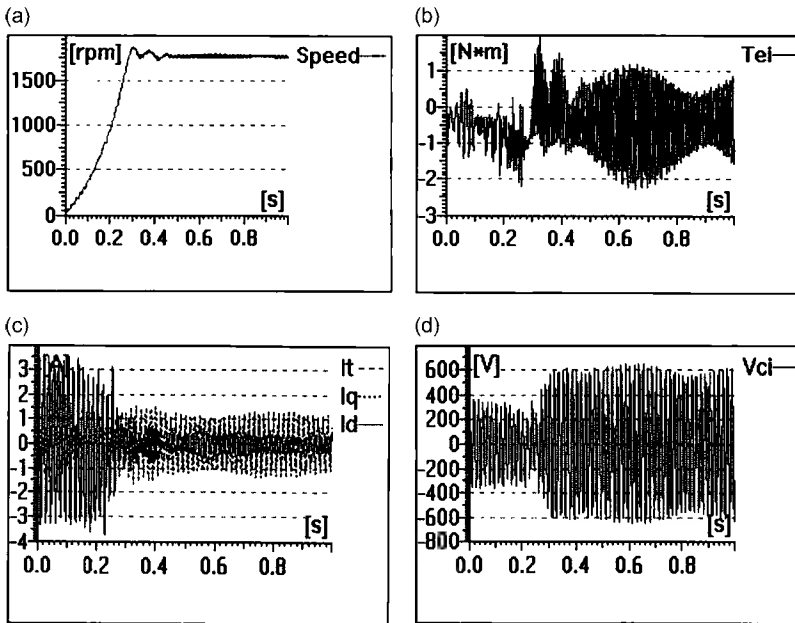


FIGURE 2.5 Starting transients. (a) Speed versus time, (b) torque versus time, (c) input current versus time, and (d) capacitor voltage versus time.

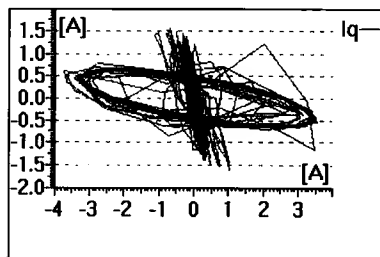


FIGURE 2.6 Current hodograph during starting.

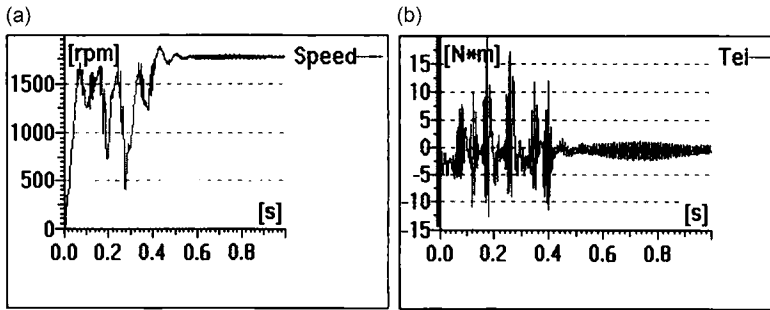


FIGURE 2.7 Starting transients for $C_s = 30 \mu\text{F}$ at start and $C_a = 5 \mu\text{F}$ from $t = 0.4 \text{ s}$ on (a) speed and (b) torque.

The hodograph should be a circle for symmetry, but in Figure 2.6, there is no such circle.

To study the influence of capacitance on the starting process, the capacitance of the starting capacitor is set to $C_s = 30 \mu\text{F}$. At $t = 0.4 \text{ s}$, the capacitance is reduced suddenly, i.e. $C_a = 5 \mu\text{F}$ (Figure 2.7).

The torque and current transients are too large when $C_s = 30 \mu\text{F}$, which indicates that the capacitor is now too large.

2.4 THE MULTIPLE-REFERENCE MODEL FOR TRANSIENTS

The d-q model has to use stator coordinates as long as the two windings are not identical. However, if the d-q model is applied separately for the instantaneous + and – (f, b) components, then, for synchronous coordinates, $+\omega_1$ (for the + (f) component) and $-\omega_1$ (for the – (b) component), the steady state means D.C. variables [2].

Consequently, the dynamic (stability) analysis may be performed via the system linearization (small deviation theory).

Stability to sinusoidal load torque pulsation such as in compressor loads can thus be handled. The superposition principle precludes the inclusion of magnetic saturation in the model [2].

2.5 INCLUDING THE SPACE HARMONICS

The existence of space harmonics causes torque pulsations, cogging, and crawling.

The space harmonics have three origins, as mentioned in Chapter 13, Vol. 1.

- Stator mmf space harmonics
- Slot opening permeance pulsations
- Main flux path saturation.

A general method to deal with space harmonics of all these three origins is presented based on the multiple magnetic circuit approach in References [6,7]. The rotor is considered bar by bar.

Toliyat and Sargolzaei [3] show that space harmonics are produced by the stator mmf, and can be dealt with the effect of skewing. Based on winding function approach, the latter method [3] also considers m windings in the stator and N_r bars in the rotor. Magnetic saturation is neglected.

The deep(s) in the torque/speed curve may be predicted by this procedure. Also the influence of the ratio of slot numbers N_s/N_r of stator and rotor and of cogging (parasitic, asynchronous) torques is clearly evident. For example, Figure 2.8 illustrates [3] the starting transients of a single-phase capacitor motor with the same number of slots per stator and rotor, without skewing. As expected, the motor is not able to start due to the strong parasitic synchronous torque at stall (Figure 2.8a).

With skewing of one slot pitch, even with $N_s = N_r$ slots, the motor can start (Figure 2.8b).

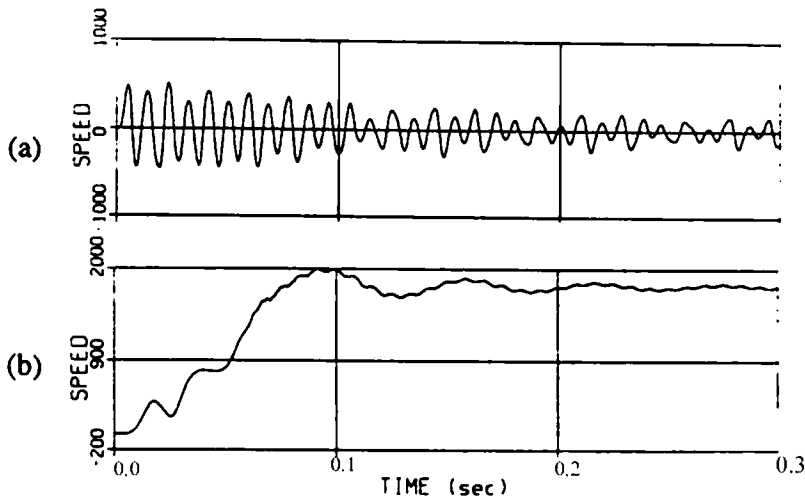


FIGURE 2.8 Starting speed transients for $N_s = N_r$ (a) without skewing and (b) with skewing. (After Ref. [3].)

2.6 SUMMARY

- Transients occur during starting or load torque perturbation operation modes.
- Also, power source voltage (and frequency) variation causes transients in the single-phase IM.
- When power converter (variable voltage, variable frequency)-fed, the single-phase IM experiences time harmonics also.
- The standard way to handle the single-phase IM is using the cross-field (d-q) model when magnetic saturation, time harmonics and skin effect may be considered simultaneously.
- Due to magnetic saturation, the main winding and total stator currents depart from sinusoidal waveforms, at both no-load and under-load conditions. The magnetic saturation level depends on the load, machine parameters and capacitance value. It seems that neglecting saturation leads to notable discrepancy between the main and total currents measured and calculated RMS values. This is the only reason why losses are not predicted correctly with sinusoidal currents, especially for low powers.
- Switching the starting capacitor off produces important speed, current, and torque transients.
- The d-q model has a straightforward numerical solution for transients, but, due to the stator winding asymmetry, it has to make use of stator coordinates to yield rotor position-independent inductances. Steady state means A.C. variables at power source frequency. Consequently, small deviation theory, to study stability, may not be used, except for the case when the auxiliary winding is open.
- The multiple-reference system theory, using $+$ $-$ (f, b) models in their synchronous coordinates ($+\omega_1$ and $-\omega_1$), may be used to investigate stability after linearization.
- Finally, besides FEM-circuit-coupled model, the winding function approach or the multiple equivalent magnetic circuit method simulates each rotor bar, and thus, asynchronous and synchronous parasitic torques may be detected.
- A FEM-circuit-coupled model (software), which is easy to handle, and computation-time competitive, for single-phase IM, is apparently not available as of this writing.
- Line-start 1 phase source split-phase capacitor self-synchronizing IMs with cage-PM-reluctance rotor have been studied recently to increase efficiency at moderate extra cost in home-like appliances [8].

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3 Super-High-Frequency Models and Behaviour of IMs

3.1 INTRODUCTION

Voltage strikes and restrikes produced during on/off switching operations of induction motors fed from standard power grid may cause severe dielectric voltage stresses on the stator induction machine (IM) windings, leading, eventually, to failure.

In industrial installations, high dielectric stresses may occur during second- and third-pole circuit breaker closure. The second- and the third-pole closure in electromagnetic power circuit breakers has been shown to occur within 0–700 μs [1].

For such situations, electrical machine modelling in the frequency range of a few KHz is required. Steep-fronted voltage waves with magnitudes up to 5 p.u. may occur at the machine terminals under certain circuit breaker operating conditions.

On the other hand, PWM voltage source inverters produce steep voltage pulses which are applied repeatedly to induction motor terminals in modern electric drives.

In insulated gate bipolar transistor (IGBT) inverters, the voltage switching rise times of 0.05–2 μs , in presence of long cables, have been shown to produce strong winding insulation stresses and premature motor bearing failures.

With short rise time IGBTs and power cables longer than a critical length l_c , repetitive voltage pulse reflection may occur at motor terminals.

The reflection process depends on the parameters of the feeding cable between motor and inverter, the IGBT voltage pulse time t_r , and the motor parameters.

The peak line-to-line terminal overvoltage (V_{pk}) at the receiving end of an initially uncharged transmission line (power cable) subjected to a single PWM pulse with rise time t_r [2] is

$$\begin{aligned} (V_{pk})_{l \geq l_c} &= (1 + \Gamma_m) V_{dc} \\ \Gamma_m &= \frac{Z_m - Z_0}{Z_m + Z_0} \end{aligned} \quad (3.1)$$

where the critical cable length l_c corresponds to the situation when the reflected wave is fully developed; V_{dc} is the D.C. link voltage in the voltage source inverter, and Γ_m is the reflection coefficient ($0 < \Gamma_m < 1$).

Z_0 is the power cable and Z_m the induction motor surge impedance. The distributed nature of a long-cable L–C parameters favours voltage pulse reflection, besides inverter short rising time. Full reflection occurs along the power cable if the voltage pulses take longer than one-third of the voltage rising time to travel from converter to motor at speed $u^* \approx 150\text{--}200 \text{ m}/\mu\text{s}$.

The voltage is then doubled, and critical length is reached [2] (Figure 3.1).

The receiving (motor) end may get $3V_{dc}$ for cable lengths greater than l_c when the transmission line (power cable) has initial trapped charges due to multiple PWM voltage pulses.

Inverter rise times of 0.1–0.2 μs lead to equivalent frequency in the MHz range.

Consequently, Super-high-frequency modelling of IMs involves frequencies in the range of 1 kHz to 10 MHz.

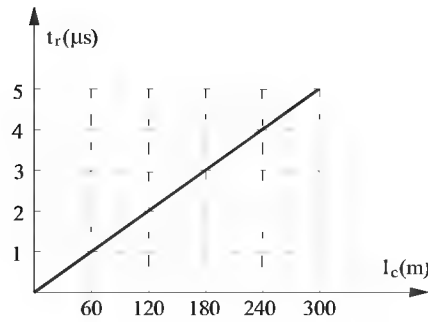


FIGURE 3.1 Critical power cable length l_c .

The effects of such fast voltage pulses on the machine windings include

- In nonuniform voltage distribution along the windings, most of the voltage drop occurs at the first 1–2 coils (connected to the terminals). Especially with random coils, the inter-turn voltage between the first and, say, the last turn, which may be located nearby, may become high enough to produce premature insulation ageing.
- The common mode voltage PWM inverter pulses, on the other hand, produce parasitic capacitive currents between stator windings and motor frame and, in parallel, through air-gap parasitic capacitance and through bearings, to motor frame. The common mode circuit is closed through the cable capacitances to ground.
- Common mode current may unwarrantly trip the null protection of the motor and damage the bearing by lubricant electrostatic breakdown.

The super-high frequency or surge impedance of the IM may be approached globally either when it is to be identified through direct tests or as a complex distributed parameter (capacitor, inductance, resistance) system, when identification from tests at the motor terminals is in fact not possible.

In such a case, either special tests are performed on a motor with added measurement points inside its electric (magnetic) circuit, or analytical or FEM methods are used to calculate the distributed parameters of the IM.

IM modelling for surge voltages may then be used to conceive methods to attenuate reflected waves and change their distribution within the motor so as to reduce insulation stress and bearing failures.

We will start with global (lumped) equivalent circuits and their estimation, and continue with distributed parameter equivalent circuits.

3.2 THREE HIGH-FREQUENCY OPERATION IMPEDANCES

When PWM inverter-fed, the IM terminals experience three pulse voltage components.

- Line-to-line voltages (e.g. phase A in series with phases B and C in parallel): V_{ab} , V_{bc} , and V_{ca}
- Line (phase)-to-neutral voltages: V_{an} , V_{bn} , and V_{cn}
- Common mode voltage V_{0in} (Figure 3.2)

$$V_{0in} = \frac{V_a + V_b + V_c}{3} \quad (3.2)$$

Assume that the zero sequence impedance of IM is Z_0 .

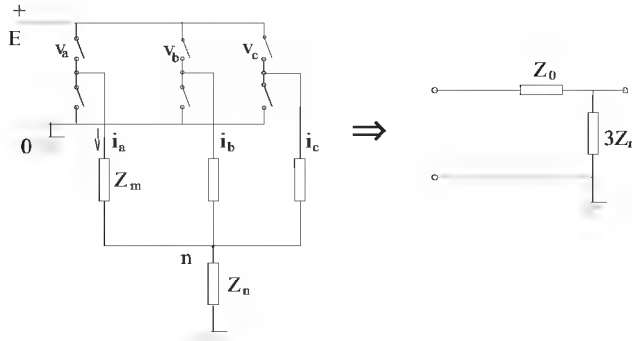


FIGURE 3.2 PWM inverter with zero sequence impedance of the load (insulated neutral point).

The zero sequence voltage and current are then

$$V_0 = \frac{(V_a - V_n) + (V_b - V_n) + (V_c - V_n)}{3} = \frac{V_a + V_b + V_c}{3} - V_n \quad (3.3)$$

Also, by definition,

$$\begin{aligned} V_n &= Z_n I_n \\ V_0 &= Z_0 I_0 \end{aligned} \quad (3.4)$$

$$I_0 = \frac{I_a + I_b + I_c}{3} = \frac{I_n}{3}$$

Eliminating V_n and V_0 from (3.3) with (3.4) yields

$$I_n = \frac{3}{Z_0 + 3Z_n} \cdot \frac{(V_a + V_b + V_c)}{3} \quad (3.5)$$

and again from (3.3),

$$V_{0in} = \frac{V_a + V_b + V_c}{3} = Z_0 I_0 + V_n \quad (3.6)$$

$$V_n = 3Z_n I_0 = Z_n I_n$$

This is how the equivalent lumped impedance (Figure 3.2) for the common voltage evolved.

It should be noted that for the differential voltage mode, the currents flow between phases and thus no interference between the differential and common modes occurs.

To measure the lumped IM parameters for the three modes, terminal connections as shown in Figure 3.3 are made.

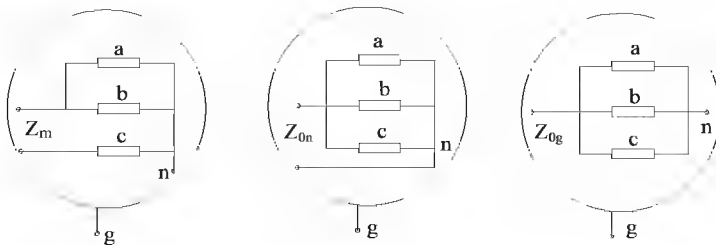


FIGURE 3.3 Line (Z_m), neutral (Z_{on}) and ground (common mode Z_{og}).

Impedances shown in Figure 3.3, referred here as differential (Z_m), zero sequence – neutral – (Z_{0n}), and common mode (Z_{og}), can be measured directly by applying single-phase A.C. voltage of various frequencies. Both the amplitude and the phase angle are of interest.

3.3 THE DIFFERENTIAL IMPEDANCE

Typical frequency responses for the differential impedance Z_m are shown in Figure 3.4 [3].

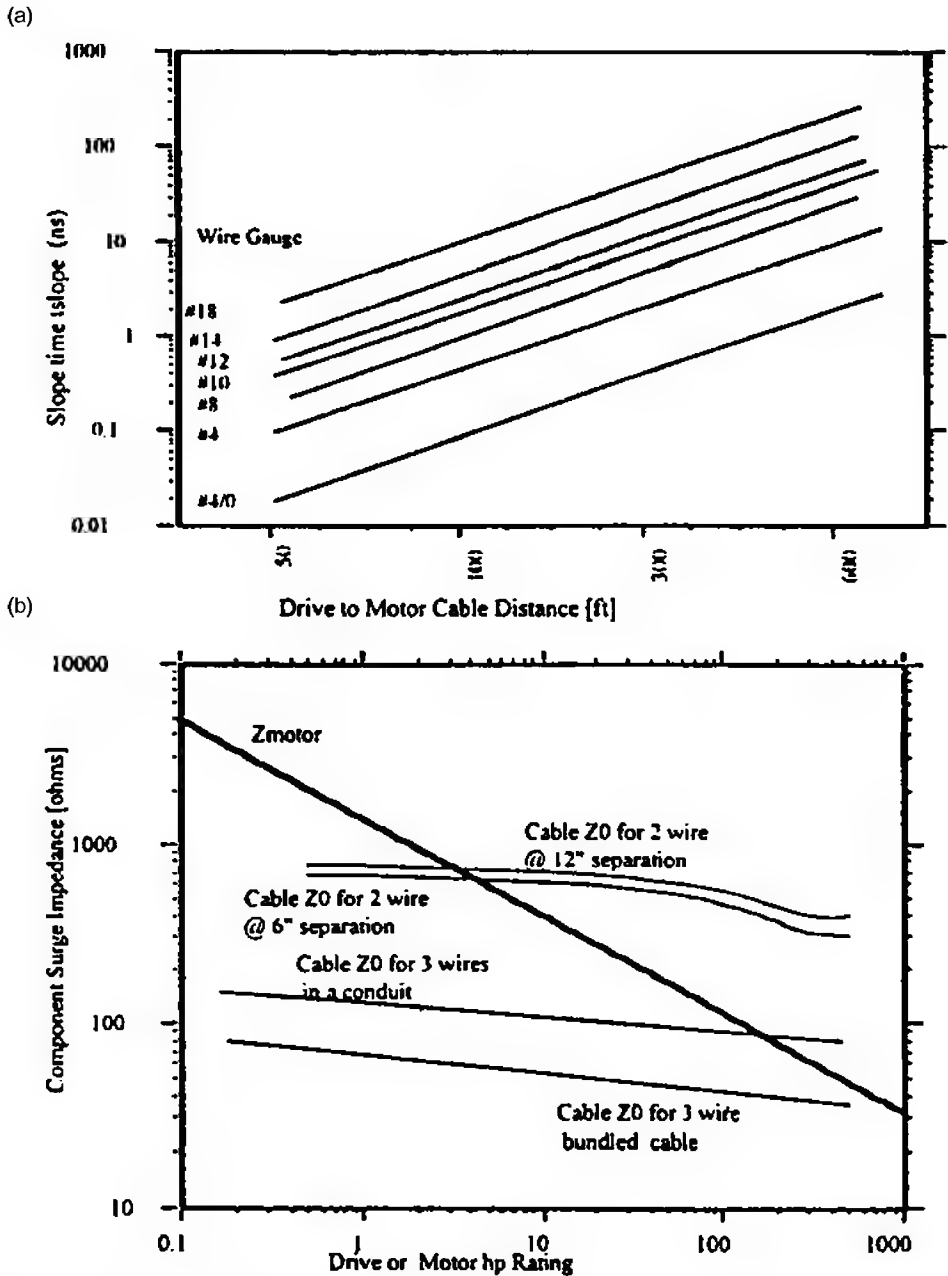


FIGURE 3.4 Differential mode impedance (Z_m). (After Ref. [3].)

Z_m has been determined by measuring the line voltage V_{ac} during the PWM sequence with phase a, b together and c switched from the + A.C. bus to the – D.C. bus. The value of Δe is the transient peak voltage V_{ac} above D.C. bus magnitude during t_{rise} , and ΔI is phase c peak transient phase current.

$$Z_m = \frac{\Delta e}{\Delta I} \quad (3.7)$$

As shown in Figure 3.4, Z_m decreases with increasing IM power. Also it has been found out that Z_m varies from manufacturer to manufacturer, for given power, as much as 5 times. When using an RLC analyser (with phases a and b in parallel, connected in series with phase c), the same impedance has been measured by a frequency response test (Figure 3.5) [3].

The phase angle of the differential impedance Z_m approaches a positive maximum between 2 and 3 kHz (Figure 3.5b). This is due to lamination skin effect which reduces the iron core A.C. inductance.

At critical core frequency f_{iron} , the field penetration depth equals the lamination thickness d_{lam} .

$$d_{lam} = \sqrt{\frac{l}{\pi f_{iron} \sigma_{iron} \mu_{rel} \mu_0}} \quad (3.8)$$

The relative iron permeability μ_{rel} is essentially determined by the fundamental magnetization current in the IM. Above f_{iron} eddy current shielding becomes important, and the iron core inductance starts to decrease until it approaches the wire self-inductance and stator air core leakage inductance at the resonance frequency $f_r = 25$ kHz (for the 1 HP motor) and 55 kHz for the 100 HP motor.

Beyond f_r , turn-to-turn and turn-to-ground capacitances of wire perimeter as well as coil-to-coil (and phase-to-phase) capacitances prevail such that the phase angle approaches now -90° (pure capacitance) around 1 MHz.

So, as expected, with increasing frequency the motor differential impedance switches character from inductive to capacitive (Figure 3.5b). Z_m is important in the computation of reflected wave voltage at motor terminals with the motor fed from a PWM inverter through a power (feeding) cable. Many simplified lumped equivalent circuits have been tried to model the experimentally obtained wide-band frequency response [4,5].

As very high-frequency phenomena are confined to the stator slots, due to the screening effect of rotor currents – and to stator end connection conductors, a rather simple line-to-line circuit motor model may be adopted to predict the line motor voltage surge currents.

The model in [3] is basically a resonant circuit to handle the wide range of frequencies involved (Figure 3.6).

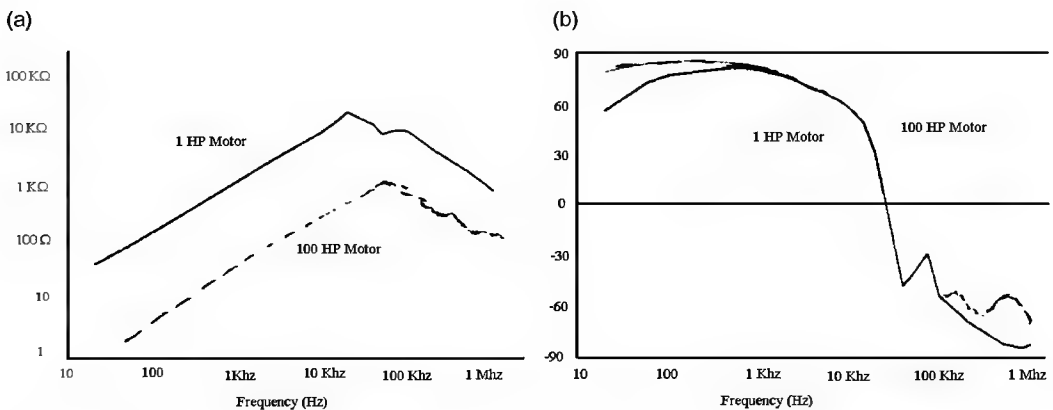


FIGURE 3.5 Differential impedance Z_m versus frequency (a) amplitude and (b) phase angle. (After Ref. [3].)

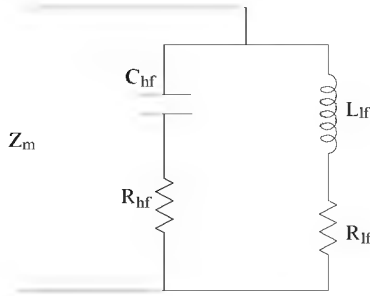


FIGURE 3.6 Simplified differential mode IM equivalent circuit.

TABLE 3.1

Differential Mode IM Model Parameters

	1 kW	10 kW	100 kW
C_{hf}	250 pF	800 pF	8.5 nF
R_{hf}	18 Ω	1.3 Ω	0.13 Ω
R_{lf}	150 Ω	300 Ω	75 Ω
L_{lf}	190 mH	80 mH	3.15 μ H

C_{hf} and R_{hf} determine the model at high frequencies (above 10 kHz in general), whereas L_{lf} and R_{lf} are responsible for lower frequency modelling. The identification of the model in Figure 3.6 from frequency response may be done through optimization (regression) methods.

Typical values of C_{hf} , R_{hf} , L_{lf} , and R_{lf} are given in Table 3.1 after Ref. [3].

A rather linear increase of C_{hf} and a rather linear decrease of R_{hf} , R_{lf} , and L_{lf} both with increasing power are shown in Table 3.1.

The high-frequency resistance R_{hf} is much smaller than the low-frequency resistance R_{lf} .

3.4 NEUTRAL AND COMMON MODE IMPEDANCE MODELS

Again we start our study from some frequency response tests for the connections shown in Figure 3.3b (for Z_{on}) and Figure 3.3c (for Z_{og}) [6]. Sample results are shown in Figure 3.7a and b.

Many lumped parameter circuits to fit results such as those shown in Figure 3.7 may be tried on. Such a simplified phase circuit is shown in Figure 3.8a [6].

Harmonics copper losses are represented by R_{sw} , μL_{sl} – stator first turn leakage inductance, C_{sw} – inter-turn equivalent capacitor, C_{ft} – total capacitance to ground, and R_{frame} – stator initial frame to ground damping resistance, which are all computable by regression methods from frequency response.

The impedances Z_{on} and Z_{og} (with the three phases in parallel) shown in Figure 3.8a are

$$Z_{on} = \frac{pL_d}{3 \left[1 + p \frac{L_d}{R_e} + p^2 L_d \frac{C_g}{2} \right]}; \quad R_e^{-1} = R_{skin}^{-1} = 0 \quad (3.9)$$

$$Z_{og} = \frac{1 + p \frac{L_d}{R_e} + p^2 L_d C_g}{6pC_g \left[1 + p \frac{L_d}{R_e} + p^2 L_d \frac{C_g}{2} \right]}; \quad R_{skin}^{-1} = 0 \quad (3.10)$$

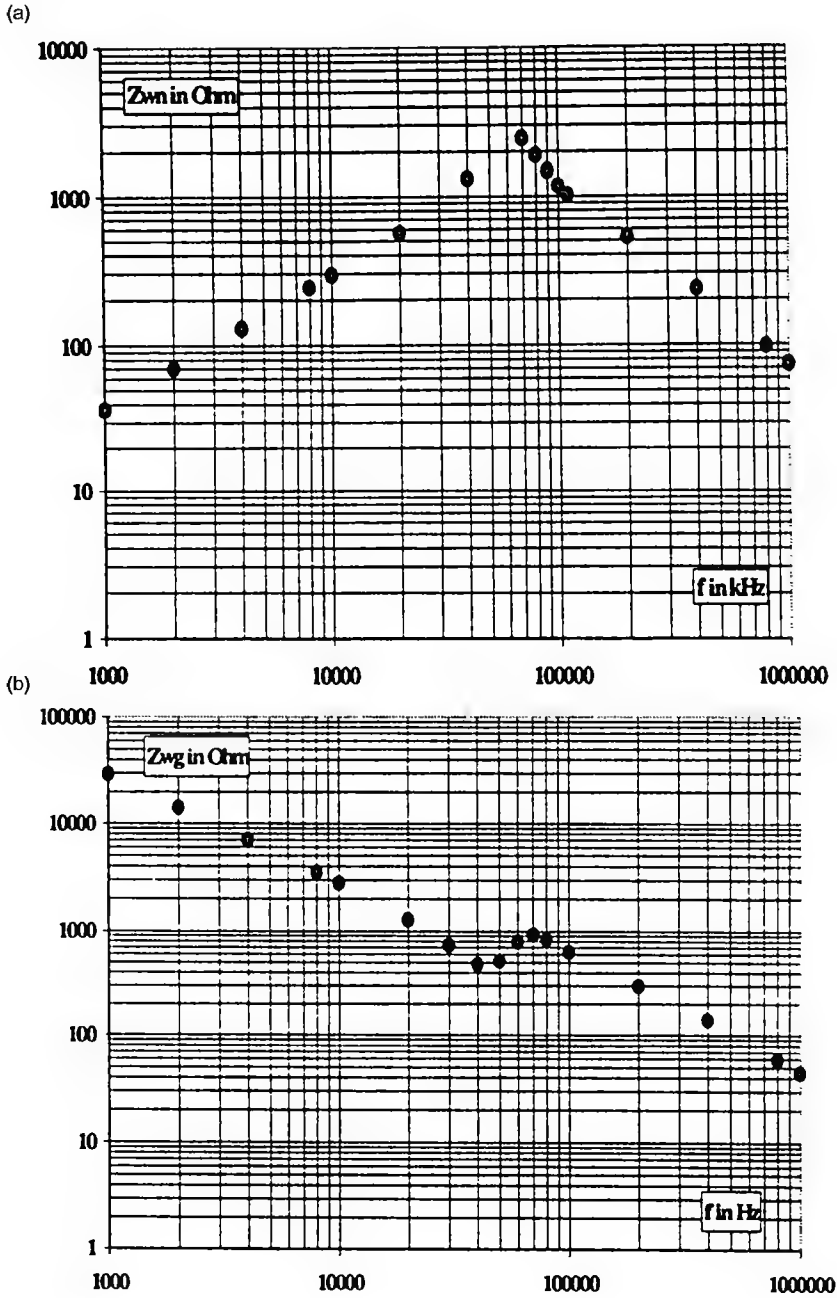


FIGURE 3.7 Frequency dependence of (a) Z_{on} and (b) Z_{og} . (After Ref. [6].)

The poles and zeros are

$$f_p(Z_{on}, Z_{og}) = \frac{1}{2\pi} \sqrt{\frac{2}{L_d C_g}} \quad (3.11)$$

$$f_z(Z_{og}) = f_p(Z_{on}, Z_{og}) / \sqrt{2} \quad (3.12)$$

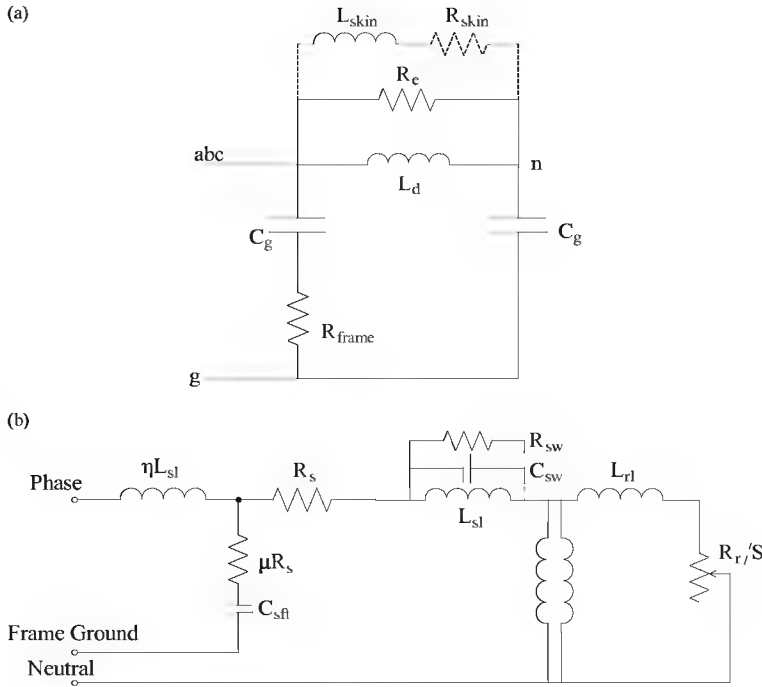


FIGURE 3.8 (a) Simplified high-frequency phase circuit and (b) universal (low- and high-frequency) circuit per phase.

At low frequency (1–10 kHz), Z_{og} is almost purely capacitive:

$$C_g = \frac{1}{6 \cdot 2\pi f (Z_{og})_f} \quad (3.13)$$

Taking $f = 1$ kHz and Z_{og} (1 kHz) from the graph shown in Figure 3.7b, the capacitance C_g is determined.

With the pole frequency f_p (Z_{on} , Z_{og}), corresponding to peak Z_{on} value, the resistance R_e is

$$R_e = 3(Z_{on})_{f_p} \quad (3.14)$$

Finally, L_d is obtained from (3.11) with f_p known from the graph shown in Figure 3.7 and C_g calculated from (3.13).

This is a high-frequency inductance.

It is thus expected that at low frequency (1–10 kHz), fitting of Z_{on} and Z_{og} with the model will not be so good.

Typical values for a 1.1 kW four-pole, 50 Hz, 220/380 V motor are $C_g = 0.25$ nF, $L_d(h_f) = 28$ mH, and $R_e = 17.5$ k Ω .

For 55 kW, $C_g = 2.17$ nF, $L_d(h_f) = 0.186$ mH, and $R_e = 0.295$ k Ω .

On a logarithmic scale, the variation of C_g and $L_d(h_f)$ with motor power in kW is approximated [6] to

$$C_g = 0.009 + 0.53 \ln(P_n(\text{kW})), [\text{nF}] \quad (3.15)$$

$$\ln(L_d(h_f)) = 2.36 - 0.1P_n(\text{kW}), [\text{mH}] \quad (3.16)$$

for 220/380 V, 50 Hz, 1–55 kW IMs.

To improve the low-frequency (between 1 and 10 kHz) fitting between the equivalent circuit and the measured frequency response, an additional low-frequency R_e , L_e branch may be added in parallel to L_d shown in Figure 3.8.

The addition of an eddy current resistance representing the motor frame R_{frame} (Figure 3.8) may also improve the equivalent circuit precision.

Test results with triangular voltage pulses and with a PWM converter and short cable have proven that the rather simple high-frequency phase equivalent circuit shown in Figure 3.8 is reliable.

More elaborated equivalent circuits for both differential and common voltage modes may be adopted for better precision [5]. For their identification however, regression methods have to be used, when the skin effect branch shown in Figure 3.8 is to be considered at least when Z_{og} is identified. Both amplitude and phase of frequency response up to 1 MHz are used for identification by regression methods [7].

Alternatively, the d-q model is placed in parallel, and thus, a general equivalent circuit acceptable for digital simulations at any frequency is obtained. This way, a universal low, wide frequency IM model is obtained (Figure 3.8b) [8].

The lumped equivalent circuits for high frequency presented here are to be identified from frequency response tests. Their configuration retains a large degree of approximation. They serve only to assess the impact of differential and common mode voltage surges at the induction motor terminals.

The voltage surge and its distribution within the IM will be addressed next.

3.5 THE SUPER-HIGH-FREQUENCY DISTRIBUTED EQUIVALENT CIRCUIT

When power grid is connected, IMs undergo surge-connection or atmospheric surge voltage pulses. When PWM voltage is surge-fed, IMs get trains of steep front voltage pulses. Their distribution, in the first few microseconds, along the winding coils, is not uniform.

Higher voltage stresses in the first 1–2 terminal coils and their first turns occur.

This nonuniform initial voltage distribution is due to the presence of stray capacitors between turns (coils) and the stator frame.

The complete distributed circuit parameters should contain individual turn-to-turn and turn-to-ground capacitances, self-turn, turn-to-turn and coil-to-coil inductances and self-turn, and eddy current resistances.

Some of these parameters may be measured through frequency response tests only if the machine is tapped adequately for the purpose. This operation may be practical for a special prototype to check design computed values of such distributed parameters.

The computation process is extremely complex, even rather impossible, in terms of turn-to-turn parameters in random wound coil windings.

Even via 3D-FEM, the complete set of distributed circuit parameters valid from 1 kHz to 1 MHz is not yet feasible.

However, at high frequencies (in the MHz range), corresponding to switching times in the order of tens of a microsecond, the magnetic core acts as a flux screen and thus most of the magnetic flux will be contained in air, as leakage flux.

The high-frequency eddy currents induced in the rotor core will confine the flux within the stator. In fact, the magnetic flux will be nonzero in the stator slot and in the stator coil end connections zone.

Let us suppose that the end connection resistances, inductances and capacitances between various turns and to the frame can be calculated separately. The skin effect is less important in this zone, and the capacitances between the end turns and the frame may be neglected, as their distance to the frame is notably larger in comparison with conductors in slots which are much closer to the slot walls.

So, in fact, the FEM may be applied within a single stator slot, conductor by conductor (Figure 3.9).

The magnetic and electrostatic field is zero outside the stator slot perimeter Γ and nonzero inside it.

The turn in the middle of the slot has a lower turn-to-ground capacitance than turns situated closer to the slot wall.

The conductors around a conductor in the middle of the slot act as an eddy current screen between turn and ground (slot wall).

In a random wound coil, positions of the first and of the last turns (n_c) are not known and thus may differ in different slots.

Computation of the inductance and resistance slot matrixes is performed separately within an eddy current FEM package [9].

The first turn current is set to 1 A at a given high frequency, whereas the current is zero in all other conductors. The mutual inductances between the active turn and the others are also calculated.

The computation process is repeated for each of the slot conductors as active.

With $I = 1$ A (peak value), the self-inductance L and resistance R of the conductor are

$$L = \frac{2W_{\text{mag}}}{\left(\frac{I_{\text{peak}}}{\sqrt{2}}\right)^2}; \quad R = \frac{P}{\left(\frac{I_{\text{peak}}}{\sqrt{2}}\right)^2} \quad (3.17)$$

The output of the eddy current FE analysis per slot is an $n_c \times n_c$ impedance matrix.

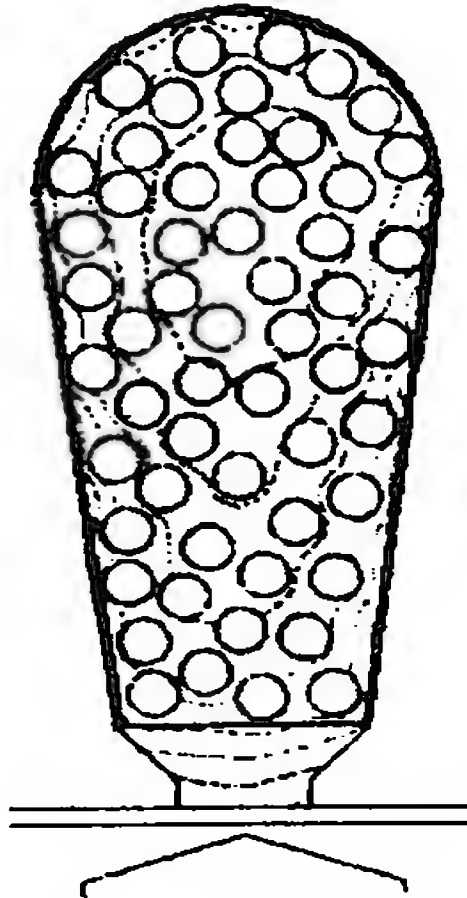


FIGURE 3.9 Flux distribution in a slot for eddy current FE analysis.

The magnetic flux lines for $n_c = 55$ turns/coil (slot), $f \approx 1/t_{\text{rise}}$ from 1 to 10 MHz, with one central active conductor ($I_{\text{peak}} = 1$ A) (Figure 3.9, [9]) show that the flux lines are indeed contained within the slot volume.

The eddy current field solver calculates A and Φ in the field equation:

$$\nabla \frac{1}{\mu\omega_r} (\nabla A) = (\sigma + j\omega\epsilon_r)(-j\omega A - \Delta\Phi) \quad (3.18)$$

where

$A(x,y)$ – the magnetic potential (Wb/m)

$\Phi(x,y)$ – the electric scalar potential (V)

μ_r – the magnetic permeability

ω – the angular frequency

σ – the electric conductivity

ϵ_r – the dielectric permittivity.

The capacitance matrix may be calculated by an electrostatic FEM package.

This time, each conductor is set to 1 V (D.C.) source, whereas the others are set to 0 V. The slot walls are defined as a zero potential boundary. The electrostatic field simulator solves now for the electric potential $\Phi(x,y)$.

$$\nabla (\epsilon_r \epsilon_0 \nabla \Phi(x,y)) = -\rho(x,y) \quad (3.19)$$

where $\rho(x,y)$ is the electric charge density.

The result is an $n_c \times n_c$ capacitance matrix per slot which contains the turn-to-turn and turn-to-slot wall capacitance of all conductors in slot. The computation process is done n_c times with always a different single conductor as the 1 V (D.C.) source.

The matrix terms C_{ij} are

$$C_{ij} = 2W_{\text{el}ij} \quad (3.20)$$

where $W_{\text{el}ij}$ is the electric field energy associated with the electric flux lines that connect charges on conductor “i” (active) and “j” (passive).

As electrostatic analysis is performed, the dependence of capacitance on frequency is neglected.

A complete analysis of a motor with 24, 36, 48, ... stator slots with each slot represented by $n_c \times n_c$ (e.g. 55×55) matrixes would be hardly practical.

A typical line-end coil simulation circuit, with the first 5 individual turns visualized, is shown in Figure 3.10 [9].

To consider extreme possibilities, the line-end coil and the last five turns of the first coil are also simulated turn-by-turn. The rest of the turns are simulated by lumped parameters. All the other coils per phase are simulated by lumped parameters. Only the diagonal terms in the impedance

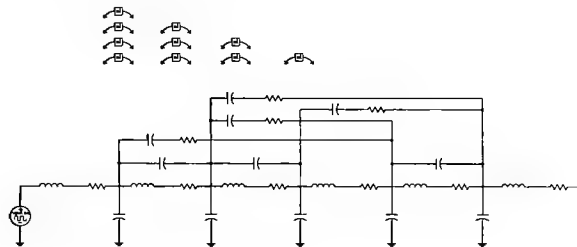


FIGURE 3.10 Equivalent circuit of the line-end coil.

and capacitance matrixes are nonzero. Saber-simulated voltage drops across the line-end coil and in its first three turns are shown in Figure 3.11a and b for a 750 V voltage pulse with $t_{\text{rise}} = 1 \mu\text{s}$ [9].

The voltage drop along the line-end coil was 280 V for $t_{\text{rise}} = 0.2 \mu\text{s}$ and only 80 V for $t_{\text{rise}} = 1 \mu\text{s}$. Notice that there are six coils per phase. Feeder cable tends to lead to 1.2–1.6 kV voltage amplitude by wave reflection and thus dangerously high electric stress may occur within the line-end coil of each phase, especially for IGBTs with $t_{\text{rise}} < 0.5 \mu\text{s}$, and random wound machines.

Thorough frequency response measurements with a tapped winding IM have been performed to measure turn–turn and turn-to-ground distributed parameters [10].

Further on, the response of windings to PWM input voltages (rise time: $0.24 \mu\text{s}$) with short and long cables have been obtained directly and calculated through the distributed electric circuit with measured parameters. Rather satisfactory but not very good agreement has been obtained [10].

It was found that the line-end coil (or coil 01) takes up 52% and the next one (coil 02) takes also a good part of input peak voltage (42%) [10]. This is in contrast to FE analysis results which tend to predict a lower stress on the second coil [9].

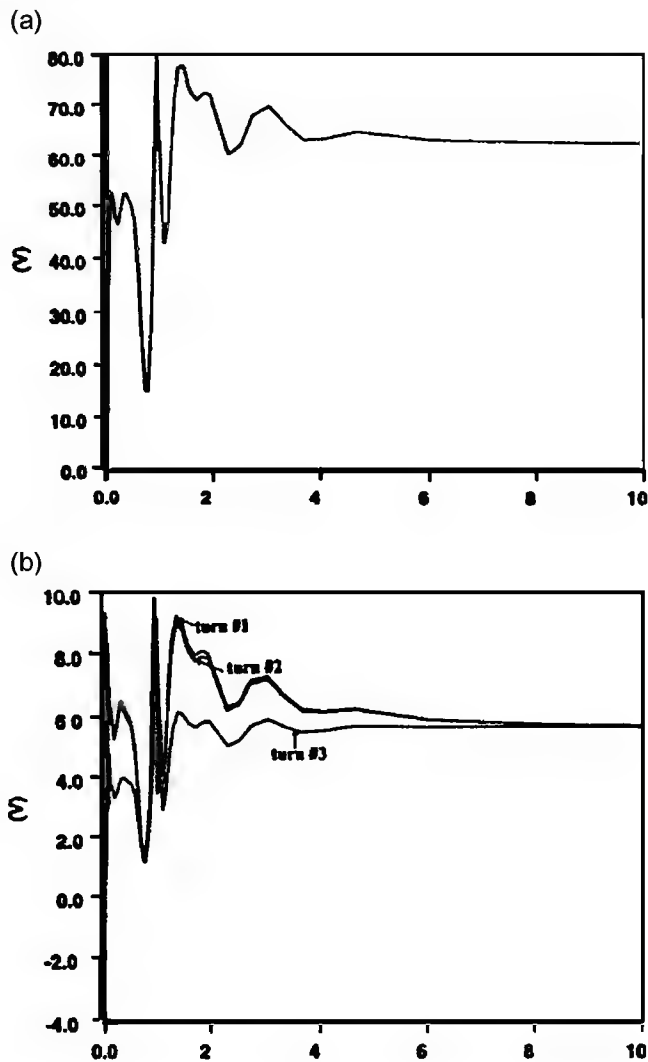


FIGURE 3.11 Voltage drop versus time for $t_{\text{rise}} = 1 \mu\text{s}$: (a) line-end coil (after Ref. [9]). (b) Turns 1–3 of line-end coil (after Ref. [9]).

As expected, long power cables tend to produce higher voltage surges at motor terminals. Consequently, the voltage drop peaks along the line-end coil and its first turn are up to two to three times higher. The voltage distribution of PWM voltage surges along the winding coils, especially along its line-end first 2 coils and the line-end first 3 turns thus obtained, is useful to winding insulation design.

Also preformed coils seem more adequate than random coils.

It is recommended that in power grid-fed IMs, the timing between the first-, second-, and third-pole power switch closure be from 0 to 700 μs . Consequently, even if the commutation voltage surge reaches 5 p.u. [11], in contrast to 2–3 p.u. for PWM inverters, the voltage drops along the first two coils and their first turns are not necessarily higher because the t_{rise} of commutation voltage surges is much larger than 0.2–1 μs . As we already mentioned, the second main effect of voltage surges on the IM is the bearing early failures with PWM inverters. Explaining this phenomenon requires, however, special lumped parameter circuits.

3.6 BEARING CURRENTS CAUSED BY PWM INVERTERS

Rotor eccentricity, homopolar flux effects, or electrostatic discharge are known causes of bearing (shaft) A.C. currents in power grid-fed IMs [12,13]. The high-frequency common mode large voltage pulse at IM terminals, when fed from PWM inverters, has been suspected to further increase bearing failure. Examination of bearing failures in PWM inverter-fed IM drives indicates fluting, induced by electrical discharge machining (EDM). Fluting is characterized by pits or transverse grooves in the bearing race which lead to premature bearing wear.

When riding the rotor, the lubricant in the bearing behaves as a capacitance. The common mode voltage may charge the shaft to a voltage that exceeds the lubricant's dielectric field rigidity believed to be around 15 $V_{\text{peak}}/\mu\text{m}$. With an average oil film thickness of 0.2–2 μm , a threshold shaft voltage of 3–30 V_{peak} is sufficient to trigger EDM.

As described in Section 3.1, a PWM inverter produces zero sequence besides positive and negative sequence voltages.

These voltages reach the motor terminals through power cables, online reactors, or common mode chokes [2]. These impedances include common mode components as well.

The behaviour of the PWM inverter IM system in the common voltage mode is suggested by the three-phase schemata shown in Figure 3.12 [14].

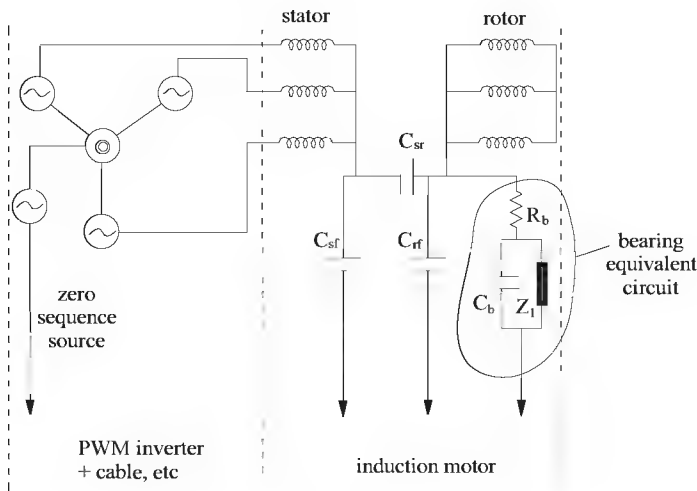


FIGURE 3.12 Three-phase PWM inverter plus IM model for bearing currents.

The common mode voltage, originating from the zero sequence PWM inverter source, is distributed between stator and rotor neutral and ground (frame):

- C_{sf} – the stator winding-frame stray capacitor,
- C_{sr} – stator-rotor winding stray capacitor (through airgap mainly)
- C_{rf} – rotor winding to motor frame stray capacitor
- R_b – bearing resistance
- C_b – bearing capacitance
- Z_l – nonlinear lubricant impedance which produces intermittent shorting of capacitor C_b through bearing film breakdown or contact point.

With the feeding cable represented by a series/parallel impedance Z_s , Z_p , the common mode voltage equivalent circuit may be extracted from Figure 3.12, as shown in Figure 3.13. R_0 and L_0 are the zero sequence impedances of IM to the inverter voltages. Calculating C_{sf} , C_{sr} , C_{rf} , R_b , C_b , and Z_l is still a formidable task.

Consequently, experimental investigation has been performed to somehow segregate the various couplings performed by C_{sf} , C_{sr} , and C_{rf} .

The physical construction to the scope implies adding an insulated bearing support sleeve to the stator for both bearings. Also brushes are mounted on the shaft to measure V_{rg} .

Grounding straps are required to short outer bearing races to the frame to simulate normal (uninsulated) bearing operation (Figure 3.14) [14].

In region A in Figure 3.15, the shaft voltage V_{rg} charges to about $20 V_{pk}$. At the end of region A, V_{sng} jumps to a higher level causing a pulse in V_{rg} . In that moment, the oil film breaks down at $35 V_{pk}$ and a $3 A_{pk}$ bearing current pulse is produced. At high temperatures, when oil film thickness is further reduced, the breakdown voltage (V_{rg}) pulse may be as low as 6–10 V.

Region B is without bearing current. Here, the bearing is charged and discharged without current.

Region C shows the rotor and bearing (V_{rg} and V_{sng}) charging to a lower voltage level. No EDM occurs this time. $V_{rg} = 0$ with V_{sng} high means that contact asperities are shorting C_b .

The shaft voltage V_{rg} , measured between the rotor brush and the ground, is a strong indicator of EDM potentiality. Test results in Figure 3.15 [2] show the shaft voltage V_{rg} , the bearing current I_b , and the stator neutral to ground voltage V_{sng} .

An indicator of shaft voltage is the bearing voltage ratio (BVR):

$$BVR = \frac{V_{rg}}{V_{sng}} = \frac{C_{sr}}{C_{sr} + C_b + C_{rf}} \quad (3.21)$$

With insulated bearings, neglecting the bearing current (if the rotor brush circuits are open and the ground of the motor is connected to the inverter frame), the ground current I_G refers to stator

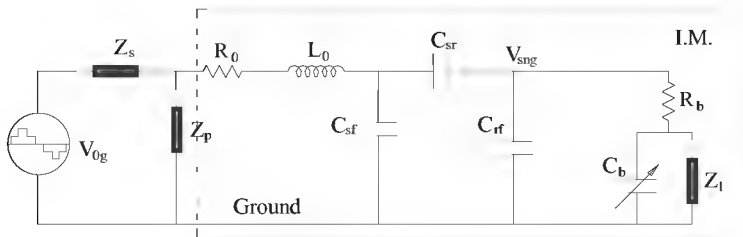


FIGURE 3.13 Common mode lumped equivalent circuit.

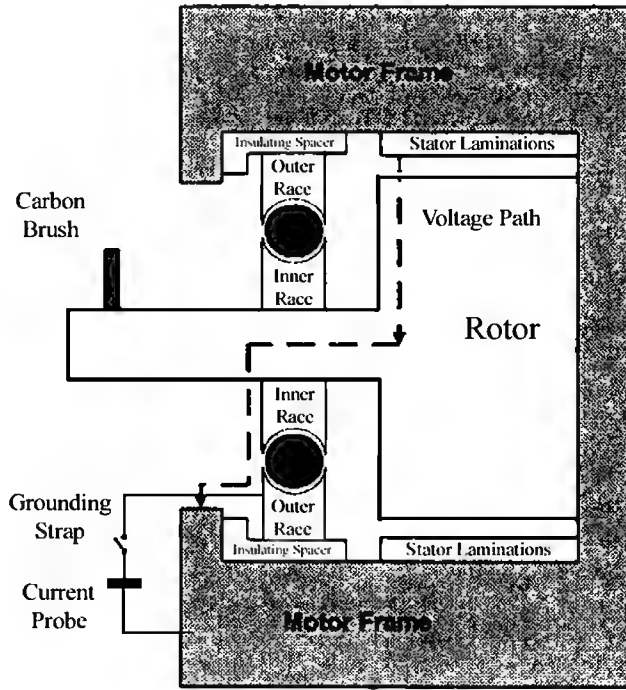


FIGURE 3.14 The test motor.

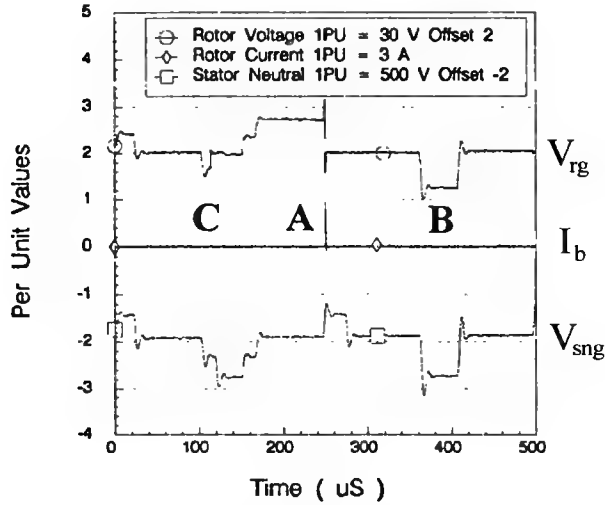


FIGURE 3.15 Bearing breakdown parameters. (After Ref. [2].)

winding to stator frame capacitance C_{sf} (Figure 3.16a). With an insulated bearing, but with both rotor brushes connected to the inverter frame, the measured current I_{AB} is related to stator winding to rotor coupling (C_{sr}) (Figure 3.16b).

In contrast, short-circuiting the bearing insulation sleeve (G') allows the measurement of initial (uninsulated) bearing current I_b (Figure 3.16c).

C_{sr} , C_{sf} , C_{rf} , R_b , and C_b can be identified through the experiments shown in Figure 3.16 [15].

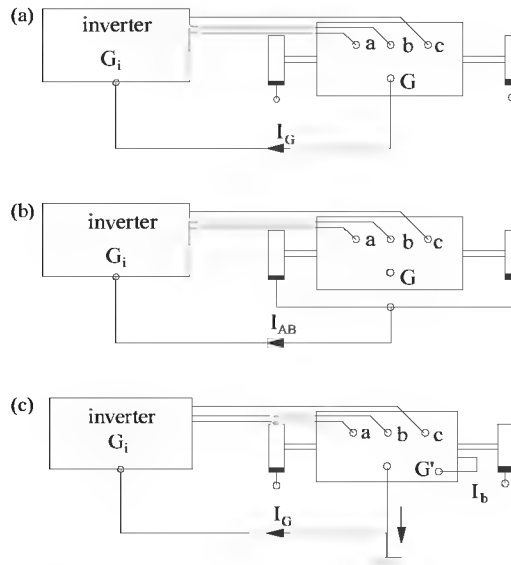


FIGURE 3.16 Capacitive coupling modes: (a) stator winding to stator, (b) stator winding to rotor, and (c) uninsulated bearing current I_b . (After Ref. [3].)

3.7 WAYS TO REDUCE PWM INVERTER BEARING CURRENTS

Reducing the shaft voltage V_{rg} to <1 to $1.5 V_{pk}$ is apparently enough to avoid EDM and thus eliminate bearing premature failure.

To do so, bearing currents should be reduced or their path be bypassed by larger capacitance path, that is, increasing C_{sf} or decreasing C_{sr} .

Three main practical procedures have evolved [14] so far:

- Properly insulated bearings (C_b decrease)
- Conducting tape on the stator in the airgap (to reduce C_{sr})
- Copper slot stick covers (or paint) and end windings shielded with nomex rings and covered with copper tape and all connected to ground (to reduce C_{sr}) (Figure 3.17).

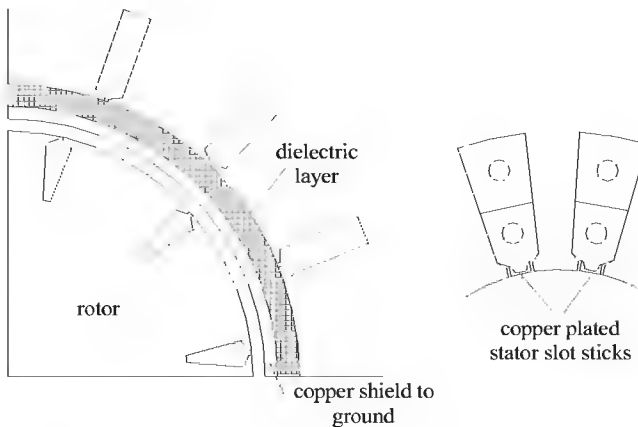


FIGURE 3.17 Conductive shields to bypass bearing currents (C_{sr} is reduced).

Various degrees of shaft voltage attenuation rates (from 50% to 100%) have been achieved with such methods depending on the relative area of the shields. Shaft voltages close to NEMA (National Electrical Manufacturers Association) specifications have been obtained.

The conductive shields do not affect notably the machine temperatures.

Besides the EDM discharge bearing current, a kind of circulating bearing current that flows axially through the stator frame has been identified [16]. Essentially, a net high-frequency axial flux is produced by the difference in stator coil end currents due to capacitance current leaks along the stack length between conductors in slots and the magnetic core. However, the relative value of this circulating current component proves to be small in comparison with EDM discharge current.

3.8 SUMMARY

- Super-high-frequency models for IMs are required to assess the voltage surge effects due to switching operations or PWM inverter-fed operation modes.
- The PWM inverter produces 0.2–2 μs rise time voltage pulses in both differential of common modes. These two modes seem independent of each other and, due to multiple reflection with long feeder cables, may reach up to 3 p.u. D.C. voltage levels.
- The distribution of voltage surges within the IM-repetitive in case of PWM inverters along the stator windings is not uniform. Most of the voltage drops along the first two line-end coils and particularly along their first 1–3 turns.
- The common mode voltage pulses seem to produce also premature bearing failure through EDM. They also produce most of electromagnetic interference effects.
- To describe the global response of IM to voltage surges, the differential impedance Z_m , the neutral impedance Z_{ng} , and the common voltage (or ground) impedance Z_{og} are defined through pertinent stator winding connections (Figure 3.3) in order to be easily estimated through direct measurements.
- The frequency response tests performed with RLC analysers suggest simplified lumped parameter equivalent circuits.
- For the differential impedance Z_m , a resonant parallel circuit with high-frequency capacitance C_{hf} and eddy current resistance R_{hf} in series is produced. The second R_{lf} , L_{lf} branch refers to lower frequency modelling (1–50 kHz or so). Such parameters are shown to vary almost linearly with motor power (Table 3.1). Care must be exercised, however, as there may be notable differences between $Z_m(p)$ from different manufacturers, for given power.
- The neutral (Z_{ng}) and ground (common voltage) Z_{og} impedances stem from the phase lumped equivalent circuit for high frequencies (Figure 3.8). Its components are derived from frequency responses through some approximations.
- The phase circuit boils down to two ground capacitors (C_g) with a high-frequency inductance in between, L_d . C_d and L_d are shown to vary simply with motor power ((3.15)–(3.16)). Skin effects components may be added.
- To model the voltage surge distribution along the stator windings, a distributed high-frequency equivalent circuit is necessary. Such a circuit should visualize turn-to-turn and turn-to-ground capacitances, turn-to-turn inductances, and eddy current resistances. The computation of such distributed parameters has been attempted by FEM, for super-high frequencies (1 MHz or so). The electromagnetic and electrostatic field is nonzero only in the stator slots and in and around stator coil end connections.
- FEM-derived distributed equivalent circuits have been used to predict voltage surge distribution for voltage surges within 0–2 μs rising time. The first coil takes most of the voltage surge. Its first 1–3 turns particularly so. The percentage of voltage drop on the line-end coil decreases from 70% for 0.1 μs rising time voltage pulses to 30% for 1 μs rising time [17].

- Detailed tests with thoroughly tapped windings have shown that as the rising time t_{rise} increases, not only the first but also the second coil takes a sizeable portion of voltage surge [8]. Improved discharge resistance (by adding oxides) insulation of magnetic wires [1] together with voltage surge reduction is the main avenues to long insulation life in IMs.
- Bearing currents, due to common mode voltage surges, are deemed responsible for occasional bearing failures in PWM inverter-fed IMs, especially when fed through long power cables.
- The so-called shaft voltage V_{rg} is a good indicator of bearing-failure propensity. In general, V_{rg} values of 15 V_{pk} at room temperature and 6–10 V_{pk} in hot motors are enough to trigger the EDM which produces bearing fluting. In essence, the high voltage pulses reduce the lubricant nonlinear impedance and thus a large bearing current pulse occurs, which further advances the fluting process towards bearing failure.
- Insulated bearings or Faraday shields (in the airgap or on slot taps, covering also the end connections) are practical solutions to avoid large bearing EDM currents. They increase the bearings life [18].
- A diverting capacitor for bearing currents has been introduced recently [19].
- Given the complexity and practical importance of super-high-frequency behaviour of IMs, much progress is expected in the near future [20], including pertinent international test standards.
- Reference [21] presents another universal high-frequency circuit model, valid for both star- and delta-phase connections and for common mode and differential mode effects with ample details and experimental proof.

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4 Motor Specifications and Design Principles

4.1 INTRODUCTION

Induction motors are used to drive loads in various industries for powers from <100 W to 10 MW and more per unit. Speeds encountered go up to tens of thousands of rpm.

There are two distinct ways to supply an induction motor to drive a load:

- Constant voltage and frequency (constant V and f) – power grid connection
- Variable voltage and frequency – PWM static converter connection.

The load is represented by its shaft torque–speed curve (envelope).

There are a few basic types of loads. Some require only constant speed (constant V and f supply), and others request variable speed (variable V and f supply).

In principle, the design specifications of the induction motor for constant and variable speeds are different from each other. Also, an existing motor, which was designed for constant V and f supply, may at some point in time be supplied from variable V and f supply for variable speed.

It is thus necessary to lay out the specifications for constant and variable V and f supply and check if the existing motor is the right choice for variable speed. Selecting an induction motor for the two cases requires special care.

Design principles are common to both constant and variable speeds. However, for the latter case, different specifications with machine special design constraints or geometrical aspects (rotor slot geometry, for example) lead to different final configurations. That is, induction motors designed for PWM static converter supplies are different.

It seems that in the near future, more and more IMs will be designed and fabricated for variable speed applications.

4.2 TYPICAL LOAD SHAFT TORQUE/SPEED ENVELOPES

Load shaft torque/speed envelopes may be placed in the first quadrant or in the second to the fourth quadrants (Figure 4.1a and b).

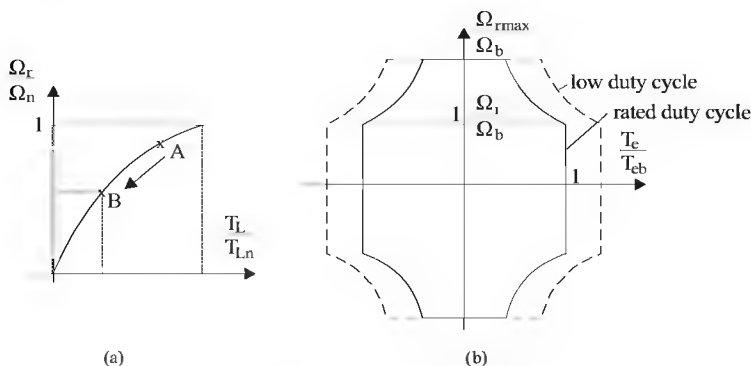


FIGURE 4.1 Single (a) and multiquadrant (b) load speed/torque envelopes.

Constant V- and f-fed induction motors may be used only for single-quadrant load torque/speed curves.

In modern applications (high-performance machine tools, robots, elevators, electric vehicles), multiquadrant operation is required. In such cases, only variable V and f (PWM static converter)-fed IMs are adequate.

Even in single-quadrant applications, variable speed may be required (from point A to point B in Figure 4.1a) in order to reduce energy consumption for lower speeds, by supplying the induction machine (IM) through a PWM static converter at variable V and f (Figure 4.2).

The load torque/speed curves may be classified into three main categories:

- Squared torque (centrifugal pumps, fans, mixers, etc.):

$$T_L = T_{Ln} \left(\frac{\Omega_r}{\Omega_n} \right)^2 \quad (4.1)$$

- Constant torque (conveyors, rollertables, elevators, extruders, cement kilns, etc.):

$$T_L = T_{Ln} = \text{Constant} \quad (4.2)$$

- Constant power (electric vehicles, etc.)

$$\begin{aligned} T &= T_{Lb} & \text{for } \Omega_r \leq \Omega_b \\ T &= T_{Lb} \frac{\Omega_b}{\Omega_r} & \text{for } \Omega_r > \Omega_b \end{aligned} \quad (4.3)$$

A generic view of the torque/speed envelopes for the three basic loads is shown in Figure 4.3.

The load torque/speed curves of Figure 4.3 show a marked diversity, and, especially, the power/speed curves indicate that the induction motor capability to meet them depends on the motor torque/speed envelope and on the temperature rise for the rated load duty cycle.

There are two main limitations concerning the torque/speed envelope deliverable by the induction motor. The first one is the mechanical characteristic of the IM itself, and the second is the temperature rise.

For a general-purpose design induction motor, when used with variable V and f supply, the torque/speed envelope for continuous duty cycle for self-ventilation (ventilator on shaft) and separate ventilator (constant speed ventilator), respectively, is shown in Figure 4.4.

Sustained operation at large torque levels and low speed is admitted only with separate (constant speed) ventilator cooling. The decrease of torque with speed reduction is caused by temperature constraints.

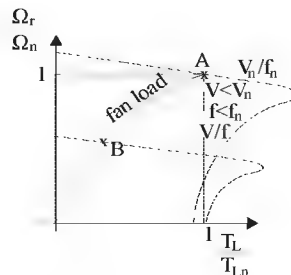


FIGURE 4.2 Variable V/f for variable speed in single-quadrant operation.

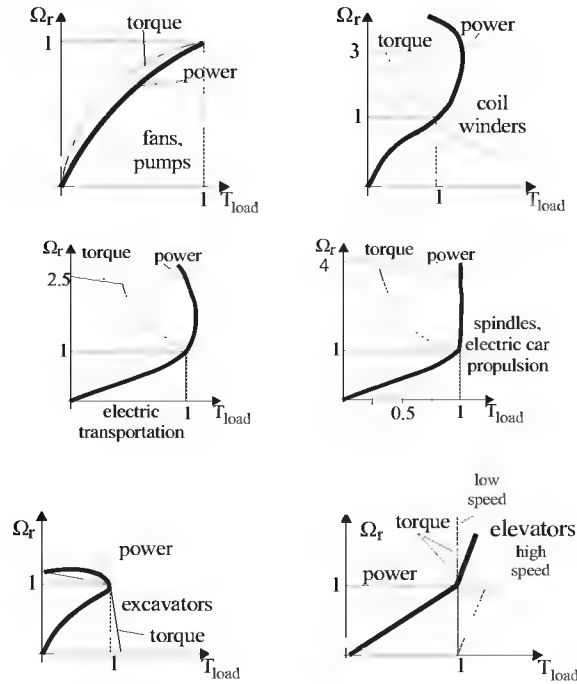


FIGURE 4.3 Typical load speed/torque curves (first quadrant shown).

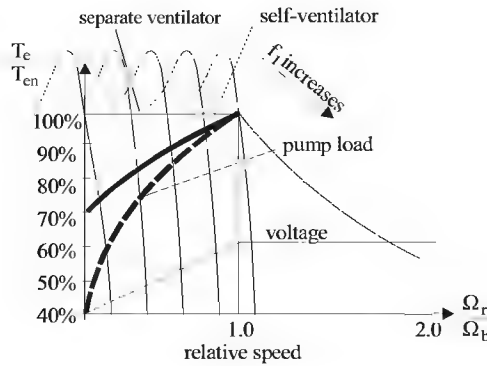


FIGURE 4.4 Standard induction motor torque/speed envelope for variable V and f supply.

As shown in Figure 4.4, the quadratic torque load (pumps, ventilators torque/speed curve) falls below the motor torque/speed envelope under rated speed (torque). For such applications only self-ventilated IM design is required.

Not so for servodrives (e.g. machine tools) where sustained operation at low speed and rated torque is necessary.

A standard motor capable of producing the extended speed/torque of Figure 4.4 has to be fed through a variable V and f source (a PWM static converter) whose voltage and frequency have to vary with speed as shown in Figure 4.5.

The voltage ceiling of the inverter is reached at base speed Ω_b . Above Ω_b , constant voltage is applied for increasing frequency. How to manage the IM flux linkage (rotor flux) to yield the maximum speed/torque envelope is a key point in designing an IM for variable speed.

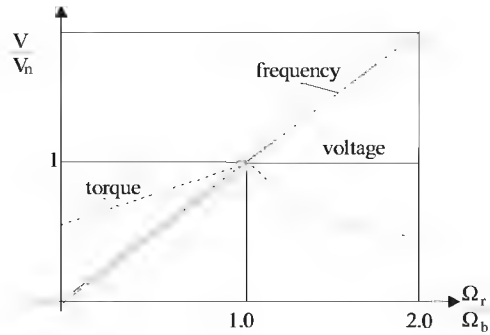


FIGURE 4.5 Voltage and frequency versus speed.

4.3 DERATING DUE TO VOLTAGE TIME HARMONICS

Derating is required when an induction motor designed for sinusoidal voltage and constant frequency is supplied from a power grid which has a notable voltage harmonic content due to increasing use of PWM static converters for other motors or due to its supply from similar static power converters. In both cases, the time harmonic content of motor input voltages is the cause of additional winding and core losses (as shown in Chapter 11, Vol. I). Such additional losses for rated power (and speed) would mean higher than rated temperature rise of windings and frame. To maintain the rated design temperature rise, the motor rating has to be reduced.

The increase in switching frequency in recent years for PWM static power converters for low- and medium-power IMs has led to a significant reduction in voltage time harmonic content at motor terminals. Consequently, the derating has been reduced. NEMA 30.01.2 suggests derating the induction motor as a function of harmonic voltage factor (HVF) (Figure 4.6).

Reducing the HVF via power filters (active or passive) becomes thus a priority as the variable speed drives extension becomes more and more important.

In a similar way, when IMs designed for sinewave power source are fed from IGBT PWM voltage source inverters, typical for induction motors now up to 5 MW or more (as of today), a certain derating is required as additional winding and core losses due to voltage harmonics occur.

This derating is not yet firmly standardized, but it should be more important when power increases as the switching frequency decreases. A value of up to 10% derating for such a situation is now common practice.

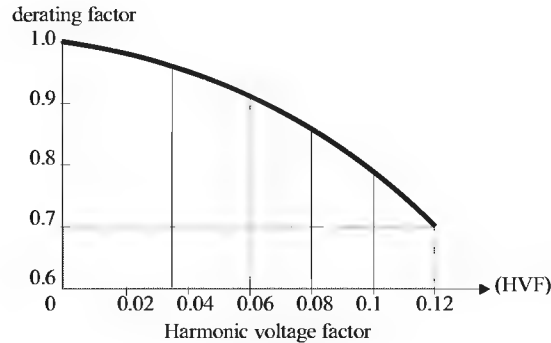


FIGURE 4.6 Derating for harmonic content of standard motors operating on sinewave power with harmonic content.

When using an IM fed from a sinewave power source with line voltage V_L through a PWM converter, the motor terminal voltage is somewhat reduced with respect to V_L due to various voltage drops in the rectifier and inverter power switches, etc.

The reduction factor is 5%–10% depending on the PWM strategy in the converter.

4.4 VOLTAGE AND FREQUENCY VARIATION

When matching an induction motor to a load, a certain supply voltage reduction has to be allowed for which the motor is still capable to produce rated power for a small temperature rise over rated value. A value of voltage variation of $\pm 10\%$ of rated value at rated frequency is considered appropriate (NEMA 12.44).

Also, a $\pm 5\%$ frequency variation at rated voltage is considered acceptable. A combined 10% sum of absolute values, with a frequency variation of $< 5\%$, has to be also handled successfully. As expected in such conditions, the motor rated speed, efficiency, and power factor, for rated power, will be slightly different from rated label values.

Through their negative sequence, unbalanced voltages may produce additional winding stator and rotor losses. In general, a 1% unbalance in voltages would produce a 6%–10% unbalance in phase currents.

The additional winding losses occurring this way would cause notable temperature increases unless the IM is derated (NEMA, Figure 4.7). A limit of about 1% in voltage unbalance is recommended for medium- and high-power motors.

4.5 SPECIFYING INDUCTION MOTORS FOR CONSTANT V AND f

Key information pertaining to motor performance, construction, and operating conditions is provided for end users' consideration when specifying induction motors.

National (NEMA in the USA [1]) and international (IEC in Europe) standards deal with such issues, to provide harmonization between manufacturers and users worldwide.

Table 4.1 summarizes most important headings and the corresponding NEMA section.

Among these numerous specifications, which show the complexity of IM design, nameplate markings are of utmost importance.

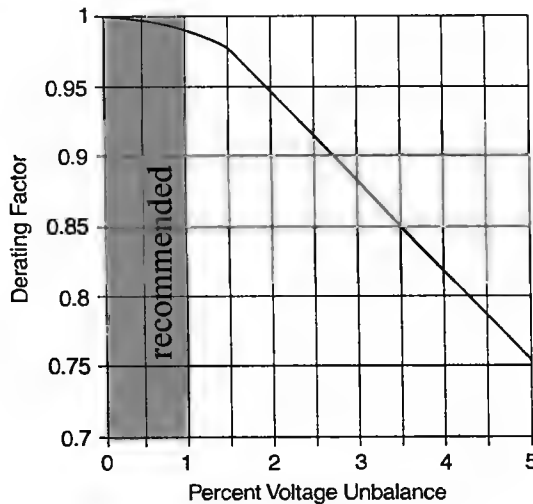


FIGURE 4.7 Derating due to voltage unbalance in %.

TABLE 4.1
NEMA Standards for Three-Phase IMs (with Cage Rotors)

Heading	NEMA Section
Nameplate markings	NEMA MG – 1 10.40
Terminal markings	NEMA MG – 1 2.60
NEMA size starters	
NEMA enclosure types	
Frame dimensions	NEMA MG – 1 11
Frame assignments	NEMA MG – 1 10
Full load current	NEC Table 430 – 150
Voltage	NEMA MG – 1 12.44, 14.35
Impact of voltage, frequency variation	
Code letter	NEMA MG – 1 10.37
Starting	NEMA MG – 1 12.44, 54
Design letter and torque	NEMA MG – 1 12
Winding temperature	NEMA MG – 1 12.43
Motor efficiency	NEMA MG – 12 – 10
Vibration	NEMA MG – 17
Testing	NEMA MG – 112, 55, 20, 49 / IEEE-112B
Harmonics	NEMA MG – 1 30
Inverter applications	NEMA MG – 1, 30, 31

The following data are given on the nameplate:

- a. Designation of manufacturer's motor type and frame
- b. kW (HP) output
- c. Time rating
- d. Maximum ambient temperature
- e. Insulation system
- f. RPM at rated load
- g. Frequency
- h. Number of phases
- i. Rated load amperes
- j. Line voltage
- k. Locked-rotor amperes or code letter for locked-rotor kVA per HP for motors of ½ HP or more
- l. Design letter (A, B, C, D)
- m. Nominal efficiency
- n. Service factor load if other than 1.0
- o. Service factor amperes when service factor exceeds 1.15
- p. Over-temperature protection followed by a type number. when over-temperature device is used
- q. Information on dual voltage and frequency operation option.

Rated power factor does not appear on NEMA nameplates, but it does so according to most European standards.

Efficiency [2,3] is perhaps the most important specification of an electric motor as the cost of energy per year even in a 1 kW motor is notably higher than the motor initial cost. Also, a 1% increase in efficiency saves energy whose costs in 3–4 years cover the initial extra motor costs.

Standard and high-efficiency IM classes have been defined and standardized by now world-wide. As expected, high-efficiency (class E) induction motors have higher efficiency than standard motors, but their size, initial cost, and locked-rotor current are higher. This latter aspect places an additional burden on the local power grid when feeding the motor upon direct starting. If soft-starting or inverter operation is used, the higher starting current does not have any effect on the local power grid rating. NEMA defines specific efficiency levels for Design B IMs (Table 4.3).

On the other hand, EU established three classes EFF1, EFF2, and EFF3 of efficiencies, giving the manufacturers an incentive to qualify for the higher classes (Table 4.2).

The torque/speed curves reveal, for constant V-/f-fed IMs, additional specifications such as starting torque, pull-up, and breaking torque for the four classes (letters: A, B, C, and D designs) of induction motors (Figure 4.8).

The performance characteristics of the B, C, and D designs from NEMA Table 2.1 with their typical applications are summarized in Table 4.3.

EFF4 class of premium efficiency was defined by IEC, but so far, only R&D IM prototypes with PM assistance apparently qualified.

4.6 MATCHING IMS TO VARIABLE SPEED/TORQUE LOADS

As IMs are, in general, designed for 60(50) Hz, when used for variable speed, with variable V and f supply, they operate at variable frequency. Below the rated frequency, the machine is capable of full flux linkage, whereas above that, flux weakening occurs.

For given load speed and load torque, with variable V and f supply we may use IMs with $2p_1 = 2, 4, 6$. Each of them, however, works at a different (distinct) frequency.

Figures 4.9 shows the case of quadratic torque (pump) load with the speed range of 0–2000 rpm, load of 150 kW at 2000 rpm, 400 V, 50 Hz (network). Two different motors are used: one of two poles and one of four poles.

At 2000 rpm, the two-pole IM works at 33.33 Hz – with full flux, whereas the four-pole IM operates at 66.66 Hz in the flux-weakening zone. Which of the two motors is used is decided by the motor costs and losses. Note, however, that the absolute torque (in Nm) of the motor has to be the same in both cases.

For a constant torque load (extruder) with the speed range of 300–1100 rpm, 50 kW at 1200 rpm (network: 400 V, 50 Hz), two motors compete. One, of four poles, will work at 40 Hz and the other, of six poles, at 60 Hz (Figure 4.10).

Again, both motors can satisfy the specifications for the entire speed range as the load torque is below the available motor torque. Again the torque in Nm is the same for both motors, and the choice between the two motors is given by motor costs and total losses.

While starting torque and current are severe design constraints for IMs designed for constant V and f supply, they are not for variable V and f supply.

Skin effect is important for constant V and f supply as it reduces the starting current and increases the starting torque. In contrast to this, for variable V and f supply, skin effect is to be reduced, especially for high-performance speed control systems.

Breakdown torque may become a much more important design factor for variable V and f supply, when a large speed zone for constant power is required. A spindle drive or an electric car drive may require more than three to one constant power speed range (Figure 4.11).

The peak torque of IM is approximately

$$T_{ek} \approx 3 \left(\frac{V_{phn}}{2\pi f_{in}} \right)^2 \left(\frac{f_{in}}{f_1} \right)^2 \frac{p_1}{2L_{sc}} = T_{ekf_{in}} \left(\frac{f_{in}}{f_1} \right)^2 \quad (4.4)$$

The peak torque for constant (rated) voltage is inversely proportional to frequency squared. To produce a 4/1 constant power speed range, the peak torque has to be four times the rated torque. Only in this case, the motor may produce at $f_{imax} = 4f_{in}$, 25% of rated torque.

TABLE 4.2
460 V, Four-Pole, Open-Frame Design B Performance and NEMA-Defined Performance

HP	Full Load Amperes (FU) per NEC Table 430-150	Maximum Locked- Rotor Amperes (IRA)		Nominal Full Load Efficiencies (%) per NEMA MG 1, Tables 12.10 and 12.11 (Open Four Pole)		Efficiency Ratio:		Minimum Locked-Rotor Torque (%) per NEMA MG 1, Tables 12.2 and 12.38.4		Minimum Breakdown Torque (%) per NEMA MG 1 Tables 12.39.1 and 12.39.3		Minimum Pull-up Torque (%) per NEMA MG 1 Tables 12.40.1 and 12.40.3	
		Design B	Design E	Design B ^a	Design E	Design B	Design E	Design B	Design E	Design B	Design E	Design B	Design E
3	4.8	32	37	86.5	89.5	1.04	1.04	215	180	250	200	150	120
5	7.6	46	61	87.5	90.2	1.03	1.03	185	170	225	200	130	120
7½	11	64	92	88.5	91	1.03	1.03	175	160	215	200	120	110
10	14	81	113	89.5	91.7	1.03	1.03	165	160	200	200	115	110
15	21	116	169	91	92.4	1.02	1.02	160	150	200	200	110	110
20	27	145	225	91	93	1.02	1.02	150	150	200	200	105	110
25	34	183	281	91.7	93.6	1.09	1.09	150	140	200	190	105	100
30	40	218	337	92.4	94.1	1.02	1.02	150	140	200	190	105	100
40	52	290	412	93	94.5	1.02	1.02	140	130	200	190	100	100
50	65	363	515	93	95.4	1.03	1.03	140	130	200	190	100	100
60	77	435	618	93.6	95.4	1.02	1.02	140	120	200	180	100	90
75	96	543	723	94.1	95.4	1.01	1.01	140	120	200	180	100	90
100	124	725	937	94.1	95.4	1.01	1.01	125	110	200	180	100	80
125	156	908	1171	94.5	95.4	1.01	1.01	110	110	200	180	100	80
150	180	1085	1405	95	95.8	1.01	1.01	110	100	200	170	100	80
200	240	1450	1873	95	95.8	1.01	1.01	100	100	200	170	90	80
250	302	1825	2344	95.4	96.2	1.01	1.01	80	90	175	170	75	70
300	361	2200	2809	95.4	96.2	1.01	1.01	80	90	175	170	75	70
350	414	2550	3277	95.4	96.5	1.01	1.01	80	75	175	160	75	60
400	477	2900	3745	95.4	96.5	1.01	1.01	80	75	175	160	75	60
450	515	3250	4214	95.8	96.8	1.01	1.01	80	75	175	160	75	60
500	590	3625	4682	95.8	96.8	1.01	1.01	80	75	175	160	75	60

^a Applies to induction motors labelled "Premium Efficiency" or "Energy Efficient"; Design E was abandoned in 2000, but the comparison holds.

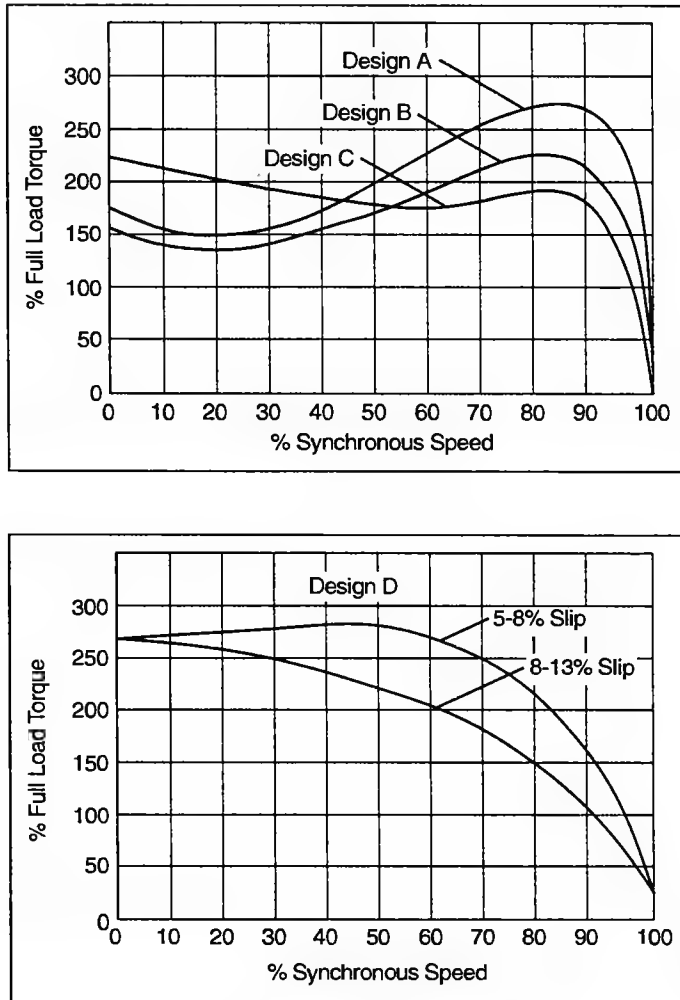


FIGURE 4.8 (a) NEMA designs A, B, C, and D (b) torque/speed curves.

Consequently, if the load maximum torque is equal to the rated torque, then at $4f_{in}$, the rated power is still produced.

In reality, a breakdown torque of 400% is hardly practical. However, efforts to reduce the short-circuit leakage inductance (L_{sc}) have led up to 300% breakdown torque.

So there are two solutions to provide the required load torque/speed envelope: increasing the motor rating (size) and costs and increasing the flux (voltage) level in the machine by switching from star to delta connection (or by reducing the number of turns per phase by switching off part of stator coils).

The above rationale was intended to suggest some basic factors that guide the IM design.

Relating the specifications to a dedicated machine geometry is the object of design (or dimensioning). This enterprise might also be called sizing the IM.

Because there are many geometrical parameters and their relationships to specifications (performance) are in general nonlinear, the design process is so complicated that it is still a combination of art and science, based solidly on existing experience (motors) with tested (proven) performance. In the process of designing an induction motor, we will define hereby a few design factors, features, and sizing principles.

TABLE 4.3
Motor Designs after NEMA and EU

Classification	Locked-Rotor Torque (% Rated Load Torque)	Breakdown Torque (% Rated Load Torque)	Locked-Rotor Current (% Rated Load Current)	Slip %	Typical Applications	Rel. η
Design B Normal locked-rotor torque and normal locked-rotor current	70–275 ^a	175–300 ^a	600–700	0.5–5	Fans, blowers, centrifugal pumps and compressors, motor–generator sets, etc., where starting torque requirements are relatively low	Medium or high
Design C High locked-rotor torque and normal locked-rotor current	200–250 ^a	190–225 ^a	600–700	1–5	Conveyors, crushers, stirring machines, agitators, reciprocating pumps and compressors, etc., where starting under load is required	Medium
Design D High locked-rotor torque and high slip	275	275	600–700		High peak loads with or without fly wheels, such as punch presses, shears, elevators, extractors, winches, hoists, oil well pumping and wire drawing machines	Medium

Note: Design A motor performance characteristics are similar to those for Design B except that the locked-rotor starting current is higher than the values shown in the table.

^a Higher values are for motors having lower horsepower ratings.

IEC60034-30 EuP Directive 2005/32/EC	Europe (50 Hz) CEMEP Voluntary Agreement	US (60 Hz) EPAct	Others Similar Local Regulations, for Example, in Countries Like
IE1 Standard efficiency	Comparable to EFF2	Below standard efficiency	AS in Australia
IE2 High efficiency	Comparable to EFF1	Identical to NEMA Energy efficiency/EPACT	NBR in Brazil GB/T in China
IE3 Premium efficiency	Extrapolated IE2 with 10%–15% lower losses	Identical to NEMA Premium efficiency	IS in India JIS in Japan MEPS in Korea

4.7 DESIGN FACTORS

Factors that influence notably the IM design are as follows.

4.7.1 COSTS

Costs in most cases is the overriding consideration in IM design. But how we define costs? It may be the costs of active materials with or without the fabrication costs. Fabrication costs depend on machine size, materials available or not in stock, manufacturing technologies, and manpower costs.

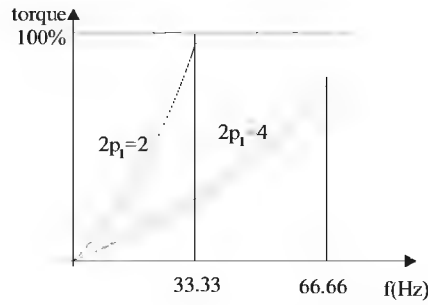


FIGURE 4.9 Torque versus motor frequency (and speed) pump load.

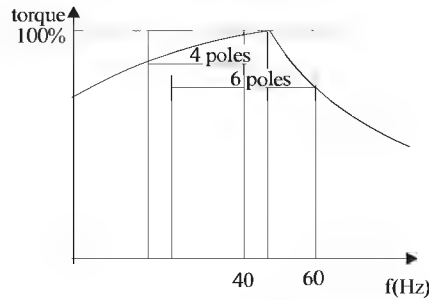


FIGURE 4.10 Torque versus motor frequency (and speed): constant torque load.

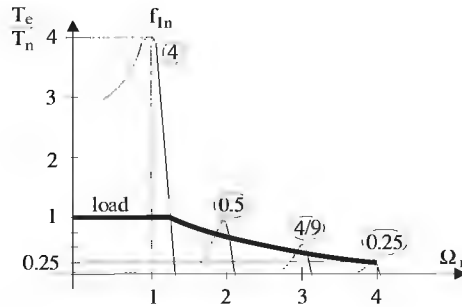


FIGURE 4.11 Induction motor torque/speed curves for various values of frequency and a 4/1 constant power speed range.

The costs of capitalized losses per entire motor active life surpass quite a few times the initial motor costs. So loss reduction (through higher efficiency or via variable V and f supply) pays off generously. This explains the rapid expansion of variable speed drives with IMs worldwide.

Finally, maintenance costs are also important but not predominant. We may now define the global costs of an IM as

$$\begin{aligned} \text{Global costs} = & \text{Material costs} + \text{Fabrication and selling costs} \\ & + \text{Losses capitalized costs} + \text{Maintenance costs} \end{aligned} \quad (4.5)$$

Global costs are also a fundamental issue when we have to choose between repairing an old motor or replacing it with a new motor (with higher efficiency and corresponding (higher) initial costs).

4.7.2 MATERIAL LIMITATIONS

The main materials used in IM fabrication are magnetic-steel laminations, copper and aluminium for windings, and insulation materials for windings in slots.

Their costs are commensurate with performance. Progress in magnetic and insulation materials has been continuous. Such new improved materials drastically affect the IM design (geometry), performance (efficiency), and costs.

Flux density, $B(T)$, losses (W/kg) in magnetic materials, current density J (A/mm^2) in conductors, and dielectric rigidity E (V/m) and thermal conductivity of insulation materials are key factors in IM design.

4.7.3 STANDARD SPECIFICATIONS

IM materials (lamination thickness, conductor diameter), performance indexes (efficiency, power factor, starting torque, starting current, breakdown torque), temperature by insulation class, frame sizes, shaft height, cooling types, service classes, protection classes, etc. are specified in national (or international) standards (NEMA, IEEE, IEC, EU, etc.) to facilitate globalization in using induction motors for various applications. They limit, to some extent, the designer's options but provide solutions that are widely accepted and economically sound.

4.7.4 SPECIAL FACTORS

In special applications, special specifications such as minimum weight and maximum reliability in aircraft applications become the main concern. Transportation applications require ease of maintaining, high reliability, and good efficiency. Circulating water home pumps requires low noise, highly reliable induction motors.

Large compressors have large inertia rotors, and thus, motor heating during frequent starts is severe. Consequently, maximum starting torque/current becomes the objective function.

4.8 DESIGN FEATURES

The major issues in designing an IM may be divided into five areas: electrical, dielectric, magnetic, thermal and mechanical.

- *Electrical design*

To supply the IM, the supply voltage, frequency, and number of phases are specified. From these data, the minimum power factor, and a target efficiency, the phase connection (start or delta), winding type, number of poles, slot numbers, and winding factors are calculated. Current densities (or current sheets) are imposed.

- *Magnetic design*

Based on output coefficients, power, speed, number of poles, and type of cooling, the rotor diameter is calculated. Then, based on a specific current loading (in A/m) and airgap flux density, the stack length is determined.

Fixing the flux densities in various parts of the magnetic circuit with given current densities and slot mmfs, the slot sizing, core height, and external stator diameter D_{out} are all calculated. After choosing D_{out} , which is standardized, the stack length is modified until the initial current density in slot is secured.

It is evident that sizing the stator and rotor core is carried out in many ways based on various criteria.

- *Insulation design*

Insulation material and its thickness, be it slot/core insulation, conductor insulation, end connection insulation, or terminal leads insulation, depend on machine voltage insulation class and on the environment in which the motor operates.

There are low line voltage (400 V/50Hz, 230 V/60Hz, 460 V/60Hz, 690 V/60Hz) or less or medium voltage machines (2.3 kV/60Hz, 4 kV/50Hz, 6 kV/50Hz). When PWM converter-fed IMs are used, care must be exercised in reducing the voltage stress on the first 20% of phase coils or to enforce their insulation or to use random wound coils.

- *Thermal design*

Extracting the heat caused by losses from the IM is imperative to keep the windings, core, and frame temperatures within safe limits. Depending on application or power level, various types of cooling are used. Air cooling is predominant, but stator water cooling of high-speed IMs (above 10,000rpm) is frequently used. Thermal designing involves calculating the loss and temperature distribution, and sizing the cooling system.

- *Mechanical design*

Mechanical design refers to critical rotating speed, noise and vibration modes, mechanical stress in the shaft, and its deformation, displacement, bearings design, inertia calculation, and forces on the winding end coils during most severe current transients.

We mention here the output coefficient as an experience-proven theoretical approach to a tentative internal stator (stator bore) diameter calculation. The standard output coefficient is $D_{is}^2 L$ where D_{is} is the stator bore diameter and L the stack length.

Besides elaborating $D_{is}^2 L$, we introduce the rotor tangential stress σ_{tan} (in N/cm^2), that is the specific tangential force at rotor surface at rated and peak torque.

This specific force criterion may be used also for linear motors. It turns out that σ_{tan} varies from 0.2–0.3 N/cm^2 for 100 W IMs to <3–4 N/cm^2 for large IMs. Not so for the output coefficient $D_{is}^2 L$, which is related to rotor volume and thus increases steadily with torque (and power).

4.9 THE OUTPUT COEFFICIENT DESIGN CONCEPT

To calculate the relationship between the $D_{is}^2 L$ and the machine power and performance, we start by calculating the airgap apparent power S_{gap} :

$$S_{gap} = 3E_1 I_{ln} \quad (4.6)$$

where E_1 is the airgap emf per phase and I_{ln} rated current (RMS values).

Based on the phasor diagram with zero stator resistance ($R_s = 0$) (Figure 4.12),

$$\underline{I}_{ln} R_s - \underline{V}_{ln} = \underline{E}_1 - jX_{ls} \underline{I}_{ln} \quad (4.7)$$

or

$$K_E = \frac{E_1}{V_{ln}} \approx 1 - x_{ls} \cdot \sin \phi_1 \quad (4.8)$$

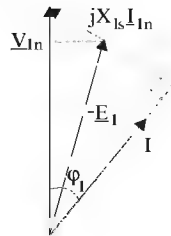


FIGURE 4.12 Simplified phasor diagram.

with

$$x_{es} = \frac{X_{ls} I_{ln}}{V_{ln}} \quad (4.9)$$

The p.u. value of stator leakage reactance increases with pole pairs p_1 and so does $\sin \phi_1$ (power factor decreases when p_1 increases).

$$K_F \approx 0.98 - 0.005p_1 \quad (4.10)$$

Also, the input apparent power S_{ln} is

$$S_{ln} = 3V_{ln}I_{ln} = \frac{P_n}{\eta_n \cos \phi_{ln}} \quad (4.11)$$

where P_n is the rated output power, and η_n and $\cos \phi_{ln}$ are the assigned values of rated efficiency and power factor based on past experience.

Typical values of efficiency for Design B (NEMA) are given in Table 4.3. Each manufacturer has its own set of data.

Efficiency increases with power and decreases with the number of poles. Efficiency of wound rotor IMs is slightly larger than that of cage-rotor IMs of the same power and speed because the rotor windings are made of copper and the total additional load (stray) losses are lower.

As efficiency is defined with stray losses p_{stray} of 0.5(1.0)% of rated power in Europe (still!) and with the latter (p_{stray}) measured in direct load tests in USA, differences in actual losses (in IMs of the same power and nameplate efficiency) of even more than 20% may be encountered when motors fabricated in Europe are compared with those made in the USA. Recently, IEC adopted the stray load loss estimation from load tests very similar to IEEE standard.

Anyways, the assigned value of efficiency is only a starting point for design as iterations are performed until the best performance is obtained.

The power factor also increases with power and decreases with the number of pole pairs with values slightly smaller than corresponding efficiency for existing motors. More data on initial efficiency and power factor data will be given in subsequent chapters on design methodologies.

The emf E_l is written as a function of airgap pole flux ϕ :

$$E_l = 4f_l K_f W_l K_{wl} \phi \quad (4.12)$$

where f_l is frequency, $1.11 > K_f > 1.02$ form factor (dependent on teeth saturation) (Figure 4.13), W_l is turns per phase, K_{wl} is the winding factor, and ϕ is the pole flux.

$$\phi = \alpha_i \tau L B_g \quad (4.13)$$

where α_i is the flux density shape factor dependent on the magnetic saturation coefficient of teeth (Figure 4.13), and B_g is the flux density in the airgap. The pole pitch τ is

$$\tau = \frac{\pi D_{is}}{2p_1}; \quad n_1 = \frac{f_l}{p_1} \quad (4.14)$$

Finally, S_{gap} is

$$S_{gap} = K_f \alpha_i K_{wl} \pi^2 D_{is}^2 L \frac{n_1}{60} A_l B_g \quad (4.15)$$

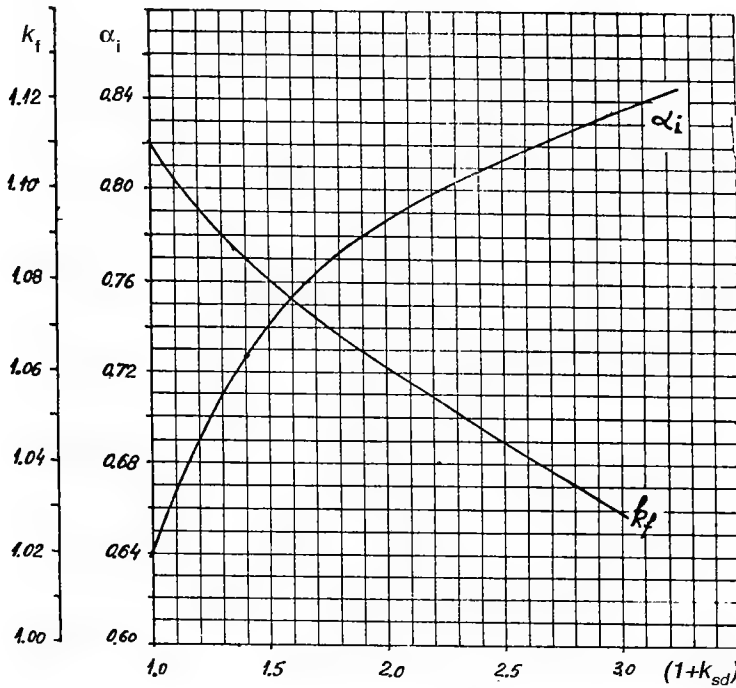


FIGURE 4.13 Form factor K_f and flux density shape factor α_i versus teeth saturation.

The specific stator current load A_1 (A/m) is

$$A_1 = \frac{6W_1 I_{1n}}{\pi D_{is}} \quad (4.16)$$

We separate the volume utilization factor C_0 (Esson's constant) as

$$C_0 = K_f \alpha_i K_{w1} \pi^2 A_1 B_g = \frac{60 S_{gap}}{D_{is}^2 L n_1}; \quad n_1 \text{ in rpm} \quad (4.17)$$

C_0 is not in fact a constant as both the values of A_1 (A/m) and airgap flux density (B_g) increase with machine power and with the number of pole pairs.

The $D_{is}^2 L$ output coefficient may be calculated from (4.17) with S_{gap} from (4.6) and (4.11):

$$D_{is}^2 L = \frac{1}{C_0} \frac{60}{n_1} \frac{K_E P_n}{\eta_n \cos \phi_{1n}} \quad (4.18)$$

Typical values of C_0 as a function S_{gap} with pole pairs p_1 as parameter for low-power IMs is given in Figure 4.14, with K_E from (4.10).

The $D_{is}^2 L$ (internal) output constant (proportional to rotor core volume) is, in fact, almost proportional to machine rated shaft torque. Torque production requires apparently less volume as the pole pair number p_1 increases, and C_0 increases with increasing p_1 (Figure 4.14).

It is standard to assign also a value λ to the stack length-to-pole pitch ratio:

$$\lambda = \frac{L}{\tau} = \frac{2LP_1}{\pi D_{is}}; \quad 0.6 < \lambda < 3.0 \quad (4.19)$$

The stator bore diameter may now be calculated from (4.18) with (4.19).

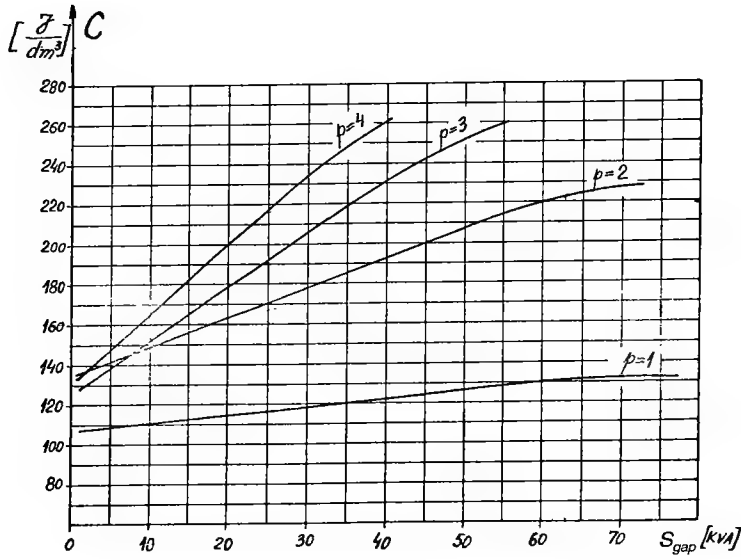


FIGURE 4.14 Esson's "constant" C_0 versus S_{gap} (airgap apparent power).

$$D_{\text{is}} = \sqrt[3]{\frac{2p_l}{\pi\lambda} \frac{1}{C_0} \frac{p_l}{f_l} \frac{K_E P_n}{\eta_n \cos \phi_{1n}}} \quad (4.20)$$

This is a standard design formula. However, it does not say enough on the machine total volume (weight). Moreover, in many designs, the stator external (frame internal) diameters are standardized.

A similar (external) output coefficient $D_{\text{out}}^2 L$ may be derived if we first adopt a design current density J_{con} (A/m²) and consider the slot fill factor (with conductors), $K_{\text{fill}} = 0.4\text{--}0.6$, given together with the tooth and stator back iron flux densities B_{ts} and B_{cs} .

With the known airgap flux density B_g and tooth flux density B_{ts} , the stator slot height h_s is approximately

$$h_s = \frac{6W_l I_n}{\frac{B_g}{B_{\text{ts}}} j_{\text{con}} K_{\text{fill}}} \frac{1}{\pi D_{\text{is}}} = \frac{A_l}{\frac{B_g}{B_{\text{ts}}} j_{\text{con}} K_{\text{fill}}} \quad (4.21)$$

Now the core (yoke) radial height h_{cs} is

$$h_{\text{cs}} = \frac{\phi}{2LB_{\text{cs}}} = \frac{\alpha_i}{2} \left(\frac{\pi D_{\text{is}}}{2P_l} \right) \frac{B_g}{B_{\text{cs}}} \quad (4.22)$$

The outer stator diameter D_{out} is

$$D_{\text{out}} = D_{\text{is}} + 2(h_s + h_{\text{cs}}) \quad (4.23)$$

We replace D_{is} from (4.23) in $D_{\text{is}}^2 L$ with h_s and h_{cs} from (4.21) and (4.22):

$$D_{\text{is}}^2 L = D_{\text{out}}^2 L \cdot f_0(D_{\text{is}}) \quad (4.24)$$

$$f_0(D_{is}) = \frac{1}{\left[1 + \frac{2(h_s + h_{cs})}{D_{is}}\right]^2} \quad (4.25)$$

And, finally,

$$f_0(D_{is}) = \frac{1}{\left[1 + \frac{2A_1 B_{ts}}{j_{con} K_{fill} B_g D_{is}} + \frac{\alpha_i}{2} \frac{\pi}{p_l} \frac{B_g}{B_{cs}}\right]^2} \quad (4.26)$$

From (4.24),

$$D_{out}^2 L = \frac{D_{is}^2 L}{f_0(D_{is})} = \frac{1}{C_0 f_0(D_{is})} \frac{P_l}{f_l} \frac{K_E P_n}{\eta_n \cos \phi_{ln}} \quad (4.27)$$

$$\text{As } L = \lambda \frac{\pi D_{is}}{2P_l} \quad (4.28)$$

$$D_{out} = \sqrt{\frac{2p_l^2}{\pi \lambda C_0 f_l} \frac{K_E P_n}{\eta_n \cos \phi_{ln}} \frac{1}{D_{is} f_0(D_{is})}} \quad (4.29)$$

Although (4.29) through the function $[D_{is} f_0(D_{is})]^{-1}$ suggests that a minimum D_{out} may be obtained for given λ , B_g/B_{co} , B_g/B_t , j_{con} , and A_1 , it seems more practical to use (4.29) to find the outer stator diameter D_{out} after the stator bore diameter was obtained from (4.20). Now if this value is not a standard one and a standard frame is a must, the aspect ratio λ is modified until D_{out} matches a standardized value.

The specific current loading A_1 depends on pole pitch τ and number of poles on D_{is} , once a certain cooling system (current density) is adopted.

In general, A_1 increases with D_{is} from values of $<10^3$ A/m for $D_{is} = 4 \cdot 10^{-2}$ m to 45,000 A/m for $D_{is} = 0.4$ m and $2p_l = 2$ poles. Smaller values are common for larger number of poles and same stator bore diameter.

On the other hand, the design current density j_{con} varies in the interval $j_{con} = (3.5-8.0)10^6$ A/m² for axial or axial-radial air cooling. Higher values are designated to high-speed IMs (lower pole pair numbers p_l) or for liquid cooling. While A_1 varies along such a large span and the slot height h_s -to-slot width b_s ratio is limited to $K_{aspect} = 3-6$, to limit the slot leakage inductance, using A_1 is avoided by calculating slot height h_s as

$$h_s = K_{aspect} b_s = K_{aspect} \frac{\pi D_{is}}{N_s} \left(1 - \frac{b_t}{\tau_{slot}}\right) = K_{aspect} \frac{\pi D_{is}}{N_s} \left(1 - \frac{B_g}{B_{ts}}\right) \quad (4.30)$$

Higher values of aspect ratios are typical to larger motors.

This way, $D_{out}^2 L$ is

$$D_{out}^2 L = \frac{1}{C_0} \frac{P_l}{f_l} \frac{K_E P_n}{\eta_n \cos \phi_{ln}} \left[1 + \frac{2K_{aspect} \pi}{N_s} \left(1 - \frac{B_g}{B_{ts}}\right) + \frac{\alpha_i}{2} \frac{\pi}{P_l} \frac{B_g}{B_{cs}}\right]^2 \quad (4.31)$$

TABLE 4.4**Outer Stator Diameter-to-Inner Stator Diameter Ratios**

$2p_1$	2	4	6	8	≥ 10
$\frac{D_{out}}{D_{is}}$	1.65–1.69	1.46–1.49	1.37–1.40	1.27–1.30	1.24–1.26

Also,

$$\frac{D_{out}}{D_{is}} \approx 1 + \frac{2K_{aspect}\pi}{N_s} \left(1 - \frac{B_g}{B_{ts}} \right) + \frac{\alpha_i}{2} \frac{\pi}{p_l} \frac{B_g}{B_{cs}} \quad (4.32)$$

To start, we may calculate D_{is}/D_{out} as a function of only pole pairs p_1 if $B_g/B_{ts} = ct$ and $B_g/B_{cs} = ct$, with K_{aspect} and N_s (slots/stator) also assigned corresponding values (Table 4.4).

The stack aspect ratio λ is assigned an initial value in a rather large interval: 0.6–3.

In general, longer stacks, allowing for a smaller stator bore diameter (for given torque), lead to shorter stator winding end connections, lower winding losses, and lower inertia, but the temperature rise along the stack length may become important. An optimal value of λ is highly dependent on IM design specifications and the design objective function taken into consideration. There are applications with space shape constraints that prevent the using of a long motor.

Example 4.1 Output Coefficient

Let us consider a 55 kW, 50 Hz, 400 V, $2p_1 = 4$ induction motor whose assigned (initial) rated efficiency and power factor are $\eta_n = 0.92$, $\cos \varphi_n = 0.92$.

Let us determine the stator internal and external diameters D_{out} and D_{is} for $\lambda = L/\tau = 1.5$.

Solution

The emf coefficient K_E (4.10) is $K_E = 0.98 - 0.005 \cdot 2 = 0.97$

The airgap apparent power S_{gap} (4.3) is

$$S_{gap} = 3K_E V_l I_{ln} = K_E \frac{P_n}{\cos \varphi_n \eta_n} = \frac{0.97 \cdot 55 \cdot 10^3}{0.92 \cdot 0.92} = 63.03 \cdot 10^3 \text{ VA}$$

Esson's constant C_0 is obtained from Figure 4.14 for $p_1 = 2$ and $S_{gap} = 63.03 \cdot 10^3 \text{ VA}$: $C_0 = 222 \cdot 10^3 \text{ J/m}^3$.

For an airgap flux density $B_g = 0.8 \text{ T}$, $K_{w1} = 0.955$, $\alpha_i = 0.74$, $K_t = 1.08$ (teeth saturation coefficient $1 + K_{st} = 1.5$, Figure 4.13). The specific current loading A_1 is (4.17)

$$A_1 = \frac{C_0}{K_t \alpha_i K_{w1} \pi^2 B_g} = \frac{222 \cdot 10^3}{1.08 \cdot 0.74 \cdot 0.955 \cdot 0.8 \pi^2} = 36.876 \cdot 10^3 \text{ A/m}$$

with $\lambda = 1.5$ from (4.20), the stator internal diameter D_{is} is obtained.

$$D_{is} = \sqrt[3]{\frac{2 \cdot 2}{2 \cdot 1.5} \cdot \frac{1}{222 \cdot 10^3} \cdot \frac{2}{50} \cdot 63.03 \cdot 10^3} = 0.2477 \text{ m}$$

The stator stack length L (4.19) is

$$L = \lambda \frac{\pi D_{is}}{2p_1} = \frac{1.5 \cdot \pi \cdot 0.2477}{2 \cdot 2} = 0.2917 \text{ m}$$

with $j_{\text{con}} = 6 \cdot 10^6 \text{ A/m}$, $K_{\text{fill}} = 0.5$, $B_{\text{ts}} = B_{\text{cs}} = 1.6 \text{ T}$, the stator slot height h_s is (4.21).

$$h_s = \frac{36.876 \cdot 10^3}{\frac{0.8}{1.6} \cdot 6 \cdot 10^6 \cdot 0.5} = 24.584 \cdot 10^{-3} \text{ m}$$

The stator back iron height h_{cs} (4.22) is

$$h_{\text{cs}} = \frac{\alpha_i}{2} \frac{\pi D_{\text{is}}}{2 p_1} \frac{B_g}{B_{\text{cs}}} = \frac{0.74 \cdot \pi \cdot 0.2477}{2 \cdot 2 \cdot 2} \cdot \frac{0.8}{1.6} \approx 36 \cdot 10^{-3} \text{ m}$$

The external stator diameter D_{out} becomes

$$D_{\text{out}} = D_{\text{is}} + 2(h_{\text{cs}} + h_s) = 0.2477 + 2(0.024584 + 0.036) = 0.3688 \text{ m}$$

With $N_s = 48$ slots/stator and a slot aspect ratio $K_{\text{aspect}} = 3.03$, the value of slot height h_s (4.30) is

$$h_s = K_{\text{aspect}} \frac{\pi D_{\text{is}}}{N_s} \left(1 - \frac{B_g}{B_{\text{ts}}}\right) = 3.03 \pi \frac{0.2477}{48} \left(1 - \frac{0.8}{1.6}\right) = 0.0246 \text{ m}$$

About the same value of h_s as above has been obtained. It is interesting to calculate the approximate value of the specific tangential force σ_{tan} .

$$\begin{aligned} \sigma_{\text{tan}} &\approx \frac{P_n}{\pi D_{\text{is}} \left(\frac{D_{\text{is}}}{2}\right) L} \frac{p_1}{2 \pi f_1} = \frac{55 \cdot 10^3}{\frac{\pi}{2} \cdot 0.2917 \cdot 0.2477^2} \cdot \frac{2}{2 \pi 50} \\ &= 1.246 \cdot 10^4 \text{ N/m}^2 = 1.246 \text{ N/cm}^2 \end{aligned}$$

This is not a high value, and the moderate-low slot aspect ratio $K_{\text{aspect}} = h_s/b_s = 3.03$ is a clear indication of this situation.

Apparently, the machine stator internal diameter may be reduced by increasing A_1 (in fact, C_0 Esson's constant). For the same λ , the stack length will be reduced, while the stator external diameter will also be slightly reduced (the back iron height h_{cs} decreases, and the slot height increases).

Given the simplicity of the above analytical approach further speculations on better (eventually optimized) designs are considered inappropriate here.

4.10 THE ROTOR TANGENTIAL STRESS DESIGN CONCEPT

The rotor tangential stress $\sigma_{\text{tan}} (\text{N/m}^2)$ may be calculated from the motor torque T_e .

$$\sigma_{\text{tan}} = \frac{T_e \cdot 2}{(\pi D_{\text{is}} L) \cdot D_{\text{is}}} \left(\text{N/m}^2 \right) \quad (4.33)$$

The electromagnetic torque T_{en} is approximately

$$T_{\text{en}} \approx \frac{p_1 P_n \left(1 + \frac{P_{\text{mec}}}{P_n}\right)}{2 \pi f_1 (1 - S_n)} \quad (4.34)$$

P_n is rated motor power; S_n = rated slip.

The rated slip is <2%–3% for most induction motors, and the mechanical losses are in general around 1% of rated power.

$$T_{en} \approx \frac{p_1 P_n \cdot 1.01}{2\pi f_1 0.98} = 0.1641 \cdot P_n \frac{p_1}{f_1} \quad (4.35)$$

Choosing σ_{tan} in the interval 0.2–5 N/cm² or 2,000–50,000 N/m², we use (4.33) directly, with $\lambda = 2p_1 L / \pi D_{is}$, to determine the internal stator diameter.

$$D_{is} = \sqrt[3]{\frac{4p_1}{\pi^2 \lambda \sigma_{tan}} \left(0.1641 \cdot P_n \cdot \frac{p_1}{f_1} \right)} \quad (4.36)$$

No apparent need occurs to adopt at this stage efficiency and power factor values for rated load.

We can now use the no-load value of airgap flux density B_{g0} :

$$B_{g0} = \frac{\mu_0 3\sqrt{2} W_1 K_{w1} I_0}{\pi p_1 K_c g (1 + K_s)} \quad (4.37)$$

where the no-load current I_0 and the number of turns/phase W_1 are unknown and the airgap g , Carter's coefficient K_c , and saturation factor K_s are assigned pertinent values.

$$\begin{aligned} g &\approx (0.1 + 0.02\sqrt[3]{P_n}) \cdot 10^{-3} [\text{m}]; \quad \text{for } P_1 = 1 \\ g &\approx (0.1 + 0.012\sqrt[3]{P_n}) \cdot 10^{-3} [\text{m}]; \quad \text{for } P_1 \geq 2 \end{aligned} \quad (4.38)$$

Typical values of airgap are 0.35, 0.4, 0.45, 0.5, 0.55 ... mm, etc. Also, $K_c \approx (1.15\text{--}1.35)$ for semi-closed slots and $K_c = 1.5\text{--}1.7$ for open stator slots (high-power induction motors). The saturation factor is typically $K_s = 0.3\text{--}0.5$ for $p_1 \geq 2$ and larger for $2p_1 = 2$.

The airgap flux density B_g is

$$\begin{aligned} B_g &= (0.5\text{--}0.7)T \quad \text{for } 2p_1 = 2 \\ B_g &= (0.65\text{--}0.75)T \quad \text{for } 2p_1 = 4 \\ B_g &= (0.7\text{--}0.8)T \quad \text{for } 2p_1 = 6 \\ B_g &= (0.75\text{--}0.85)T \quad \text{for } 2p_1 = 8 \end{aligned} \quad (4.39)$$

The larger values correspond to larger motors.

The product, $W_1 I_0$, is thus obtained from (4.37). The number of turns W_1 may be calculated from the emf E_1 (4.12) and (4.13).

$$W_1 = \frac{E_1}{4f_1 K_f K_{w1} \phi} = \frac{E_1 2p_1}{4f_1 K_f K_{w1} \alpha_i \pi D_{is} L B_g} \quad (4.40)$$

with $W_1 I_0$ and W_1 known, the no-load (magnetization) current I_0 may be obtained. The airgap active power P_{gap} is

$$P_{gap} = T_{en} \frac{2\pi f_1}{p_1} = 3K_E V_1 I_T \quad (4.41)$$

where I_T is the stator current torque component (in phase with E_1). With I_T determined from (4.41), we now calculate the stator rated current I_{In} :

$$I_{In} \approx \sqrt{I_0^2 + I_T^2} \quad (4.42)$$

The rotor bar current (for a cage rotor) I_b is

$$I_b \approx \frac{2mW_l K_{wl} I_T}{N_r} \quad (4.43)$$

N_r – number of rotor slots, m – number of stator phases.

We now check the product $\eta_n \cos \varphi_n$:

$$\eta_n \cos \varphi_n = \frac{P_n}{3V_l I_{ln}} < 1 \quad (4.44)$$

The linear current loading A_l is also checked:

$$A_l = \frac{2mW_l I_{ln}}{\pi D_{is}} \quad (4.45)$$

and eventually compare with data from existing similar motors.

With all these data available, the sizing of stator and rotor slots and their windings is feasible. Then, the machine reactances and resistances and the steady-state performance are calculated. Knowing the motor geometry and the loss breakdown, the thermal aspects (design) may be approached. Finally, if the temperature rise or other performance is not satisfactory, the design process is repeated.

Given the complexity of such an enterprise, some coherent methodologies are in order. They will be explained in subsequent chapters.

Example 4.2 Tangential Stress

Let us consider the motor data from Example 4.1, assume $\sigma_{tan} = 1.5 \cdot 10^4 \text{ N/m}^2$, and determine the values of D_{is} , L , W_l , I_0 , I_{ln} , and $\eta_n \cos \varphi_n$.

Solution

With $p_1 = 2$, $P_n = 55 \text{ kW}$, $f_1 = 50 \text{ Hz}$, $\lambda = 1.5$, from (4.36),

$$D_{is} = \sqrt[3]{\frac{4 \cdot 2 \cdot 0.1641 \cdot 55 \cdot 10^3 \cdot 2}{\pi^2 1.5 \cdot 1.5 \cdot 10^4 \cdot 50}} = 0.2352 \text{ m}$$

The stack length L is

$$L = \lambda \frac{\pi D_{is}}{2p_1} = 1.5 \frac{\pi \cdot 0.2352}{2 \cdot 2} = 0.277 \text{ m}$$

with $B_g = 0.8 \text{ T}$, $K_f = 1.08$, $\alpha_i = 0.74$, $K_{wl} = 0.955$, $K_f = 0.97$, and from (4.40), the number of turns per phase W_l is

$$W_l = \frac{0.97 \cdot \left(\frac{400}{\sqrt{3}} \right) \cdot 2 \cdot 2}{4 \cdot 50 \cdot 1.08 \cdot 0.955 \cdot 0.74 \cdot \pi \cdot 0.2352 \cdot 0.277 \cdot 0.8} = 36 \text{ turns/phase}$$

The rated electromagnetic torque T_{en} (4.35) is

$$T_{en} = 0.1641 \cdot P_n \frac{P_l}{f_1} = 0.1641 \cdot 55 \cdot 10^3 \cdot \frac{2}{50} = 361.02 \text{ Nm}$$

Now, from (4.41), the torque current component I_T is

$$I_T = \frac{T_{en} \cdot 2\pi f_1}{3K_E V_{IP1}} = \frac{361.02 \cdot 2\pi 50}{3 \cdot 0.97 \cdot \frac{400}{\sqrt{3}} \cdot 2} = 84.24 \text{ A}$$

The magnetization current I_0 is obtained from (4.37):

$$I_0 = \frac{B_{g0} \pi P_1 K_c g (1 + K_s)}{\mu_0 3\sqrt{2} W_1 K_{wl}} = \frac{0.8 \cdot \pi \cdot 2 \cdot 1.25 \cdot 0.55 \cdot (1 + 0.5) \cdot 10^{-3}}{1.256 \cdot 10^{-6} \cdot 36 \cdot 0.955 \cdot 3\sqrt{2}} = 28.36 \text{ A}$$

The airgap g (4.38) is

$$g = (0.1 + 0.012\sqrt{55,000}) \cdot 10^{-3} = 0.55 \cdot 10^{-3} \text{ m}$$

The stator rated current I_{In} is

$$I_{In} = \sqrt{I_0^2 + I_T^2} = \sqrt{28.36^2 + 84.24^2} = 88.887 \text{ A}$$

$$\eta_n \cos \phi_{In} = \frac{P_n}{3V_1 I_{In}} = \frac{55 \cdot 10^3}{3 \cdot \frac{400}{\sqrt{3}} \cdot 88.887} = 0.894$$

This corresponds to a rather high (say) $\eta_n = \cos \phi_{In} = 0.9455$.

Note that these values appear at design starting, before all the losses in the machine have been assessed. They provide a design start without Esson's (output) constant which increased continuously over the last decades as material quality and cooling systems improved steadily.

4.11 SUMMARY

- Mechanical loads are characterized by torque/speed curves.
- Single-quadrant and multiquadrant load torque/speed curves are typical.
- Constant V and f supply IMs are suitable only for constant speed single-quadrant loads.
- For single and multiquadrant variable speed loads, variable V and f supply IMs are required. They result in energy savings commensurable with speed control range.
- Three load torque/speed curves are typical: quadratic torque/speed (pumps), constant torque (elevators), and constant power (machine tool, spindles, traction, etc.).
- The standard IM design torque/speed envelope, to match the load, includes two regions: below and above base speed Ω_b . For base speed full voltage, full torque is delivered at rated service cycle and rated temperature rise.
- With self-ventilation, the machine over-temperature restriction leads to torque reduction with speed reduction. For constant torque below base speed, separate (constant speed) ventilation is required.
- Above base speed, with constant voltage and increasing frequency, the torque available decreases and so does the flux linkage in the machine.
- A 2/1 constant power speed range (from Ω_b to $2\Omega_b$) is typical with standard IM designs at constant voltage.
- When an induction motor designed for sine wave power is faced with a notable harmonic content in the power grid due to the presence of power electronic equipment nearby, it has to be derated. In general, a HVF of less than 3% is considered harmless (Figure 4.6).

- A standard sine wave IM, when fed from a PWM voltage source inverter, due to the additional (time harmonics) core and winding losses, has to be derated. A derating of up to 10% is considered acceptable with today's IGBT converters.
- Further on, the presence of a static power converter leads to a 5% voltage reduction at motor terminals with respect to the power grid voltage.
- Finally, an additional derating occurs due to unbalanced power grid voltage. The derating is significant for voltage unbalance above 2% (Figure 4.7).
- Induction motor specifications for constant V and f motors are lined up in pertinent standards. Nameplate markings refer to a myriad of specifications for the user's convenience.
- Efficiency is the most important nameplate marking as the cost of losses per year is about (or more than) 30%–40% of initial motor costs.
- Standard and high-efficiency motors are now available. EU regulations refer to high-efficiency thresholds (Table 4.3).
- Designs A, B, C, and D reveal through their torque–speed curves, the starting, pull-up, and breakdown torques which are important factors in most constant V and f supply IMs.
- Matching a constant V and f IM to a load refers to equality of load and motor torque at rated speed and lower load torque below rated speed.
- For variable speed drives, two different pole pairs count motors at two different frequencies: one below base speed (full flux zone) and one above base speed (flux-weakening, constant voltage zone) may be used.
- For constant power large speed ranges $\Omega_{\max}/\Omega_b > 3$, very large breakdown torque designs are required (above 300%). Alternatively, the voltage per phase is increased above base speed by star/delta connection or a larger torque (larger size) IM is chosen.
- Designing of an IM involves sizing the motor for given specifications of power, supply parameters, and load torque/speed envelope.
- Main design factors are costs of active materials, fabrication and selling, losses capitalized costs, maintenance costs, material limitations (magnetic, electric, dielectric, thermal, mechanical), and special application specifications.
- The IM design features five issues:
 - Electric design
 - Dielectric design
 - Magnetic design
 - Thermal design
 - Mechanical design.
- IM sizing is both a science and an art based on prior experience.
- $D_{is}^2 L$ output coefficient design concept has gained widespread acceptance due to Esson's output constant, and with efficiency and power factor known, the stator bore diameter D_{is} , may be calculated for given power, speed, and stack length L per pitch τ ratio λ given.
- Further on, with given stator winding current density, airgap, stator teeth, and back core flux densities B_g , B_{ts} , B_{cs} , the outer stator diameter is obtained. Based on these data, the stator/rotor slot sizing, wire gauge, machine parameters, performance, losses, and temperatures may be determined. Such a complex enterprise requires coherent methodologies, to be developed in subsequent chapters.
- The rotor tangential (shear) stress, $\sigma_{tan} = (0.2\text{--}4) \text{ N/cm}^2$, is defined as a more general design concept valid for both rotary and linear induction motors (LIM). This way, there is no need to assign initial values to efficiency and power factor to perform the complete design (sizing) process.
- More on design principles in References [4–9].

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5 IM Design below 100 KW and Constant V and f (Size Your Own IM)

5.1 INTRODUCTION

The power of 100kW is traditionally considered the border between small- and medium-power induction machines (IMs). In general, sub-100kW motors use a single stator and rotor stack (no radial cooling channels) and a finned frame washed by air from an externally mounted at shaft end ventilator (Figure 5.1). It has an aluminum cast-cage rotor and, in general, random wound stator coils made of round magnetic wire with one to six elementary conductors (diameter ≤ 2.5 mm) in parallel and one to three current paths in parallel, depending on the number of pole pairs. The number of pole pairs $p_1 = 1, 2, 3, \dots 6$.

Induction motors with power below 100kW constitute a sizeable portion of electric motor world markets. Their design for standard or high efficiency is by now a mature mixture of art and science, at least in the preoptimization stage. Design optimization will be dealt with separately in a dedicated chapter.

For most part, IM design methodologies are proprietary.

Here, we present a sample of such methodologies next. For further reference, see also [1].

5.2 DESIGN SPECIFICATIONS BY EXAMPLE

Standard design specifications are

- Rated power: $P_n[\text{W}] = 5.5\text{kW}$
- Synchronous speed: $n_1[\text{rpm}] = 1800$
- Line supply voltage: $V_l[\text{V}] = 460\text{V}$
- Supply frequency: $f_l[\text{Hz}] = 60$
- Number of phases $m = 3$
- Phase connections: star

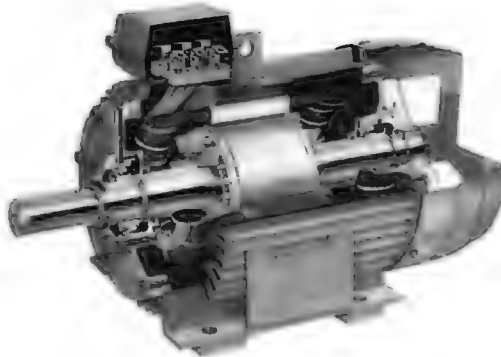


FIGURE 5.1 Low-power three-phase IM with cage rotor.

- Targeted power factor: $\cos \varphi_n = 0.83$
- Targeted efficiency: $\eta_n = 0.895$ (high efficiency motor)
- p.u. locked rotor torque: $t_{LR} = 1.75$
- p.u. locked rotor current: $i_{LR} = 6$
- p.u. breakdown torque: $t_{bK} = 2.5$
- Insulation class: F; temperature rise: class B
- Protection degree: IP55 – IC411
- Service factor load: 1.0
- Environment conditions: standard (no derating)
- Configuration (vertical or horizontal shaft, etc.): horizontal shaft.

5.3 THE ALGORITHM

The main steps in IM design are shown in Figure 5.2. The design process may start with (1), design specs, and assigned values of flux densities and current densities – and in (2), the stator bore diameter D_{is} , stack length, stator slots, and stator outer diameter D_{out} are calculated after

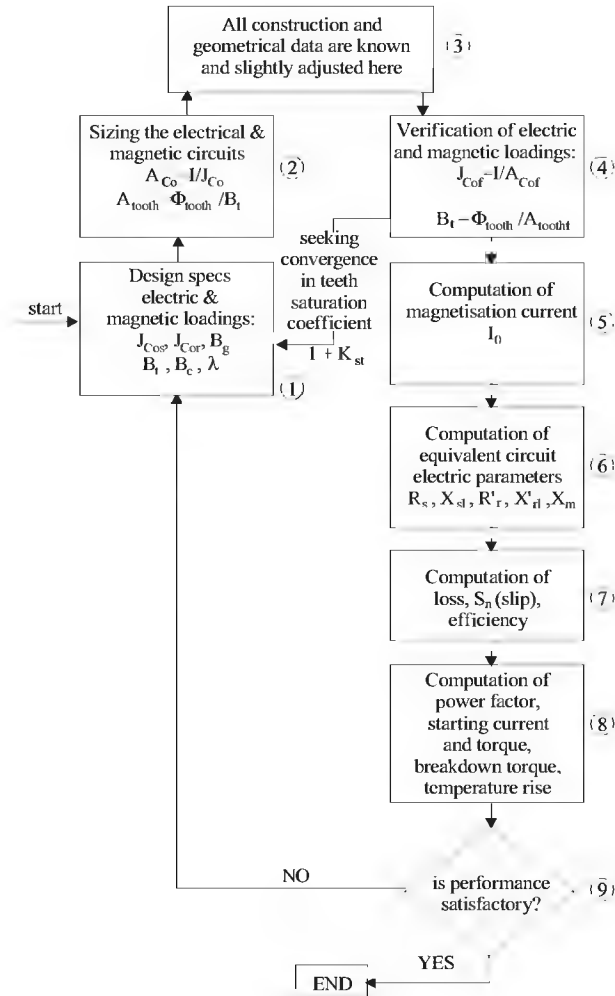


FIGURE 5.2 The design algorithm.

stator and rotor currents are determined. The rotor slots, back iron height, and cage sizing are also determined in (2).

All dimensions are adjusted in (3) to standardized values (stator outer diameter, stator winding wire gauge, etc.). Then in (4), the actual magnetic and electric loadings (current and flux densities) are verified.

If the magnetic saturation coefficients ($1 + K_{st}$) of stator and rotor tooth are not equal to assigned values, the design restarts (1) with adjusted values of tooth flux densities until sufficient convergence is obtained in $1 + K_{st}$.

Once this loop is surpassed, the stages (5) to (8) are traveled by computing the magnetization current I_0 (5); equivalent circuit parameters are calculated in (6); losses, rated slip S_n , and efficiency are determined in (7); and then, power factor, locked rotor current and torque, breakdown torque, and temperature rise are assessed in (8).

In (9), all this performance is checked and, if found unsatisfactory, the whole process is restarted in (1) with new values of flux densities and/or current densities and stack aspect ratio $\lambda = L/\tau$ (τ – pole pitch).

The decision in (9) may be made based on an optimization method which might result in going back to (1) or directly to (3) when the chosen construction and geometrical data are altered according to an optimization method (deterministic or evolutionary) as shown in Chapter 8, Vol. 2.

So, IM design is basically an iterative procedure [2] whose output – the resultant machine to be built – depends on the objective function(s) to be minimized and on the corroborating constraints related to temperature rise, starting current (torque), breakdown torque, etc.

The objective function may be weight or costs or (efficiency)⁻¹ or global costs of active materials or a weighted combination of them.

But before treating the optimization stage in Chapter 8, Vol. 2, let us perform here a practical design.

5.4 MAIN DIMENSIONS OF STATOR CORE

We are going to use the widely accepted $D_{is}^2 L$ output constant concept unfolded in the previous chapter. For completely new designs, the rotor tangential stress concept can be used.

Based on this, the stator bore diameter D_{is} (4.15) is

$$D_{is} = \sqrt[3]{\frac{2p_l}{\pi\lambda} \frac{p_l}{f_l} \frac{S_{gap}}{C_0}}; \quad K_E = 0.98 - 0.005p_l = 0.97 \quad (5.1)$$

$$\text{with } S_{gap} = \frac{K_E P_n}{\eta_n \cos \phi_{1n}}; \quad \lambda = L \left(\frac{2p_l}{\pi D_{is}} \right) = \frac{L}{\tau} \quad (5.2)$$

From past experience, λ is given in Table 5.1.

From (5.2), the apparent airgap power S_{gap} is

$$S_{gap} = \frac{0.97 \cdot 5.5 \cdot 10^3}{0.895 \cdot 0.83} = 7181.8 \text{ VA}$$

TABLE 5.1
Stack Aspect Ratio λ

$2p_l$	2	4	6	8
λ	0.6–1.0	1.2–1.8	1.6–2.2	2–3

C_0 is extracted from Figure 4.14 for $S_{\text{gap}} = 7181.8 \text{ VA}$, $C_0 = 147 \cdot 10^3 \text{ J/m}^3$ and $\lambda = 1.5$, $f_1 = 60 \text{ Hz}$, $p_1 = 2$. So D_{is} from (5.1) is

$$D_{\text{is}} = \sqrt[3]{\frac{2 \cdot 2 \cdot 2}{\pi \cdot 15 \cdot 60} \cdot \frac{7181.8}{147 \cdot 10^3}} = 0.1116 \text{ m}$$

The stack length L (from 5.2) is

$$L = \frac{1.5\pi \cdot 0.1116}{2 \cdot 2} = 0.1315 \text{ m}$$

The pole pitch

$$\tau = \frac{\pi \cdot 0.1116}{2 \cdot 2} = 0.0876 \text{ m}$$

The number of stator slots per pole $3q$ may be $3 \cdot 2 = 6$ or $3 \cdot 3 = 9$. For $q = 3$, the slot pitch τ_s will be around

$$\tau_s = \frac{\tau}{3q} = \frac{0.0876}{3 \cdot 3} = 9.734 \cdot 10^{-3} \text{ m} \quad (5.3)$$

In general, the larger q gives better performance (space field harmonics and losses are smaller).

The slot width at airgap is to be around 5–5.3 mm with a tooth of 4.7–4.4 mm, which is mechanically feasible.

From past experience (or from optimal lamination concept, introduced later in this chapter), the ratio of the internal to external stator diameter $D_{\text{is}}/D_{\text{out}}$, below 100 kW for standard motors, is given in Table 5.2.

With $2p_1 = 4$, D_{out} can be determined by substituting the value of $K_D = 0.62$ in the following equation:

$$D_{\text{out}} = \frac{D_{\text{is}}}{K_D} = \frac{0.1116}{0.62} = 0.18 \text{ m} \quad (5.4)$$

Let us suppose that this value is standardized. The airgap value has been introduced also in Chapter 4, Vol. 1, as

$$\begin{aligned} g &= (0.1 + 0.02 \cdot \sqrt[3]{P_n}) \cdot 10^{-3} \text{ m} \quad \text{for } 2p_1 = 2 \\ g &= (0.1 + 0.012 \cdot \sqrt[3]{P_n}) \cdot 10^{-3} \text{ m} \quad \text{for } 2p_1 \geq 2 \end{aligned} \quad (5.5)$$

TABLE 5.2
Inner/Outer Stator Diameter Ratio

$2p_1$	2	4	6	8
$\frac{D_{\text{is}}}{D_{\text{out}}}$	0.54–0.58	0.61–0.63	0.68–0.71	0.72–0.74

In our case,

$$g = (0.1 + 0.012 \cdot \sqrt[3]{5500}) \cdot 10^{-3} = 0.3111 \cdot 10^{-3} \approx 0.35 \cdot 10^{-3} \text{ m}$$

As known, a very small airgap would produce large space airgap field harmonics and additional losses, whereas a very large one would reduce the power factor and efficiency.

5.5 THE STATOR WINDING

Induction motor windings are presented in Chapter 4, Vol. 1. Based on such knowledge, we choose the number of stator slots N_s :

$$N_s = 2p_1 q m = 2 \cdot 2 \cdot 3 \cdot 3 = 36 \quad (5.6)$$

A two-layer winding with chorded coils: $y/\tau = 7/9$ is chosen as $7/9 = 0.777$ which is close to 0.8, which would reduce the first (fifth-order) stator mmf space harmonic.

The electrical angle between emfs in neighboring slots α_{ec} is

$$\alpha_{ec} = \frac{2\pi p_1}{N_s} = \frac{2\pi 2}{36} = \frac{\pi}{9} \quad (5.7)$$

The largest common divisor of N_s and p_1 (36, 2) is $t = p_1 = 2$, and thus, the number of distinct stator slot emfs $N_s/t = 36/2 = 18$. The star of emf phasors has 18 arrows (Figure 5.3A), and the distribution of phases in slots is shown in Figure 5.3B.

The zone factor K_{q1} is

$$K_{q1} = \frac{\sin \frac{\pi}{6}}{q \sin \left(\frac{\pi}{6q} \right)} = \frac{0.5}{3 \sin \left(\frac{\pi}{18} \right)} = 0.9598 \quad (5.8)$$

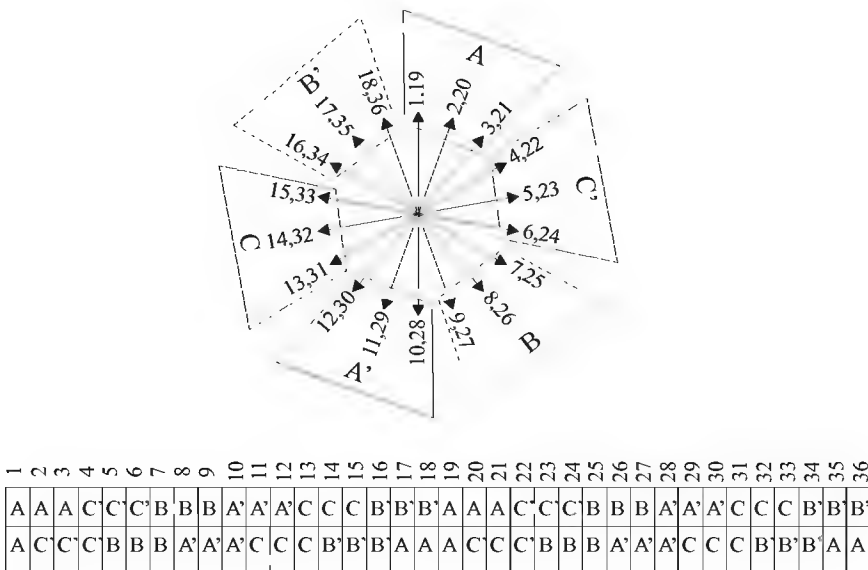


FIGURE 5.3 A 36 slots, $2p_1 = 4$ poles, 2 layer, chorded coils ($y/\tau = 7/9$) three-phase winding.

The chording factor K_{y1} is

$$K_{y1} = \sin \frac{\pi y}{2 \tau} = \sin \frac{\pi 7}{2 \cdot 9} = 0.9397 \quad (5.9)$$

So, the stator winding factor K_{w1} becomes

$$K_{w1} = K_{q1} K_{y1} = 0.9598 \cdot 0.9397 = 0.9019$$

The number of turns per phase is based on the pole flux ϕ :

$$\phi = \alpha_i \tau L B_g \quad (5.10)$$

The airgap flux density is recommended in the intervals:

$$\begin{aligned} B_g &= (0.5 - 0.75)T \quad \text{for } 2p_1 = 2 \\ B_g &= (0.65 - 0.78)T \quad \text{for } 2p_1 = 4 \\ B_g &= (0.7 - 0.82)T \quad \text{for } 2p_1 = 6 \\ B_g &= (0.75 - 0.85)T \quad \text{for } 2p_1 = 8 \end{aligned} \quad (5.11)$$

The pole spanning coefficient α_i (Figure 4.13) depends on the tooth saturation factor $1 + K_{st}$.

Let us consider $1 + K_{st} = 1.4$, with $\alpha_i = 0.729$, $K_f = 1.085$. Now from (5.10) with $B_g = 0.7$ T,

$$\phi = 0.729 \cdot 0.0876 \cdot 0.1315 \cdot 0.7 = 5.878 \cdot 10^{-3} \text{ Wb}$$

The number of turns per phase W_1 (from Chapter 4, Vol. 1, (4.9)) is

$$W_1 = \frac{K_E V_{1ph}}{4 K_f K_{w1} f_1 \phi} = \frac{0.97 \cdot \left(\frac{460}{\sqrt{3}} \right)}{4 \cdot 1.085 \cdot 0.902 \cdot 60 \cdot 5.878 \cdot 10^{-3}} = 186.8 \text{ turns/phase} \quad (5.12)$$

The number of conductors per slot n_s is

$$n_s = \frac{a_1 W_1}{p_1 q} \quad (5.13)$$

where a_1 is the number of current paths in parallel.

In our case, $a_1 = 1$ and

$$n_s = \frac{1 \cdot 186.8}{2 \cdot 3} = 31.33 \quad (5.14)$$

It should be an even number as there are two distinct coils per slot in a double-layer winding, $n_s = 30$. Consequently, $W_1 = p_1 q n_s = 2 \cdot 3 \cdot 30 = 180$.

Going back to (5.12), we have to recalculate the actual airgap flux density B_g :

$$B_g = 0.7 \cdot \frac{186.8}{180} = 0.726 \text{ T} \quad (5.15)$$

The rated current I_{ln} is

$$I_{ln} = \frac{P_n}{\eta_n \cos \varphi_n \sqrt{3} V_l} = \frac{5500}{0.895 \cdot 0.83 \cdot 1.73 \cdot 460} = 9.303 \text{ A} \quad (5.16)$$

As high efficiency is required, and in general, at this power level and speed, winding losses are predominant, from the recommended current densities:

$$\begin{aligned} J_{cos} &= (4 \cdots 7) \text{ A/mm}^2 \quad \text{for } 2p_1 = 2, 4, \\ J_{cos} &= (5 \cdots 8) \text{ A/mm}^2 \quad \text{for } 2p_1 = 6, 8 \end{aligned} \quad (5.17)$$

we choose $J_{cos} = 4.5 \text{ A/mm}^2$.

The magnetic wire cross section A_{Co} is

$$A_{Co} = \frac{I_{ln}}{J_{cos} a_1} = \frac{9.303}{4.5 \cdot 1} = 2.06733 \text{ mm}^2 \quad (5.18)$$

With the wire gauge diameter d_{Co}

$$d_{Co} = \sqrt{\frac{4A_{Co}}{\pi}} = \sqrt{\frac{4 \cdot 2.06733}{\pi}} = 1.622 \text{ mm} \quad (5.19)$$

In general, if $d_{Co} > 1.3 \text{ mm}$, in low-power IMs, we may use a few conductors in parallel a_p .

$$d'_{Co} = \sqrt{\frac{4A_{Co}}{\pi a_p}} = \sqrt{\frac{4 \cdot 2.06733}{\pi \cdot 2}} = 1.15 \text{ mm} \quad (5.20)$$

Now, we have to choose a standardized bare wire diameter from Table 5.3.

The value of 1.15 mm is standardized, so each coil is made of 15 turns and each turn contains 2 elementary conductors in parallel (diameter $d'_{Co} = 1.15 \text{ mm}$).

If the number of conductors in parallel $a_p > 4$, the number of current paths in parallel has to be increased. If, even in this case, a solution is not found, rectangular cross-sectional magnetic wire is used.

5.6 STATOR SLOT SIZING

As we know by now the number of turns per slot n_s and the number of conductors in parallel a_p with the wire diameter d'_{Co} , we can calculate the useful slot area A_{su} provided we use a slot fill factor K_{fill} . For round wire, $K_{fill} \approx 0.35\text{--}0.4$ below 10kW and 0.4 to 0.44 above 10kW.

$$A_{su} = \frac{\pi d_{Co}'^2 a_p n_s}{4 K_{fill}} = \frac{\pi \cdot 1.15^2 \cdot 2 \cdot 30}{4 \cdot 0.40} = 155.7 \text{ mm}^2 \quad (5.21)$$

For the case in point, trapezoidal or rounded semiclosed stator slot shape is recommended. (Figure 5.4).

For such slot shapes, the stator tooth is rectangular (Figure 5.5). The variables b_{os} , h_{os} , and h_w are assigned values from past experience: $b_{os} = 2 \text{ to } 3 \text{ mm} \leq 6g$, $h_{os} = 0.5\text{--}1.0 \text{ mm}$, and wedge height $h_w = 1\text{--}4 \text{ mm}$.

The stator slot pitch τ_s (from 5.3) is $\tau_s = 9.734 \text{ mm}$.

TABLE 5.3
Standardized Magnetic Wire Diameters

Rated Diameter (mm)	Insulated Diameter (mm)
0.3	0.327
0.32	0.348
0.33	0.359
0.35	0.3795
0.38	0.4105
0.40	0.4315
0.42	0.4625
0.45	0.4835
0.48	0.515
0.50	0.536
0.53	0.567
0.55	0.5875
0.58	0.6185
0.60	0.639
0.63	0.6705
0.65	0.691
0.67	0.7145
0.70	0.742
0.71	0.7525
0.75	0.749
0.80	0.8455
0.85	0.897
0.90	0.948
0.95	1.0
1.0	1.051
1.05	1.102
1.10	1.153
1.12	1.173
1.15	1.2035
1.18	1.2345
1.20	1.305
1.25	1.305
1.30	1.356
1.32	1.3765
1.35	1.407
1.40	1.4575
1.45	1.508
1.5	1.559

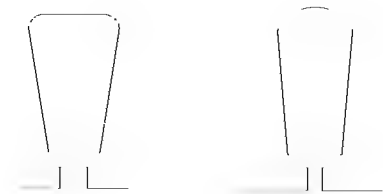


FIGURE 5.4 Recommended stator slot shapes.

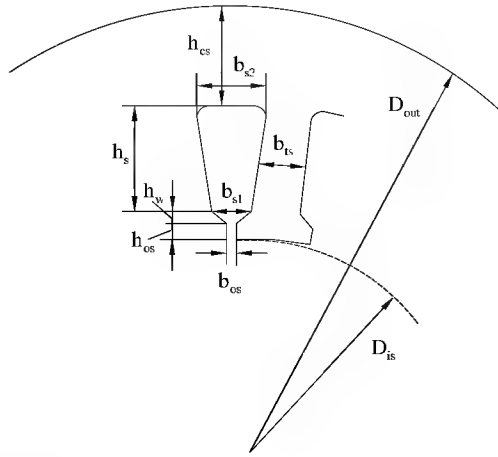


FIGURE 5.5 Stator slot geometry.

Assuming that all the airgap flux passes through the stator teeth:

$$B_g \tau_s L \approx B_{ts} b_{ts} L K_{Fe} \quad (5.22)$$

$K_{Fe} \approx 0.96$ for 0.5 mm thick lamination considers the influence of lamination insulation thickness.

With $B_{ts} = 1.5\text{--}1.65$ T, ($B_{ts} = 1.55$ T), from (5.22) the tooth width b_{ts} can be determined:

$$b_{ts} = \frac{0.726 \cdot 9.734 \cdot 10^{-3}}{1.55 \cdot 0.96} = 4.75 \cdot 10^{-3} \text{ m}$$

From technological limitations, the tooth width should not be under $3.5 \cdot 10^{-3}$ m.

With $b_{os} = 2.2 \cdot 10^{-3}$ m, $h_{os} = 1 \cdot 10^{-3}$ m, and $h_w = 1.5 \cdot 10^{-3}$ m, the slot lower width b_{sl} is

$$\begin{aligned} b_{sl} &= \frac{\pi(D_{is} + 2h_{os} + 2h_w)}{N_s} - b_{ts} \\ &= \frac{\pi(111.6 + 2 \cdot 1 + 2 \cdot 1.5) \cdot 10^{-3}}{36} - 4.75 \cdot 10^{-3} = 5.42 \cdot 10^{-3} \text{ m} \end{aligned} \quad (5.23)$$

The useful area of slot A_{su} can be expressed as

$$A_{su} = h_s \frac{(b_{sl} + b_{s2})}{2} \quad (5.24)$$

Also,

$$b_{s2} \approx b_{sl} + 2h_s \tan \frac{\pi}{N_s} \quad (5.25)$$

From these two equations, the unknowns b_{s2} and h_s can be determined as follows:

$$b_{s2}^2 - b_{sl}^2 = 4A_{su} \tan \frac{\pi}{N_s} \quad (5.26)$$

$$b_{s2} = \sqrt{4A_{su} \tan \frac{\pi}{N_s} + b_{sl}^2} = 10^{-3} \sqrt{4 \cdot 155.72 \tan \frac{\pi}{36} + 5.42^2} \approx 9.16 \cdot 10^{-3} \text{ m} \quad (5.27)$$

Using (5.24), the slot useful height h_s can be determined as

$$h_s = \frac{2A_{su}}{b_{s1} + b_{s2}} = \frac{2 \cdot 155.72}{5.42 + 9.16} \cdot 10^{-3} = 21.36 \cdot 10^{-3} \text{ m} \quad (5.28)$$

Now, we proceed in calculating the teeth saturation factor $1 + K_{st}$ by assuming that stator and rotor tooth produce same effects in this respect:

$$1 + K_{st} = 1 + \frac{F_{mts} + F_{mtr}}{F_{mg}} \quad (5.29)$$

The airgap mmf F_{mg} , with $K_c = 1.2$, is

$$F_{mg} \approx 1.2 \cdot g \cdot \frac{B_g}{\mu_0} = 1.2 \cdot 0.35 \cdot 10^{-3} \cdot \frac{0.726}{1.256 \cdot 10^{-6}} = 242.77 \text{ Aturns} \quad (5.29')$$

with $B_{ts} = 1.55 \text{ T}$, from the magnetization curve table (Table 5.4), $H_{ts} = 1760 \text{ A/m}$. Consequently, the stator tooth mmf F_{mts} is

$$F_{mts} = H_{ts} (h_s + h_{os} + h_w) = 1760(21.36 + 1 + 1.5) \cdot 10^{-3} = 41.99 \text{ Aturns} \quad (5.30)$$

From (5.29), we may calculate the value of rotor tooth mmf F_{mtr} which corresponds to $1 + K_{st} = 1.4$.

$$F_{mtr} = K_{st} F_{mg} - F_{mts} = 0.4 \cdot 242.77 - 41.99 = 55.11 \text{ Aturns} \quad (5.31)$$

As this value is only slightly larger than that of stator tooth, we may go on with the design process.

TABLE 5.4
Lamination Magnetization Curve $B_m(H_m)$

B (T)	H (A/m)	B (T)	H (A/m)
0.05	22.8	1.05	237
0.1	35	1.1	273
0.15	45	1.15	310
0.2	49	1.2	356
0.25	57	1.25	417
0.3	65	1.3	482
0.35	70	1.35	585
0.4	76	1.4	760
0.45	83	1.45	1050
0.5	90	1.5	1340
0.55	98	1.55	1760
0.6	106	1.6	2460
0.65	115	1.65	3460
0.7	124	1.7	4800
0.75	135	1.75	6160
0.8	148	1.8	8270
0.85	162	1.85	11,170
0.9	177	1.9	15,220
0.95	198	1.95	22,000
1.0	220	2.0	34,000

However, if $F_{mtr} \ll F_{mts}$ (or negative) in (5.31), it would mean that for given $1 + K_{st}$, a smaller value of flux density B_g is required.

Consequently, the whole design procedure has to be brought back to Equation (5.10). The iterative procedure is closed for now when $F_{mtr} \approx F_{mts}$.

As the outer diameter of stator has been calculated in (5.4) at $D_{out} = 0.18$ m, the stator back iron height h_{cs} becomes

$$\begin{aligned} h_{cs} &= \frac{D_{out} - (D_{is} + 2(h_{cs} + h_w + h_s))}{2} \\ &= \frac{180 - (111.6 + 2(21.36 + 1.5 + 1))}{2} = 10.34 \text{ mm} \end{aligned} \quad (5.32)$$

The back core flux density B_{cs} has to be verified here, with $\phi = 5.878 \cdot 10^{-3}$ Wb (from 5.10).

$$B_{cs} = \frac{\phi}{2Lh_{cs}} = \frac{5.878 \cdot 10^{-3}}{2 \cdot 0.1315 \cdot 10.34 \cdot 10^{-3}} = 2.16 \text{ T!!} \quad (5.33)$$

Evidently B_{cs} is too large. There are three main ways to solve this problem. One is to simply increase the stator outer diameter until $B_{cs} \approx 1.4\text{--}1.7$ T. The second solution is going back to the design start (Equation 5.1) and introducing a larger stack aspect ratio λ which eventually would result in a smaller D_{is} , and, finally, a larger back iron height b_{cs} and thus a lower B_{cs} . The third solution is to increase current density and thus reduce slot height h_s . However, if high efficiency is the target, such a solution is to be used cautiously.

Here, we decide to modify the stator outer diameter to $D'_{out} = 0.190$ m and thus obtain

$$B_{cs} = 2.16 \frac{b_{cs}}{b_{cs} + \frac{0.190 - 0.180}{2}} = \frac{2.16 \cdot 10.34 \cdot 10^{-3}}{(10.34 + 5) \cdot 10^{-3}} = 1.456 \text{ T}$$

This is considered here a reasonable value.

From now on, the outer stator diameter will be $D'_{out} = 0.190$ m.

5.7 ROTOR SLOTS

For cage rotors, as shown in Chapters 10 and 11, Vol. 1, care must be exercised in choosing the correspondence between the stator and rotor numbers of slots to reduce parasitic torque, additional losses, radial forces, noise, and vibration. Based on past experience (Chapters 10 and 11, Vol. 1, back this up by pertinent explanations), the most adequate number of stator and rotor slot combinations is given in Table 5.5.

For our case, let us choose $N_s \neq N_r$; $N_r = 28$.

As the starting current is rather large – high efficiency is targeted – the skin effect is not very pronounced. Also, as the locked rotor torque is large, the leakage inductance is not to be large. Consequently, from the six typical slot shapes shown in Figure 5.6, that shown in Figure 5.6c is used for the rotor.

First, we need the value of rated rotor bar current I_b :

$$I_b = K_1 \frac{2mW_l K_{wl}}{N_r} I_{ln} \quad (5.34)$$

TABLE 5.5
Stator/Rotor Slot Numbers

$2p_1$	N_s	N_r – Skewed Rotor Slots
2	24	18, 20, 22, 28, 30, 33, 34
	36	25, 27, 28, 29, 30, 43
	48	30, 37, 39, 40, 41
4	24	16, 18, 20, 30, 33, 34, 35, 36
	36	28, 30, 32, 34, 45, 48
	48	36, 40, 44, 57, 59
	72	42, 48, 54, 56, 60, 61, 62, 68, 76
6	36	20, 22, 28, 44, 47, 49
	54	34, 36, 38, 40, 44, 46
	72	44, 46, 50, 60, 61, 62, 82, 83
8	48	26, 30, 34, 35, 36, 38, 58
	72	42, 46, 48, 50, 52, 56, 60
12	72	69, 75, 80
	90	86, 87, 93, 94

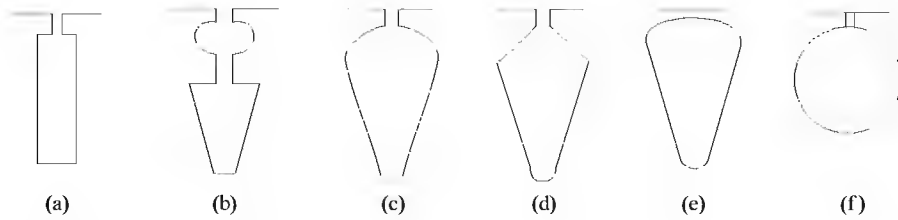


FIGURE 5.6 Typical rotor cage slots.

with $K_1 = 1$, the rotor and stator mmf would have equal magnitudes. In reality, the stator mmf is slightly larger:

$$K_1 \approx 0.8 \cdot \cos \phi_{in} + 0.2 = 0.8 \cdot 0.83 + 0.2 = 0.864 \quad (5.35)$$

From (5.34), the bar current I_b is

$$I_b = \frac{0.864 \cdot 2 \cdot 3 \cdot 180 \cdot 0.9019 \cdot 9.303}{28} = 279.6 \text{ A}$$

For high efficiency, the current density in the rotor bar $j_b = 3.42 \text{ A/mm}^2$. The rotor slot area A_b is

$$A_b = \frac{I_b}{j_b} = \frac{279.6}{3.42 \cdot 10^6} = 81.65 \cdot 10^{-6} \text{ m}^2 \quad (5.36)$$

The end-ring current I_{er} is

$$I_{er} = \frac{I_b}{2 \sin \frac{\pi p_1}{N_r}} = \frac{279.6}{2 \sin \left(\frac{2\pi}{28} \right)} = 628.255 \text{ A} \quad (5.37)$$

The current density in the end ring $J_{er} = (0.75-0.8)J_b$. The higher values correspond to end rings attached to the rotor stack, as part of heat is transferred directly to rotor core.

With $J_{er} = 0.75 \cdot J_b = 0.75 \cdot 342 \cdot 10^6 = 2.55 \cdot 10^6 \text{ A/m}^2$, the end-ring cross section, A_{er} , is

$$A_{er} = \frac{I_{er}}{J_{er}} = \frac{628.255}{2.565 \cdot 10^6} = 245 \cdot 10^{-6} \text{ m}^2 \quad (5.38)$$

We may proceed now to rotor slot sizing based on the variables defined in Figure 5.7.

The rotor slot pitch τ_r is

$$\tau_r = \frac{\pi(D_{is} - 2g)}{N_r} = \frac{\pi(111.6 - 0.7) \cdot 10^{-3}}{28} = 12.436 \cdot 10^{-3} \text{ m} \quad (5.39)$$

With the rotor tooth flux density $B_{tr} = 1.60 \text{ T}$, the tooth width b_{tr} is

$$b_{tr} \approx \frac{B_g}{K_{Fe} B_{tr}} \cdot \tau_r = \frac{0.726}{0.96 \cdot 1.6} \cdot 12.436 \cdot 10^{-3} = 5.88 \cdot 10^{-3} \text{ m} \quad (5.40)$$

The diameter d_1 is obtained from

$$\frac{\pi(D_{re} - 2h_{or} - d_1)}{N_r} = d_1 + b_{tr} \quad (5.41)$$

$$\begin{aligned} d_1 &= \frac{\pi(D_{re} - 2h_{or}) - N_r b_{tr}}{\pi + N_r} \\ &= \frac{\pi(111.6 - 0.7 - 1) - 28 \cdot 5.88 \cdot 10^{-3}}{\pi + 28} = 5.70 \cdot 10^{-3} \text{ m}; \quad h_{or} = 0.5 \cdot 10^{-3} \text{ m} \end{aligned} \quad (5.42)$$

To define completely the rotor slot geometry, we use the slot area equations:

$$A_b = \frac{\pi}{8}(d_1^2 + d_2^2) + \frac{(d_1 + d_2)h_r}{2} \quad (5.43)$$

$$d_1 - d_2 = 2h_r \tan \frac{\pi}{N_r} \quad (5.44)$$

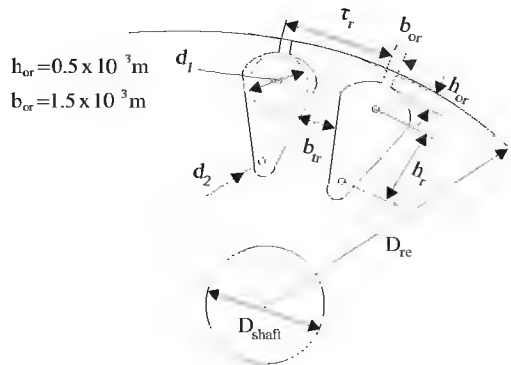


FIGURE 5.7 Rotor slot geometry.

Solving (5.43) and (5.44), we obtain d_2 and h_r (with $d_1 = 5.70 \cdot 10^{-3} \text{ m}$, $A_b = 81.65 \cdot 10^{-6} \text{ m}^2$) as $d_2 = 1.2 \cdot 10^{-3} \text{ m}$ and $h_r = 20 \cdot 10^{-3} \text{ m}$.

Now, we have to verify the rotor teeth mmf F_{mtr} for $B_{\text{tr}} = 1.6 \text{ T}$, $H_{\text{tr}} = 2460 \text{ A/m}$ (Table 5.4).

$$\begin{aligned} F_{\text{mtr}} &= H_{\text{tr}} \left(h_r + h_{\text{or}} + \frac{(d_1 + d_2)}{2} \right) \\ &= 2460 \left(20 + 0.5 + \frac{(1.2 + 5.70)}{2} \right) \cdot 10^{-3} \\ &= 60.134 \text{ Aturns} \end{aligned} \quad (5.45)$$

This is rather close to the value of $F_{\text{mtr}} = 55.11 \text{ Aturns}$ of (5.31). So the design is acceptable so far.

If F_{mtr} had been too large, we might have reduced the flux density and thus increase tooth width b_{tr} and the bar current density. Increasing the slot height is not practical as already $d_2 = 1.2 \cdot 10^{-3} \text{ m}$. This bar current density increase could reduce the efficiency below the target value. We may alternatively increase $1 + K_{\text{st}}$ and redo the design from (5.10).

When the power factor constraint is not too tight, this is a good solution. To maintain the same efficiency, the stator bore diameter has to be increased. So the design should restart from Equation (5.1). The process ends when F_{mtr} is within bounds.

When F_{mtr} is too small, we may increase B_{tr} and return to (5.40) until sufficient convergence is obtained. The required rotor back core may be calculated after allowing for a given flux density $B_{\text{cr}} = 1.4\text{--}1.7 \text{ T}$. With $B_{\text{cr}} = 1.65 \text{ T}$, the rotor back core height h_{cr} is

$$h_{\text{cr}} = \frac{\phi}{2 L \cdot B_{\text{cr}}} = \frac{5.878 \cdot 10^{-3}}{2 \cdot 0.1315 \cdot 1.65} = 13.55 \cdot 10^{-3} \text{ m} \quad (5.46)$$

The maximum diameter of the shaft D_{shaft} is

$$\begin{aligned} (D_{\text{shaft}})_{\text{max}} &\leq D_{\text{is}} - 2g - 2 \left(h_{\text{or}} + \frac{d_1 + d_2}{2} + h_r + h_{\text{cr}} \right) \\ &= \left(111.6 - 2 \cdot 0.35 - 2 \left(1.5 + \frac{(1.2 + 5.69)}{2} + 20 + 13.55 \right) \right) \cdot 10^{-3} \approx 35 \cdot 10^{-3} \text{ m} \end{aligned} \quad (5.47)$$

The shaft diameter corresponds to the rated torque and is given in tables based on mechanical design and past experience. The rated torque is approximately

$$T_{\text{en}} = \frac{P_n}{2\pi \frac{f_l}{p_l} (1 - S_n)} \approx \frac{5.5 \cdot 10^{-3}}{2\pi \frac{60}{2} (1 - 0.02)} = 33.56 \text{ Nm} \quad (5.48)$$

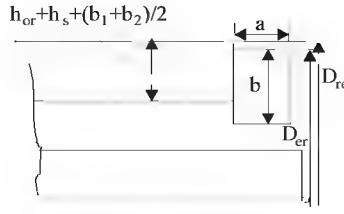
For the case in point, the $35 \cdot 10^{-3} \text{ m}$ left for the shaft diameter suffices.

The end-ring cross section is shown in Figure 5.8.

In general, $D_{\text{re}} - D_{\text{er}} = (3\text{--}4)10^{-3} \text{ m}$.

Also,

$$b = (1.0\text{--}1.2) \left(h_r + h_{\text{or}} + \frac{(b_1 + b_2)}{2} \right) \quad (5.49)$$

**FIGURE 5.8** End-ring cross section.

For

$$b = 1.0 \cdot \left(1 + 20 + \frac{(5.69 + 1.2)}{2} \right) = 24.445 \cdot 10^{-3} \text{ m} \quad (5.50)$$

the value of a is

$$a = \frac{A_{er}}{b} = \frac{245 \cdot 10^{-6}}{24.445 \cdot 10^{-3}} = 10.02 \cdot 10^{-3} \text{ m} \quad (5.51)$$

5.8 THE MAGNETIZATION CURRENT

The total magnetization mmf F_{lm} is

$$F_{lm} = 2 \left(K_c g \frac{B_g}{\mu_0} + F_{mts} + F_{mtr} + F_{mcs} + F_{mcr} \right) \quad (5.52)$$

So far we have considered $K_c = 1.2$ in F_{mg} (5.29). We do know now all variables to calculate Carter's coefficient K_c :

$$\gamma_1 = \frac{b_{os}^2}{5g + b_{os}} = \frac{2.2^2 \cdot 10^{-3}}{5 \cdot 0.35 + 2.2} = 1.2253 \cdot 10^{-3} \text{ m} \quad (5.53)$$

$$\gamma_2 = \frac{b_{or}^2}{5g + b_{or}} = \frac{1.5^2 \cdot 10^{-3}}{5 \cdot 0.35 + 1.5} = 0.692 \cdot 10^{-3} \text{ m} \quad (5.54)$$

$$K_{c1} = \frac{\tau_s}{\tau_s - \gamma_1} = \frac{9.734}{9.734 - 1.2253} = 1.144 \quad (5.55)$$

$$K_{c2} = \frac{\tau_r}{\tau_r - \gamma_2} = \frac{12.436}{12.436 - 0.692} = 1.059 \quad (5.56)$$

The total Carter coefficient K_c is

$$K_c = K_{c1} K_{c2} = 1.144 \cdot 1.059 = 1.2115 \quad (5.57)$$

This is very close to the assigned value of 1.2, so it holds. With $F_{mts} = 42$ Aturns (5.30) and $F_{mtr} = 60.134$ Aturns (5.41) as definitive values (for $(1 + K_{st}) = 1.4$), we still have to calculate the back core mmfs F_{mcs} and F_{mcr} .

$$F_{mcs} = C_{cs} \frac{\pi(D_{out} - h_{cs})}{2p_1} H_{cs}(B_{cs}) \quad (5.58)$$

$$F_{\text{mcr}} = C_{\text{cr}} \frac{\pi(D_{\text{shaft}} + h_{\text{cr}})}{2p_1} H_{\text{cr}}(B_{\text{cr}}) \quad (5.59)$$

C_{cs} and C_{cr} are subunitary empirical coefficients that define an average length of flux path in the back core. They can be calculated by using analytical iterative model (AIM), Chapter 5, Vol. 1.

$$C_{\text{cs},\text{r}} \approx 0.88 \cdot e^{-0.4 B_{\text{cs},\text{r}}^2} \quad (5.60)$$

with $B_{\text{cs}} = 1.456$ T and $B_{\text{cr}} = 1.6$ T from Table 5.4 $H_{\text{cs}} = 1050$ A/m, $H_{\text{cr}} = 2460$ A/m. From (5.58) and (5.59),

$$F_{\text{mcs}} = 0.88 \cdot e^{-0.4 \cdot 1.456^2} \frac{\pi(190 - 15.34) \cdot 10^{-3}}{2 \cdot 2} 1050 = 54.22 \text{ Atums}$$

$$F_{\text{mcr}} = 0.88 \cdot e^{-0.4 \cdot 1.6^2} \frac{\pi(36 + 13.55) \cdot 10^{-3}}{2 \cdot 2} 2460 = 23.04 \text{ Atums}$$

Finally, from (5.52) and (5.29),

$$F_{\text{lm}} = 2 \cdot (242.77 + 42 + 60.134 + 54.22 + 23.04) = 844.328$$

The total saturation factor K_s comes from

$$K_s = \frac{F_{\text{lm}}}{2F_{\text{mg}}} - 1 = \frac{844.328}{2 \cdot 242.77} - 1 = 1.739 \quad (5.61)$$

The magnetization current I_μ is

$$I_\mu = \frac{\pi p_1 (F_{\text{lm}} / 2)}{3\sqrt{2} W_1 K_{w1}} = \frac{\pi \cdot 2 \cdot 844.328}{6\sqrt{2} \cdot 180 \cdot 0.9019} = 3.86 \text{ A} \quad (5.62)$$

The relative (p.u.) value of I_μ is

$$i_\mu = \frac{I_\mu}{I_{\text{ln}}} = \frac{3.86}{9.303} = 0.415 = 41.5\% \quad (5.62')$$

5.9 RESISTANCES AND INDUCTANCES

The resistances and inductances refer to the equivalent circuit (Figure 5.9).

The stator phase resistance is

$$R_s = \rho_{\text{co}} \frac{l_c W_l}{A_{\text{co}} a_l} \quad (5.63)$$

The coil length l_s includes the active part $2L$ and the end connection part $2l_{\text{end}}$

$$l_c = 2(L + l_{\text{end}}) \quad (5.64)$$

The end connection length depends on the coil span y , number of poles, shape of coils, and number of layers in the winding.

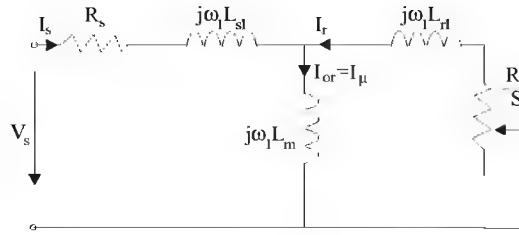


FIGURE 5.9 The T equivalent circuit (core losses not evident).

In general, manufacturing companies developed empirical formulas such as

$$\begin{aligned}
 l_{\text{end}} &= 2y - 0.04 \text{ m} & \text{for } 2p_1 = 2 \\
 l_{\text{end}} &= 2y - 0.02 \text{ m} & \text{for } 2p_1 = 4 \\
 l_{\text{end}} &= \frac{\pi}{2}y + 0.018 \text{ m} & \text{for } 2p_1 = 6 \\
 l_{\text{end}} &= 2.2y - 0.012 \text{ m} & \text{for } 2p_1 = 8
 \end{aligned} \tag{5.65}$$

$\frac{y}{\tau} = \beta$ with β the chording factor. In general,

$$\frac{2}{3} \leq \beta \leq 1 \tag{5.66}$$

In our case for $\frac{y}{\tau} = \frac{7}{9}$, we do have

$$y = \frac{7}{9}\tau = \frac{7}{9} \cdot 0.0876 = 0.06813 \text{ m} \tag{5.67}$$

And from (5.65) for $2p_1 = 4$,

$$l_{\text{end}} = 2y - 0.02 = 2 \cdot 0.06813 - 0.02 = 0.11626 \text{ m} \tag{5.68}$$

The copper resistivity at 20°C and 115°C is $(\rho_{\text{Co}})_{20^\circ\text{C}} = 1.78 \cdot 10^{-8} \Omega\text{m}$ and $(\rho_{\text{Co}})_{115^\circ\text{C}} = 1.37(\rho_{\text{Co}})_{20^\circ\text{C}}$. We do not know yet the rated stator temperature, but the high-efficiency target indicates that the winding temperature should not be too large even if the insulation class is F. We use here $(\rho_{\text{Co}})_{80^\circ\text{C}}$.

$$(\rho_{\text{Co}})_{80^\circ\text{C}} = (\rho_{\text{Co}})_{20^\circ\text{C}} \left(1 + \frac{1}{273} (80 - 20) \right) = 2.1712 \cdot 10^{-8} \Omega\text{m} \tag{5.69}$$

From (5.63):

$$R_s = 2.1712 \cdot 10^{-8} \cdot \frac{2(0.1315 + 0.11626) \cdot 180}{2.06733 \cdot 10^{-6}} = 0.93675 \Omega$$

The rotor bar/end-ring segment equivalent resistance R_{be} is

$$R_{be} = \rho_{Al} \left[\frac{L}{A_b} K_R + \frac{l_{er}}{2A_{er} \sin^2 \left(\frac{\pi P_1}{N_r} \right)} \right] \quad (5.70)$$

The cast aluminum resistivity at 20°C $(\rho_{Al})_{20^\circ\text{C}} = 3.1 \cdot 10^{-8} \Omega\text{m}$, and the end-ring segment length l_{er} is

$$l_{er} = \frac{\pi(D_{er} - b)}{N_r} = \frac{\pi(111.6 - 2 \cdot 0.35 - 6 - 24.445) \cdot 10^{-3}}{28} = 9.022 \cdot 10^{-3} \text{ m} \quad (5.71)$$

K_R , the skin effect resistance coefficient for the bar (Chapter 9, Vol. 1, Equation 9.1), is approximately (as for a rectangular bar)

$$K_R = \xi \frac{(\sinh 2\xi + \sin 2\xi)}{(\cosh 2\xi - \cos 2\xi)} \approx \xi \quad (5.72)$$

$$\xi = \beta_s h_r \sqrt{S}; \quad \beta_s = \sqrt{\frac{\omega_l \mu_0}{2\rho_{Al}}} = \sqrt{\frac{2\pi 60 \cdot 1.25 \cdot 10^{-6}}{2 \cdot 3.1 \cdot 10^{-8}}} = 87(\text{m})^{-1} \quad (5.73)$$

For $h_r = 20 \cdot 10^{-3} \text{ m}$ and $S = 1$, $\xi = 87 \cdot 20 \cdot 10^{-3} \cdot 1 = 1.74$; $K_r \approx 1.74$. From (5.70), the value of R_{be} is

$$\begin{aligned} R_{be}^{S=1} &= 3.1 \cdot 10^{-8} \left(1 + \frac{1}{273} (80 - 20) \right) \left(\frac{0.1315 \cdot 1.73}{81.65 \cdot 10^{-6}} + \frac{9.022 \cdot 10^{-3}}{2 \cdot 245 \cdot 10^{-6} \sin^2 \left(\frac{2\pi}{28} \right)} \right) \\ &= 1.194 \cdot 10^{-4} \Omega \end{aligned}$$

The rotor cage resistance reduced to the stator R'_r is

$$(R'_r)_{S=1} = \frac{4m_1}{N_r} (W_1 K_{w1})^2 R_{be 80^\circ} = \frac{4 \cdot 3}{28} (180 \cdot 0.9019)^2 \cdot 1.194 \cdot 10^{-4} = 1.1295 \Omega \quad (5.74)$$

The stator phase leakage reactance X_{sl} is

$$X_{sl} = 2\mu_0 \omega_l L \frac{W_1^2}{p_1 q} (\lambda_s + \lambda_{ds} + \lambda_{ec}); \quad \beta = \frac{y}{\tau} \quad (5.75)$$

λ_s , λ_{ds} , and λ_{ec} are the slot, differential, and end-ring connection coefficients:

$$\begin{aligned} \lambda_s &= \left[\frac{2}{3} \frac{h_s}{(b_{s1} + b_2)} + \frac{2h_w}{(b_{os} + b_{s1})} + \frac{h_{os}}{b_{os}} \right] \left(\frac{1 + 3\beta}{4} \right) \\ &= \left[\frac{2}{3} \frac{21.36}{(5.42 + 9.16)} + \frac{2 \cdot 1.5}{(2.2 + 5.42)} + \frac{1}{2.2} \right] \left(\frac{1 + 3(7/9)}{4} \right) = 1.523 \end{aligned} \quad (5.76)$$

The expression of λ_{ds} has been developed in Chapter 9. Vol. 1, Equation (9.85). An alternative approximation is given here:

$$\lambda_{ds} \approx \frac{0.9\tau_s q^2 K_{wl}^2 C_s \gamma_{ds}}{K_{cg}(1+K_{st})} \quad (5.77)$$

$$C_s = 1 - 0.033 \frac{b_{cs}^2}{g\tau_s}$$

$$\begin{aligned} \gamma_{ds} &= (0.11 \sin \varphi_l + 0.28) \cdot 10^{-2}; \quad \text{for } q = 8 \\ \gamma_{ds} &= (0.11 \sin \varphi_l + 0.41) \cdot 10^{-2}; \quad \text{for } q = 6 \\ \gamma_{ds} &= (0.14 \sin \varphi_l + 0.76) \cdot 10^{-2}; \quad \text{for } q = 4 \\ \gamma_{ds} &= (0.18 \sin \varphi_l + 1.24) \cdot 10^{-2}; \quad \text{for } q = 3 \\ \gamma_{ds} &= (0.25 \sin \varphi_l + 2.6) \cdot 10^{-2}; \quad \text{for } q = 2 \\ \gamma_{ds} &= 9.5 \cdot 10^{-2}; \quad \text{for } q = 1 \\ \varphi_l &= \pi(6\beta - 5.5) \end{aligned} \quad (5.78)$$

For $\beta = 7/9$ and $q = 3$, γ_{ds} (from (5.78)) is

$$\gamma_{ds} = \left[0.18 \sin \left(\pi \left(\frac{6 \cdot 7}{9} - 5.5 \right) \right) + 1.24 \right] \cdot 10^{-2} = 1.15 \cdot 10^{-2}$$

$$C_s = 1 - 0.033 \frac{2.2^2}{0.35 \cdot 9.734} = 0.953$$

From Equation (5.77),

$$\lambda_{ds} = \frac{0.9 \cdot 9.734 \cdot 10^{-3} \cdot 3^2 \cdot 0.9019^2 \cdot 0.953 \cdot 1.15 \cdot 10^{-2}}{1.21 \cdot 0.35 \cdot 10^{-3} (1 + 0.4)} = 1.18$$

For two-layer windings, the end connection-specific geometric permeance coefficient λ_{ec} is

$$\begin{aligned} \lambda_{ec} &= 0.34 \frac{q}{L} (l_{end} - 0.64 \cdot \beta \cdot \tau) \\ &= 0.34 \cdot \frac{3}{0.1315} \left(0.11626 - 0.64 \cdot \frac{7}{9} \cdot 0.0876 \right) = 0.5274 \end{aligned} \quad (5.79)$$

From Equation (5.75), the stator phase reactance X_{sl} is

$$X_{sl} = 2 \cdot 1.126 \cdot 10^{-6} \cdot 2\pi 60 \cdot 0.1315 \cdot \frac{180^2}{2 \cdot 3} (1.523 + 0.18 + 0.5274) = 2.17 \, \Omega \quad (5.80)$$

The equivalent rotor bar leakage reactance X_{be} is

$$X_{be} = 2\pi f_l \mu_0 L (\lambda_r K_x + \lambda_{dr} + \lambda_{er}) \quad (5.80')$$

where λ_r are the rotor slot, differential, and end-ring permeance coefficient. For the rounded slot shown in Figure 5.7 (see Chapter 6, Vol. 1),

$$\lambda_r \approx 0.66 + \frac{2h_r}{3(d_1 + d_2)} + \frac{h_{or}}{b_{or}} = 0.66 + \frac{2 \cdot 20}{3(5.7 + 1.2)} + \frac{0.5}{1.5} = 2.922 \quad (5.81)$$

The value of λ_{dr} is (Chapter 6, Vol. 1)

$$\lambda_{dr} = \frac{0.9\tau_r \gamma_{dr}}{K_c g} \left(\frac{N_r}{6p_1} \right)^2; \quad \gamma_{dr} = 9 \left(\frac{6p_1}{N_r} \right)^2 10^{-2} \quad (5.82)$$

$$\gamma_{dr} = 9 \left(\frac{6 \cdot 2}{28} \right)^2 10^{-2} = 1.653 \cdot 10^{-2}$$

$$\lambda_{dr} = \frac{0.9 \cdot 12.436}{1.21 \cdot 0.35} \cdot 1.653 \cdot 10^{-2} \left(\frac{28}{6 \cdot 2} \right)^2 = 2.378$$

$$\begin{aligned} \lambda_{er} &= \frac{2.3(D_{er} - b)}{N_r \cdot L \cdot 4 \sin^2 \left(\frac{\pi p_1}{N_r} \right)} \log \frac{4 \cdot 7(D_{er} - b)}{b + 2a} \\ &= \frac{2.3 \cdot 80.455}{28 \cdot 131.5 \cdot 4 \sin^2 \left(\frac{\pi \cdot 2}{28} \right)} \log \left(\frac{4 \cdot 7 \cdot 80.455}{24.445 + 20} \right) = 0.2255 \end{aligned} \quad (5.83)$$

The skin effect coefficient for the leakage reactance K_x is, for $\xi = 1.74$.

$$K_x \approx \frac{3}{2\xi} \frac{(\sinh(2\xi) - \sin(2\xi))}{(\cosh(2\xi) - \cos(2\xi))} \approx \frac{3}{2\xi} = 0.862 \quad (5.84)$$

From (5.80), X_{be} is

$$(X_{be})_{S=1} = 2\pi 60 \cdot 1.256 \cdot 10^{-6} \cdot 0.1315(2.922 \cdot 0.862 + 2.378 + 0.2255) = 3.1877 \cdot 10^{-4} \Omega$$

The rotor leakage reactance X_{rl} becomes

$$(X_{rl})_{S=1} = 4m \frac{(W_l K_{wl})^2}{N_r} X_{be} = 12 \cdot \frac{(180 \cdot 0.9019)^2}{28} 3.1387 \cdot 10^{-4} = 3.6506 \Omega \quad (5.85)$$

For zero speed ($S = 1$), both stator and rotor leakage reactances are reduced due to leakage flux path saturation. This aspect has been treated in detail in Chapter 9, Vol. 1. For the power levels of interest here, with semiclosed stator and rotor slots:

$$\begin{aligned} (X_{sl})_{sat}^{S=1} &= X_{sl}(0.7 - 0.8) \approx 2.17 \cdot 0.75 = 1.625 \Omega \\ (X_{rl})_{sat}^{S=1} &= X_{rl}(0.6 - 0.7) \approx 3.938 \cdot 0.65 = 2.56 \Omega \end{aligned} \quad (5.86)$$

For rated slip (speed), both skin and leakage saturation effects have to be eliminated ($K_R = K_x = 1$).

From (5.70), R_{be80° is

$$\begin{aligned} (R_{be80^\circ})_{S_n} &= 3.1 \cdot 10^{-8} \left(1 + \frac{1}{273} (80 - 20) \right) \left[\frac{0.1315 \cdot 1}{81.65 \cdot 10^{-6}} + \frac{9.022 \cdot 10^{-3}}{2.245 \cdot 10^{-6} \sin^2 \left(\frac{\pi \cdot 2}{28} \right)} \right] \\ &= 0.7495 \cdot 10^{-4} \Omega \end{aligned}$$

So the rotor resistance $(R'_r)_{S_n}$ is

$$(R'_r)_{S_n} = (R'_r)_{S=1} \cdot \frac{R_{be80^\circ}^{S=S_n}}{R_{be80^\circ}^{S=1}} = 1.1295 \cdot \frac{0.7495 \cdot 10^{-4}}{1.194 \cdot 10^{-4}} = 0.709 \Omega \quad (5.87)$$

In a similar way, the equivalent rotor leakage reactance at rated slip S_n , $X_{rl}^{S=S_n} = 3.938 \Omega$.

The magnetization X_m is

$$X_m = \sqrt{\left(\frac{V_{ph}}{I_u} \right)^2 - R_s^2} - X_{sl} = \sqrt{\left(\frac{460}{3.86\sqrt{3}} \right)^2 - 0.93675^2} - 2.17 = 66.70 \Omega \quad (5.88)$$

5.9.1 SKEWING EFFECT ON REACTANCES

In general, the rotor slots are skewed. A skewing c of one stator slot pitch τ_s is typical ($c = \tau_s$).

The change in parameters due to skewing is discussed in detail in Chapter 9, Vol. 1. Here, an approximation is used:

$$X_m = X_m K_{skew} \quad (5.89)$$

$$K_{skew} = \frac{\sin \frac{\pi c}{2 \tau}}{\frac{\pi c}{2 \tau}} = \frac{\sin \frac{\pi \tau_s}{2 \tau}}{\frac{\pi \tau_s}{2 \tau}} = \frac{\sin \frac{\pi}{2 \cdot 3q}}{\frac{\pi}{2 \cdot 3q}} = \frac{\sin \frac{\pi}{18}}{\frac{\pi}{18}} = 0.9954 \quad (5.90)$$

Now with (5.88) and (5.89),

$$X_m = 66.70 \cdot 0.9954 = 66.3955 \Omega$$

Also, as suggested in Chapter 9, Vol. 1, the rotor leakage inductance (reactance) is augmented by a new term " X'_{rlskew} ":

$$X'_{rlskew} = X_m (1 - K_{skew}^2) = 66.70 (1 - 0.9954^2) = 0.6055 \Omega \quad (5.91)$$

So, the final values of rotor leakage reactance at $S = 1$ and $S = S_n$, respectively, are

$$(X_{rl})_{skew}^{S=1} = (X_{rl})_{sat}^{S=1} + X_{rlskew} = 2.56 + 0.6055 = 3.1655 \Omega \quad (5.92)$$

$$(X_{rl})_{skew}^{S=S_n} = X_{rl} + X_{rlskew} = 3.6506 + 0.6055 = 4.256 \Omega \quad (5.93)$$

5.10 LOSSES AND EFFICIENCY

The efficiency is defined in general as output per input power:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{in}} + \sum \text{losses}} \quad (5.94)$$

The loss components are

$$\sum \text{losses} = p_{\text{Co}} + p_{\text{Al}} + p_{\text{iron}} + p_{\text{mv}} + p_{\text{stray}} \quad (5.95)$$

p_{Co} , the stator winding loss component, is

$$p_{\text{Co}} = 3R_s I_{\text{in}}^2 = 3 \cdot 0.93675 \cdot 9.303^2 = 243.215 \text{ W} \quad (5.96)$$

p_{Al} , rotor cage loss component (at $S = S_n$), is

$$p_{\text{Al}} = 3(R_r)_{S_n} I_m^2 = 3R_r K_f^2 I_{\text{in}}^2 \quad (5.97)$$

With (5.87) and (5.35), we get

$$p_{\text{Al}} = 3 \cdot 0.709 \cdot 0.864^2 \cdot 9.303^2 = 137.417 \text{ W}$$

The mechanical/ventilation losses are considered here $p_{\text{mv}} = 0.03P_n$ for $p_1 = 1$, $0.012P_n$ for $p_1 = 2$, and $0.008P_n$ for $p_1 = 3, 4$.

The stray losses p_{stray} are explained in detail in Chapter 11, Vol. 1. Here, their standard value $p_{\text{stray}} = 0.01P_n$ is considered.

The core loss p_{iron} is made of fundamental p_{iron}^l and additional (harmonics) p_{iron}^h iron loss.

The fundamental core losses occur only in the teeth and back iron (p_{tl} , p_{yl}) of the stator as the rotor (slip) frequency is low ($f_2 < 3\text{--}4 \text{ Hz}$).

The stator teeth fundamental losses (see Chapter 11, Vol. 1) are

$$p_{\text{tl}} \approx K_t p_{10} \left(\frac{f_1}{50} \right)^{1.3} B_{\text{ts}}^{1.7} G_{\text{tl}} \quad (5.98)$$

where p_{10} is the specific losses in W/Kg at 1.0 T and 50 Hz ($p_{10} = 2\text{--}3 \text{ W/Kg}$; it is a catalogue data for the lamination manufacturer). $K_t = 1.6\text{--}1.8$ accounts for core loss augmentation due to mechanical machining (K_t value depends on the quality of the material, sharpening of the cutting tools, etc.).

G_{tl} , the stator tooth weight, is

$$\begin{aligned} G_{\text{tl}} &= \gamma_{\text{iron}} \cdot N_s \cdot b_{\text{ts}} \cdot (h_s + h_w + h_{\text{os}}) \cdot L \cdot K_{\text{Fe}} \\ &= 7800 \cdot 36 \cdot 4.75 \cdot 10^{-3} \cdot (21.36 + 1.5 + 1) \cdot 10^{-3} \cdot 0.1315 \cdot 0.95 = 3.975 \text{ Kg} \end{aligned} \quad (5.99)$$

With $B_{\text{ts}} = 1.55 \text{ T}$ and $f_1 = 60 \text{ Hz}$, from (5.98), p_{tl} is

$$p_{\text{tl}} = 1.7 \cdot 2 \cdot \left(\frac{60}{50} \right)^{1.3} \cdot 1.55^{1.7} \cdot 3.975 = 36.08 \text{ W}$$

In a similar way, the stator back iron (yoke) fundamental loss component p_{y1} is

$$p_{y1} = K_y p_{10} \left(\frac{f_1}{50} \right)^{1.3} B_{cs}^{1.7} G_{y1} \quad (5.100)$$

Again, $K_y = 1.6$ – 1.9 takes care of the influence of mechanical machining, and the yoke weight G_{y1} is

$$\begin{aligned} G_{y1} &= \gamma_{iron} \frac{\pi}{4} \left[D_{out}^2 - (D_{out} - 2h_{cs})^2 \right] \cdot L \cdot K_{Fe} \\ &= 7800 \frac{\pi}{4} \left[0.19^2 - (0.19 - 2 \cdot 15.34 \cdot 10^{-3})^2 \right] \cdot 0.1315 \cdot 0.95 = 8.275 \text{ Kg} \end{aligned} \quad (5.101)$$

with $K_y = 1.6$, $B_{cs} = 1.6 \text{ T}$, p_{y1} from (5.100) is

$$p_{y1} = 1.6 \cdot 2 \cdot \left(\frac{60}{50} \right)^{1.3} \cdot 1.6^{1.7} \cdot 8.275 = 74.62 \text{ W}$$

So, the fundamental iron loss component p_{iron}^I is

$$p_{iron}^I = p_{t1} + p_{y1} = 36.08 + 74.62 = 110.70 \text{ W} \quad (5.102)$$

The tooth flux pulsation core loss constitutes the main components of stray losses (Chapter 11, Vol. 1) [1].

$$p_{iron}^s \approx 0.5 \cdot 10^{-4} \left[\left(N_r \frac{f_1}{p_l} K_{ps} B_{ps} \right)^2 G_{ts} + \left(N_s \frac{f_1}{p_l} K_{pr} B_{pr} \right)^2 G_{tr} \right] \quad (5.103)$$

$$K_{ps} \approx \frac{1}{2.2 - B_{ts}} = \frac{1}{2.2 - 1.55} = 1.5385 \quad (5.104)$$

$$K_{pr} = \frac{1}{2.2 - B_{tr}} = \frac{1}{2.2 - 1.6} = 1.666$$

$$B_{ps} \approx (K_{c2} - 1) B_g = (1.059 - 1.0) \cdot 0.726 = 0.0428 \text{ T} \quad (5.105)$$

$$B_{pr} \approx (K_{c1} - 1) B_g = (1.144 - 1.0) \cdot 0.726 = 0.1045 \text{ T} \quad (5.106)$$

The rotor teeth weight G_{tr} is

$$\begin{aligned} G_{tr} &= \gamma_{iron} \cdot L \cdot K_{Fe} \cdot N_r \cdot \left(h_r + \frac{d_1 + d_2}{2} \right) \cdot b_{tr} \\ &= 7800 \cdot 0.1315 \cdot 0.95 \cdot 28 \cdot \left(20 + \frac{5.7 + 1.2}{2} \right) \cdot 10^{-3} \cdot 5.88 \cdot 10^{-3} = 3.710 \text{ Kg} \end{aligned} \quad (5.107)$$

Now, from (5.103),

$$\begin{aligned} p_{iron}^s &= 0.5 \cdot 10^{-4} \left[\left(28 \cdot \frac{60}{2} \cdot 1.538 \cdot 0.0429 \right)^2 3.975 + \left(36 \cdot \frac{60}{2} \cdot 1.666 \cdot 0.1045 \right)^2 3.710 \right] \\ &= 7.2 \text{ W} \end{aligned}$$

The total core loss p_{iron} is

$$p_{\text{iron}} = p_{\text{iron}}^l + p_{\text{iron}}^s = 110.70 + 7.2 = 117.90 \text{ W} \quad (5.108)$$

Total losses (5.95) is

$$\sum \text{losses} = 243.215 + 137.417 + 117.90 + 2.2 \cdot 10^{-2} \cdot 5500 = 609.6 \text{ W}$$

The efficiency η_n (from 5.87) becomes

$$\eta_n = \frac{5500}{5500 + 609.6} = 0.9002!$$

The targeted efficiency was 0.895, so the design holds. If the efficiency had been smaller than the target value, the design should have returned to square one by adopting a larger stator bore diameter D_{is} , then smaller current densities, etc. A larger size machine would have been obtained, in general.

5.11 OPERATION CHARACTERISTICS

The operation characteristics are defined here as active no-load current I_{0a} , rated slip S_n , rated torque T_n , breakdown slip and torque S_k , T_{bk} , current I_s and power factor versus slip, starting current, and torque I_{LR} , T_{LR} .

The no-load active current is given by the no-load losses:

$$I_{0a} = \frac{p_{\text{iron}} + p_{\text{mv}} + 3 \cdot I_{\mu}^2 \cdot R_s}{3V_{\text{ph}}} = \frac{117.9 + 56.1 + 3 \cdot 3.86^2 \cdot 0.936}{3(460/\sqrt{3})} = 0.271 \text{ A} \quad (5.109)$$

The rated slip S_n is

$$S_n = \frac{p_{\text{Al}}}{P_N + p_{\text{Al}} + p_{\text{mv}} + p_{\text{stray}}} = \frac{137.417}{5500 + 137.417 + 56.1 + 27.5} = 0.024! \quad (5.110)$$

The rated shaft torque T_n is

$$T_n = \frac{P_n}{2\pi \frac{f_l}{p_l} (1 - S_n)} = \frac{5500}{2\pi \frac{60}{2} (1 - 0.024)} = 29.91 \text{ Nm} \quad (5.111)$$

The approximate expressions of torque versus slip (Chapter 7, Vol. 1) are

$$T_e = \frac{3p_l}{\omega_l} \frac{V_{\text{ph}}^2 \frac{R_r}{S}}{\left(R_s + C_m \frac{R_l}{S}\right)^2 + (X_{sl} + C_m X_{rl})^2} \quad (5.112)$$

$$\text{with } C_m \approx 1 + \frac{X_{sl}}{X_m} = 1 + \frac{2.17}{66.4} = 1.0327 \quad (5.113)$$

From (5.112), the breakdown torque T_{bk} for motoring is

$$\begin{aligned}
T_{bk} &= \frac{3p_l}{2\omega_l} \frac{V_{ph}^2}{\left[R_s + \sqrt{R_s^2 + (X_{sl} + C_l X_{fl})^2} \right]} \\
&= \frac{3 \cdot 2}{2 \cdot 2\pi 60} \frac{(460/\sqrt{3})^2}{\left[0.936 + \sqrt{0.936^2 + (2.17 + 4.256)^2} \right]} = 75.48 \text{ Nm}
\end{aligned} \tag{5.114}$$

The starting current I_{LR} is

$$\begin{aligned}
I_{LR} &= \frac{V_{ph}}{\sqrt{(R_s + R_r^{S=1})^2 + (X_{sl}^{S=1} + X_{fl}^{S=1})^2}} \\
&= \frac{460/\sqrt{3}}{\sqrt{(0.936 + 1.1295)^2 + (1.6275 + 3.165)^2}} = 54.44 \text{ A}
\end{aligned} \tag{5.115}$$

The starting torque T_{LR} may now be computed as

$$T_{LR} = \frac{3R_r^{S=1} I_{LR}^2}{\omega_l} P_l = \frac{3 \cdot 1.1295 \cdot 54.44^2 \cdot 2}{2\pi 60} = 53.305 \text{ Nm} \tag{5.116}$$

In the specifications, the rated power factor $\cos \varphi_{In}$, T_{bk}/T_{en} , T_{LR}/T_{en} , and I_{LR}/I_{In} are given. So we have to check the final design values of these constraints:

$$\cos \varphi_{In} = \frac{P_n}{3V_{ph} I_{In} \eta_{In}} = \frac{5500}{3 \frac{460}{\sqrt{3}} 9.303 \cdot 0.90} = 0.825 < 0.83 \tag{5.117}$$

$$t_{bk} = \frac{T_{bk}}{T_{en}} = \frac{75.48}{29.91} = 2.523 \approx 2.5 \tag{5.118}$$

$$t_{LR} = \frac{T_{LR}}{T_{en}} = \frac{53.305}{29.91} = 1.7828 \approx 1.75 \tag{5.119}$$

$$i_{LR} = \frac{I_{LR}}{I_{In}} = \frac{54.44}{9.303} = 5.85 < 6.0 \tag{5.120}$$

Apparently, the design does not need any iterations. This is pure coincidence “combined” with standard specifications and designer’s experience! Higher breakdown or starting torque ratios t_{bk} , t_{LR} would, for example, need lower rotor leakage inductance and higher rotor resistance as influenced by skin effect. A larger stator bore diameter is again required. In general, it is not easy to make a few simple changes to get the desired operation characteristics. Here, the optimization design methods come into play.

5.12 TEMPERATURE RISE

Any electromagnetic design has to be thermally valid (Chapter 12, Vol. 1). Only a coarse verification of temperature rise is given here.

First, the temperature differential $\Delta\theta_{co}$ between the conductors in slots and the slot wall is calculated:

$$\Delta\theta_{co} \approx \frac{P_{co}}{\alpha_{cond} A_{ls}} \tag{5.121}$$

Then, the frame temperature rise $\Delta\theta_{\text{frame}}$ with respect to ambient air is determined:

$$\Delta\theta_{\text{frame}} = \frac{\sum \text{losses}}{\alpha_{\text{conv}} A_{\text{frame}}} \quad (5.122)$$

For IMs with self-ventilators placed outside the motor (below 100kW),

$$\alpha_{\text{conv}} \left(\text{W/m}^2 \text{ K} \right) = \begin{cases} 60; & \text{for } 2p_1 = 2; \\ 50; & \text{for } 2p_1 = 4; \\ 40; & \text{for } 2p_1 = 6; \\ 32; & \text{for } 2p_1 = 8; \end{cases} \quad (5.123)$$

More precise values are given in Chapter 12, Vol. 1.

The slot insulation conductivity plus its thickness are lumped into α_{cond} :

$$\alpha_{\text{cond}} = \frac{\lambda_{\text{ins}}}{h_{\text{ins}}} = \frac{0.25}{0.3 \cdot 10^{-3}} = 833 \text{ W/m}^2 \text{ K} \quad (5.124)$$

λ_{ins} is the insulation thermal conductivity in (W/m K) and h_{ins} the total insulation thickness from the slot middle to teeth wall.

The stator slot lateral area A_{ls} is

$$A_{\text{ls}} \approx (2h_s + b_{s2}) \cdot L \cdot N_s = (2 \cdot 21.36 + 9.16) \cdot 10^{-3} \cdot 0.1315 \cdot 36 = 0.2456 \text{ m}^2 \quad (5.125)$$

The frame area S_{frame} (including the Finns area) is

$$A_{\text{frame}} = \pi D_{\text{out}} (L + \tau) \cdot K_{\text{fin}} = \pi \cdot 190 (0.1315 + 0.0876) \cdot 3.0 = 0.392 \text{ m}^2 \quad (5.126)$$

Now, from (5.121)

$$\Delta\theta_{\text{co}} = \frac{234.215}{833 \cdot 0.2456} = 1.18^\circ\text{C}$$

and from (5.122),

$$\Delta\theta_{\text{frame}} = \frac{609.6}{50 \cdot 0.392} = 31.10^\circ\text{C}$$

Suppose that the ambient temperature is $\theta_{\text{amb}} = 40^\circ\text{C}$.

In this case, the winding temperature is

$$\theta_{\text{co}} = \theta_{\text{amb}} + \Delta\theta_{\text{co}} + \Delta\theta_{\text{frame}} = 40 + 1.18 + 31.10 = 72.28^\circ\text{C} < 80^\circ\text{C} \quad (5.127)$$

For this particular design, the $K_{\text{fin}} = 3.0$, which represents the frame area multiplication by fins, provided to increase heat transfer, may be somewhat reduced, especially if the ambient temperature would be 20°C as usual.

5.13 SUMMARY

- 100kW is, in general, considered the border between low- and medium-power IMs.
- Low-power IMs use a single stator and rotor stack and a finned frame cooled by an external ventilator mounted on the motor shaft.
- Low-power IMs use a rotor cage made of cast aluminum and a random wound coil stator winding made of round (in general) magnetic wire with 1–4(6) elementary conductors in parallel and 1–3 current paths in parallel.
- The number of poles is $2p_1 = 2, 4, 6, 8, 10$, in general.
- In designing an IM, we understand sizing of the IM; the breakdown torque, starting torque, and current are essential among design specifications.
- The design algorithm contains nine main stages: design specs; sizing the electric and magnetic circuits; adjustment of sizing data; verification of electric and magnetic loading (with eventual return to step one); computation of magnetization current, of equivalent circuit parameters, of losses, rated slip, and efficiency, of power factor; temperature rise; and performance check with eventual returns to step one, with adjusted electric and magnetic loadings.
- With assigned values to efficiency and power factor, based on past experience (Esson's output coefficient), the stator bore diameter D_{is} is calculated after assigning a value (dependent on and increasing with the number of poles) for $\lambda = \text{stack length/pole pitch} = 0.6\text{--}3$.
- The airgap should be small to increase power factor, but not too small to avoid too high stray losses.
- Double-layer, chorded coils stator windings are used in most cases.
- The essence of slot design is the design current density, which depends on the IM design letters A, B, C, and D, type of cooling, power level, and number of poles. Values in the range of 4–8 A/mm² are typical below 100kW.
- Semiclosed trapezoidal (rounded) stator and rotor slots with rectangular teeth are most adequate below 100kW.
- A key factor in the design is the teeth saturation factor $1 + K_{st}$ which is assigned an initial value which has to be met after a few design iterations.
- There are special stator/rotor slot number combinations to avoid synchronous parasitic torques, radial forces, noise, and vibration. They are given in Table 5.5.
- An essential design variable is the ratio $K_D = \text{outer/inner stator diameter}$. Its recommended value intervals decrease with the number of poles, based on past experience.
- When the lamination cross section is designed such that to produce the maximum airgap flux density for a given rated current density in the stator via optimal lamination algorithm (OLA) [3], the ratio K_D is

$2p_1$	2	4	6	8
K_D	0.58	0.65	0.69	0.72

Using values around these would most probably yield practical designs.

- Care must be exercised to correct the rotor resistance and leakage reactances for skin effect and leakage magnetic saturation at zero speed ($S = 1$). Otherwise, both starting torque and starting current calculations will contain notable errors.
- No electromagnetic design is practical unless the temperature rise corresponds to the winding insulation class. Detailed thermal design methodologies are presented in Chapter 12, Vol. 1.
- For more on IM design basics, see [4–6].
- Design optimization methods will be introduced in Chapter 8, Vol. 2.

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6 Induction Motor Design above 100 KW and Constant V and f (Size Your Own IM)

6.1 INTRODUCTION

Induction motors above 100kW are built for low voltage (480 V/50Hz, 460 V/60Hz, 690 V/50Hz) or medium voltages, 2.4–6kV and 12kV in special cases.

The advent of power electronic converters, especially those using insulated gate bipolar transistors (IGBTs), caused the rise of power/unit limit for low-voltage induction machines (IMs), 400 V/50Hz to 690 V/60Hz, to more than 2 MW. Although we are interested here in constant V- and f-fed IMs, this trend has to be observed.

Higher voltage, for given power, means lower cross section easier to wind stator windings. It also means lower cross-sectional feeding cables. However, it means thicker insulation in slots, etc. and thus a low slot-fill factor, and a slightly larger size machine. Also, a higher voltage power switch tends to be costly. Insulated coils are used. Radial–axial cooling is typical, so radial ventilation channels are provided. In contrast, low-voltage IMs above 100kW are easy to build, especially with round conductor coils (a few conductors in parallel with copper diameter below 3.0mm) and, as power goes up, with more than one current path, $a_1 > 1$. This is feasible when the number of poles increases with power: for $2p_1 = 6, 8, 10, 12$. If $2p_1 = 2, 4$ as power goes up, the current goes up and preformed coils made of stranded rectangular conductors, eventually with one to two turns/coil only, are required. Rigid coils are used, and slot insulation is provided.

Axial cooling, finned-frame, unistack configuration low-voltage IMs have been recently introduced up to 2.2 MW for low voltages (690 V/60Hz and less).

Most IMs are built with cage rotors, but, for heavy starting, or limited speed-control applications, wound rotors are used.

To cover most of these practical cases, we will unfold a design methodology treating the case of the same machine with medium-voltage stator and a low-voltage stator, and deep bar cage rotor, double-cage rotor, and wound rotor.

The electromagnetic design algorithm is similar to that applied below 100kW. However, the slot shape and stator coil shape, insulation arrangements, and parameters expressions accounting for saturation and skin effects are slightly, or more, different with the three types of rotors.

Knowledge in Chapters 9 and 11, Vol. 1, on skin and saturation effects, respectively, and for stray losses is directly applied throughout the design algorithm.

The deep bar and double-cage rotors will be designed based on fulfilment of breakdown torque and starting torque and current, to reduce drastically the number of iterations required. Even when optimization design is completed, the latter will be much less time-consuming, as the “initial” design is meeting approximately the main constraints. Unusually, high breakdown/rated torque ratios ($t_{be} = T_{bk}/T_{en} > 2.5$) are to be approached with open stator slots and larger l/τ ratios to obtain low stator leakage inductance values.

$$T_{bk} \approx \frac{3p_1}{2} \left(\frac{V_{ph}}{\omega_1} \right)^2 \frac{1}{L_{sc}}; \quad L_{sc} = L_{sl} + L_{rl} \quad (6.1)$$

where L_{sl} is the stator leakage, and L_{rl} is the rotor leakage inductance at breakdown torque. It may be argued that, in reality, the current at breakdown torque is rather large ($I_k/I_{ln} \geq T_{bk}/T_{en}$) and thus both leakage flux paths saturate notably and, consequently, both leakage inductances are somewhat reduced by 10%–15%. While this is true, it only means that ignoring the phenomenon in (6.1) will yield conservative (safe) results.

The starting torque T_{LR} and current I_{LR} are

$$T_{LR} \approx \frac{3(R_r)_{S=1} K_{istart}^2 I_{LR}^2 p_l}{\omega_l} \quad (6.2)$$

$$I_{LR} \approx \frac{V_{lph}}{\sqrt{(R_s + (R_r)_{S=1})^2 + \omega_l^2 ((L_{sl})_{S=1} + (L_{rl})_{S=1})^2}} \quad (6.3)$$

In general, $K_{istart} = 0.9\text{--}0.975$ for powers above 100 kW. Once the stator design, based on rated performance requirements, is done, with R_s and L_{sl} known, Equations (6.1) through (6.3) yield unique values for $(R_r)_{S=1}$, $(L_{rl})_{S=1}^{sat}$ and $(L_{rl})_{S=S_n}$. For a targeted efficiency and with the stator design done and core loss calculated, the rotor resistance at rated power (slip) can be calculated approximately:

$$(R_r)_{S=S_n} = \left(\frac{P_n}{\eta_{ln}} - 3R_s I_{ln}^2 - p_{iron} - p_{stray} - p_{mec} \right) \frac{1}{3(K_i I_{ln})^2} \quad (6.4)$$

$$\text{with } K_i = \frac{(I_r)_{S=S_n}}{I_{ln}} \approx 0.8 \cos \phi_{ln} + 0.2; \quad I_{ln} = \frac{P_n}{\sqrt{3} V_{ln} \cos \phi_{ln} \eta_{ln}} \quad (6.5)$$

We may assume that rotor bar resistance and leakage inductance at $S = 1$ represent 0.80–0.95 of their values calculated from (6.1) through (6.4).

$$(R_{be})_{S=1} = (0.85\text{--}0.95) \frac{(R_r)_{S=1}}{K_{bs}}; \quad K_{bs} = \frac{4m(W_l K_{wl})^2}{N_r} \quad (6.6)$$

$$(L_{be})_{S=1} = (0.75\text{--}0.80) \frac{(L_{rl})_{S=1}^{sat}}{K_{bs}} \quad (6.7)$$

Their values for rated slip are

$$(R_{be})_{S=S_n} = (0.7\text{--}0.85) \frac{(R_r)_{S=S_n}}{K_{bs}} \quad (6.8)$$

$$(L_{be})_{S=S_n} = (0.8\text{--}0.85) \frac{(L_{rl})_{S=S_n}}{K_{bs}} \quad (6.9)$$

With rectangular semiclosed rotor slots, the skin effect K_R and K_x coefficients are

$$K_R = \frac{(R_{be})_{S=1}}{(R_{be})_{S=S_n}} \approx \xi = \beta_{skin} h_r; \quad \beta_{skin} = \sqrt{\frac{\pi f_l \mu_0}{\rho_{Al}}} \quad (6.10)$$

$$\frac{(L_{be})_{S=1}^{unsat}}{(L_{be})_{S=S_n}} \approx \frac{\frac{h_r}{3b_r} K_x + \frac{h_{or}}{b_{or}}}{\frac{h_r}{3b_r} + \frac{h_{or}}{b_{or}}} \quad (6.11)$$

where h_{or} – rotor slot neck height, b_{or} – rotor slot opening
 h_r – active rotor slot height, b_r – rotor slot width.

Apparently, by assigning a value for h_{or}/b_{or} , Equation (6.11) allows us to calculate b_r because

$$K_x \approx \frac{3}{2\beta_{skin} h_r} \quad (6.12)$$

Now, the bar cross section for given rotor current density j_{AL} , ($A_b = h_r \cdot b_r$) is

$$A_b = \frac{I_b}{j_{AL}} = \frac{K_i I_{In}}{K_{bi} j_{AL}}; \quad K_{bi} = \frac{2m_l W_l K_{wl}}{N_r} \quad (6.13)$$

If A_b from (6.13) is too far away from $h_r \cdot b_r$, a more complex than rectangular slot shape is to be looked for to satisfy the values of K_R and K_X calculated from 6.10 and 6.11.

It should be noted that the rotor leakage inductance has also a differential component which has not been considered in (6.9) and (6.11).

Consequently, the above rationale is merely a basis for a closer-to-target rotor design start from the point of view of breakdown and starting torques, and starting current.

A similar approach may be taken for the double-cage rotor, but to separate the effects of the two cages, the starting and rated power conditions are taken to design the starting and working cage, respectively.

6.2 MEDIUM-VOLTAGE STATOR DESIGN

To save space, the design methodology will be unfolded simultaneously with a numerical example of an IM with the following specifications:

- $P_n = 736 \text{ kW}$ (1000 HP)
- Targeted efficiency: 0.96
- $V_{In} = 4 \text{ kV}$ (Δ)
- $f_l = 60 \text{ Hz}$, $2p_l = 4$ poles, $m = 3$ phases;

Service: Si_l continuous, insulation class F, temperature rise for class B (maximum 80 K).

The rotor will be designed separately for three cases: deep bar cage, double-cage, and wound rotor configurations.

6.2.1 MAIN STATOR DIMENSIONS

As we are going to use again Esson's constant (Chapter 4, Vol. 1), we need the apparent airgap power S_{gap} .

$$S_{gap} = 3EI_{In} = 3K_E V_{lph} I_{In} \quad (6.14)$$

$$\text{with } K_E = 0.98 - 0.005 \cdot p_l = 0.98 - 0.005 \cdot 2 = 0.97. \quad (6.15)$$

The rated current I_{ln} is

$$I_{ln} = \frac{P_n}{\sqrt{3} V_{ln} \cos \varphi_n \eta_n} \quad (6.16)$$

To find I_{ln} , we need to assign target values to rated efficiency η_n and power factor $\cos \varphi_n$, based on past experience and design objectives.

Although the design literature uses graphs of η_n , $\cos \varphi_n$ versus power and number of pole pairs p_1 , continuous progress in materials and technologies makes the η_n graphs quickly obsolete. However, the power factor data tend to be less dependent on material properties and more dependent on air-gap/pole-pitch ratio and on the leakage/magnetization inductance ratio (L_{sc}/L_m) as

$$(\cos \varphi)_{\substack{\text{max} \\ \text{zero loss}}} \approx \frac{1 - \frac{L_{sc}}{L_m + L_{sl}}}{1 + \frac{L_{sc}}{L_m + L_{sl}}} \quad (6.17)$$

Because L_{sc}/L_m ratio increases with the number of poles, the power factor decreases with the number of poles increasing. Also, as the power goes up, the ratio L_{sc}/L_m goes down, for given $2p_1$, and thus, $\cos \varphi_n$ increases with power.

Furthermore, for high breakdown torque, L_{sc} has to be small and thus the maximum power factor increases. Adopting a rated power factor is not easy. Data shown in Figure 6.1 are to be taken as purely orientative.

Corroborating (6.1) with (6.17), for given breakdown torque, the maximum ideal power factor $(\cos \varphi)_{\text{max}}$ can be obtained.

For our case, $\cos \varphi_n = 0.92\text{--}0.93$.

Rated efficiency may be purely assigned a desired, though realistic, value. Higher values are typical for high-efficiency motors. However, for $2p_1 < 8$, and $P_n > 100\text{ kW}$ the efficiency is above 0.9 and goes up to more than 0.95 for $P_n > 2000\text{ kW}$. For high-efficiency motors, efficiency at 2000 kW goes as high as 0.97 with recent designs.

With $\eta_n = 0.96$ and $\cos \varphi_n = 0.92$, the rated phase current I_{lnf} (6.16) is

$$I_{lnf} = \frac{736 \cdot 10^{-3}}{3 \cdot 4 \cdot 10^3 \cdot 0.92 \cdot 0.96} = \frac{120.42}{\sqrt{3}} \text{ A}$$

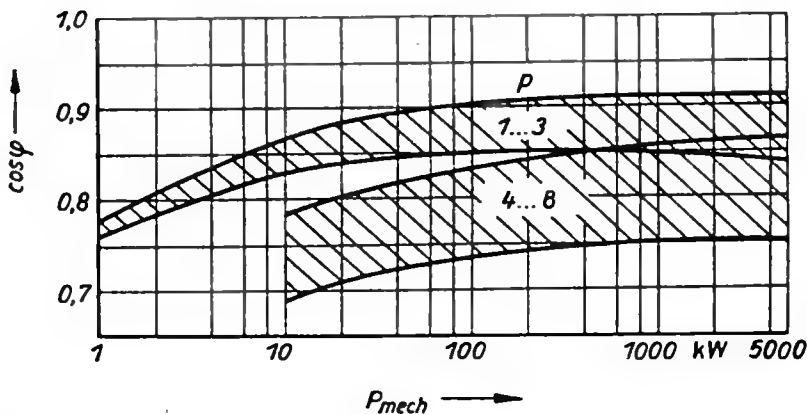


FIGURE 6.1 Typical power factor of cage-rotor IMs.

From (6.14), the airgap apparent power S_{gap} becomes

$$S_{\text{gap}} = \sqrt{3} \cdot 0.97 \cdot 4000 \cdot 120.42 = 808.307 \cdot 10^3 \text{ VA}$$

6.2.2 STATOR MAIN DIMENSIONS

The stator bore diameter D_{is} can be determined using Esson's constant from Equation (5.1):

$$D_{\text{is}} = \sqrt[3]{\frac{2p_1}{\pi\lambda_1} \frac{p_1}{f_1} \frac{S_{\text{gap}}}{C_0}} \quad (6.18)$$

From Figure 4.14, $C_0 = 265 \cdot 10^3 \text{ J/m}^3$, $\lambda = 1.1 = \text{stack length/pole pitch}$ (Table 5.1, Chapter 5) with (6.18), D_{is} is

$$D_{\text{is}} = \sqrt[3]{\frac{2 \cdot 2}{\pi \cdot 1.1} \frac{2}{60} \frac{808.307 \cdot 10^3}{265 \cdot 10^3}} = 0.49 \text{ m}$$

The airgap is chosen at $g = 1.5 \cdot 10^{-3} \text{ m}$ as a compromise between mechanical constraints and limitation of surface and tooth flux pulsation core losses.

The stack length l_i is

$$l_i = \lambda \tau = \lambda \cdot \frac{\pi D_{\text{is}}}{2p_1} = 1.1 \frac{\pi \cdot 0.49}{2 \cdot 2} = 0.423 \text{ m} \quad (6.19)$$

6.2.3 CORE CONSTRUCTION

Traditionally, the core is divided between a few elementary ones with radial ventilation channels in between. Such a configuration is typical for radial–axial cooling (Figure 6.2) [1].

Recently, the unistack core concept, rather standard for low power (below 100 kW), has been extended up to more than 2000 kW both for medium- and low-voltage stator IMs. In this case, axial air cooling of the finned motor frame is provided by a ventilator on motor shaft, outside bearings (Figure 6.3) [2].

As both concepts are in use and as, in Chapter 5, the unistack case has been considered, the divided stack configuration will be considered here for high-voltage stator case.

The inner/outer stator diameter ratio intervals have been recommended in Table 5.2. For $2p_1 = 4$, let us consider $K_D = 0.63$.

Complete range of enclosures and cooling arrangements can be built from the same basic design by using modular construction.

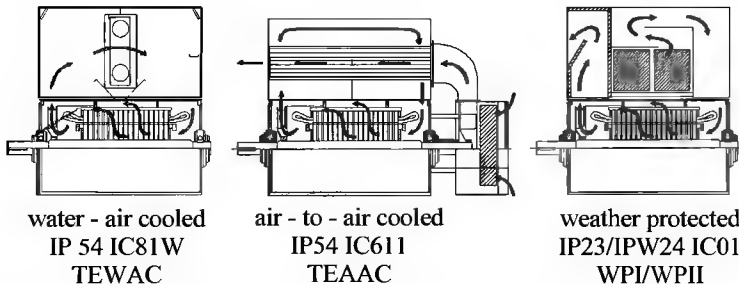


FIGURE 6.2 Divided core with radial–axial air cooling. (Source: ABB.)



FIGURE 6.3 Unistack with axial air cooling. (Source: ABB.)

Consequently, the outer stator diameter D_{out} is

$$D_{out} = \frac{D_{is}}{K_D} = \frac{0.5}{0.63} = 0.78\text{m} = 780\text{mm} \quad (6.20)$$

The airgap flux density is taken as $B_g = 0.7$ T. From Equation (4.14), C_0 is

$$C_0 = K_B \alpha_i K_{wl} \pi A_l B_g 2p_1 \quad (6.21)$$

Assuming a tooth saturation factor $(1 + K_{st}) = 1.25$, from Figure 4.13, $K_B = 1.1$, $\alpha_i = 0.65$. The winding factor is given a value $K_{wl} \approx 0.925$. With $B_g = 0.7$ T, $2p_1 = 4$ and $C_0 = 265 \cdot 103 \text{ J/m}^3$, the stator rated current sheet A_l is

$$A_l = \frac{265 \cdot 10^3}{1.1 \cdot 0.69 \cdot 0.925 \cdot \pi \cdot 0.7 \cdot 4} = 42.93 \cdot 10^3 \text{ Aturns/m}$$

This is a moderate value.

The pole flux ϕ is

$$\phi = \alpha_i \tau l_i B_g; \quad \tau = \frac{\pi D_{is}}{2p_1} \quad (6.22)$$

$$\tau = \frac{\pi \cdot 0.5}{2 \cdot 2} = 0.3925\text{m}; \quad \phi = 0.65 \cdot 0.3925 \cdot 0.423 \cdot 0.7 = 0.0755\text{Wb}$$

The number of turns per phase W_1 ($a_l = 1$ current paths) is

$$W_1 = \frac{K_E V_{ph}}{4 K_B f_i K_{wl} \phi} = \frac{0.97 \cdot 4000}{4 \cdot 1.1 \cdot 60 \cdot 0.925 \cdot 0.0755} = 210 \quad (6.23)$$

The number of conductors per slot n_s is written as

$$n_s = \frac{2m_1 a_l W_1}{N_s} \quad (6.24)$$

The number of stator slots. N_s , for $2p_1 = 4$ and $q = 6$, becomes

$$N_s = 2p_1 q_1 m_1 = 2 \cdot 2 \cdot 6 \cdot 3 = 72 \quad (6.25)$$

$$\text{So } n_s = \frac{2 \cdot 3 \cdot 1 \cdot 210}{72} = 17.50$$

We choose $n_s = 18$ conductors/slot, but we have to decrease the ideal stack length l_i to

$$l_i = l_i \cdot \frac{17.50}{18} = 0.423 \cdot \frac{17.50}{18} \approx 0.411 \text{ m}$$

$$\text{The flux per pole } \phi \approx \phi \cdot \frac{18}{17.50} = 0.07765 \text{ Wb}$$

The airgap flux density remains unchanged ($B_g = 0.7 \text{ T}$).

As the ideal stack length l_i is final (provided the teeth saturation factor K_{st} is confirmed later on), the former may be divided into a few parts.

Let us consider $n_{ch} = 6$ radial channels each 10^{-2} m wide ($b_{ch} = 10^{-2} \text{ m}$). Due to axial flux fringing, its equivalent width $b'_{ch} \approx 0.75 b_{ch} = 7.5 \cdot 10^{-3} \text{ m}$ ($g = 1.5 \text{ mm}$). So the total geometrical length L_{geo} is

$$L_{geo} = l_i + n_{ch} b'_{ch} = 0.411 + 6 \cdot 0.0075 = 0.456 \text{ m} \quad (6.26)$$

On the other hand, the length of each elementary stack is

$$l_s = \frac{L_{geo} - n_{ch} b_{ch}}{n_{ch} + 1} = \frac{0.456 - 6 \cdot 0.01}{6 + 1} \approx 0.05667 \text{ m} \quad (6.27)$$

As laminations are 0.5 mm thick, the number of laminations required to make l_s is easy to match. So there are seven stacks each 56 mm long (axially).

6.2.4 THE STATOR WINDING

For medium-voltage IMs, the winding is made of form-wound (rigid) coils. The slots are open in the stator so that the coils may be introduced in slots after prefabrication (Figure 6.4). The number of slots per pole/phase q_1 is to be chosen rather large as the slots are open and the airgap is only $g = 1.5 \cdot 10^{-3} \text{ m}$.

- EPOXY RESIN with mica insulation - Compact Industry Insulation System (MCI)
- voidless insulation construction
- tight fit in the slots
- free of partial discharges
- high thermal conductivity
- good moisture resistance

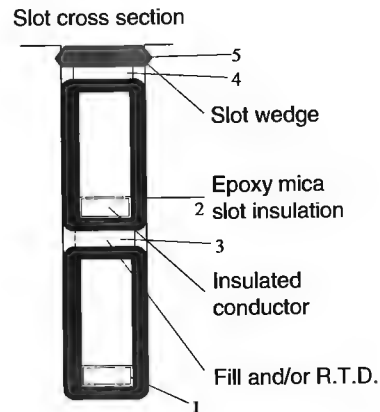


FIGURE 6.4 Open stator slot for high-voltage winding with form-wound (rigid) coils. (Source: ABB.)

The stator slot pitch τ_s is

$$\tau_s = \frac{\pi D_{is}}{N_s} = \frac{\pi \cdot 0.49}{72} = 0.02137 \text{ m} \quad (6.28)$$

The coil throw is taken as $y/\tau = 15/18 = 5/6$ ($q_l = 6$). There are 18 slots per pole to reduce drastically the fifth mmf space harmonic.

The winding factor K_{wl} is

$$K_{wl} = \frac{\sin \frac{\pi}{6}}{6 \cdot \sin \frac{\pi}{6 \cdot 6}} \sin \frac{\pi}{2} \frac{5}{6} = 0.9235$$

The winding is fully symmetric with $N_s/m_l a_l = 24$ (integer), $2p_l/a_l = 4/1$ (integer). Also $t = \text{g.c.d}(N_s, p_l) = p_l = 2$, and $N_s/m_l t = 72/(3 \cdot 2) = 12$ (integer).

The conductor cross section A_{Co} is (delta connection)

$$A_{Co} = \frac{I_{Inf}}{a_l J_{Co}}; \quad a_l = 1, \quad J_{Co} = 6.3 \text{ A/mm}^2; \quad I_{Inf} = 69.36 \text{ A} \quad (6.29)$$

$$A_{Co} = \frac{120.42}{1 \cdot 6.3 \sqrt{3}} = 11.048 \text{ mm}^2 = a_c \cdot b_c$$

A rectangular cross-sectional conductor will be used. The rectangular slot width b_s is

$$b_s = \tau_s \cdot (0.36 \div 0.5) = 0.021375 \cdot (0.36 \div 0.5) = 7.7 \div 10.7 \text{ mm} \quad (6.30)$$

Before choosing the slot width, it is useful to discuss the various insulation layers (Table 6.1).

The available conductor width in slot a_c is

$$a_c = b_s - b_{ins} = 10.0 - 4.4 = 5.6 \text{ mm} \quad (6.31)$$

This is a standardized value, and it was considered when adopting $b_s = 10 \text{ mm}$ (6.30). From (6.19), the conductor height b_c becomes

$$b_c = \frac{A_{Co}}{a_c} = \frac{11.048}{5.6} \approx 2 \text{ mm} \quad (6.32)$$

So the conductor size is $2 \times 5.6 \text{ mm}^2$.

TABLE 6.1
Stator Slot Insulation at 4 kV

Figure 6.4	Denomination	Thickness (mm)	
		Tangential	Radial
1	Conductor insulation (both sides)	$1 \cdot 0.4 = 0.4$	$18 \cdot 0.4 = 7.2$
2	Epoxy mica coil and slot insulation	4	$4 \cdot 2 = 8.0$
3	Interlayer insulation	-	$2 \cdot 1 = 2$
4	Wedge	-	$1 \cdot 4 = 4$
	Total	$b_{ins} = 4.4$	$h_{ins} = 21.2$

The slot height h_s is written as

$$h_s = h_{ins} + n_s b_c = 21.2 + 18 \cdot 2 = 57.2 \text{ mm} \quad (6.33)$$

Now the back iron radial thickness h_{cs} is

$$h_{cs} = \frac{D_{out} - D_{is}}{2} - h_s = \frac{780 - 490}{2} - 57.2 = 87.8 \text{ mm} \quad (6.34)$$

The back iron flux density B_{cs} is

$$B_{cs} = \frac{\phi}{2l_i h_{cs}} = \frac{0.0755}{2 \cdot 0.411 \cdot 0.0878} \approx 1.00 \text{ T} \quad (6.35)$$

This value is too small, so we may reduce the outer diameter to a lower value: $D_{out} = 730 \text{ mm}$; the back core flux density will be now close to 1.4 T.

The maximum tooth flux density B_{tmax} is

$$B_{tmax} = \frac{\tau_s B_g}{\tau_s - b_s} = \frac{21.37 \cdot 0.7}{21.37 - 10} = 1.315 \text{ T} \quad (6.36)$$

This is acceptable although even higher values (up to 1.8 T) are used as the tooth gets wider with radius and the average tooth flux density will be notably lower than B_{tmax} .

The stator design is now complete, but it is not definitive. After the rotor is designed, performance is computed. Design iterations may be required, at least to converge K_{st} (teeth saturation factor), if not for observing various constraints (related to performance or temperature rise).

6.3 LOW-VOLTAGE STATOR DESIGN

Traditionally, low-voltage stator IMs above 100kW have been built with round conductors (a few in parallel) in cases where the number of poles is large so that many current paths in parallel are feasible ($a_l = p_l$).

Recently, extension of variable speed IM drives tends to lead to the conclusion that low-voltage IMs up to 2000kW and more at 690 V/60 Hz (660 V, 50 Hz) or at (460 V/50 Hz, 400 V/50 Hz) are to be designed for constant V and f, but having in view the possibility of being used in general-purpose variable speed drives with voltage source PWM IGBT converter supplies. To this end, the machine insulation is enforced by using almost exclusively form-wound coils and open stator slots as for medium-voltage IMs. Also, insulated bearings are used to reduce bearing stray currents from PWM converters (at switching frequency).

Low-voltage PWM converters have a cost advantage. Also, a single stator stack is used (Figure 6.5).

The form-wound (rigid) coils (Figure 6.5) have a small number of turns (conductors) and a kind of crude transposition occurs in the end-connections zone to reduce the skin effect. For high-powers and $2p_l = 2, 4$, even 2–3 elementary conductors in parallel and $a_l = 2, 4$ current paths may be used to keep the elementary conductors within a size with limited skin effect (Chapter 9, Vol. 1).

In any case, skin effect calculations are required as the power goes up and the conductor cross section follows path. For a few elementary conductors or current paths in parallel, additional (circulating current) losses occur as detailed in Sections 9.2–9.3, Vol. 1.

Aside from these small differences, the stator design follows the same path as for medium-voltage stators.

This is why it will not be further treated here.

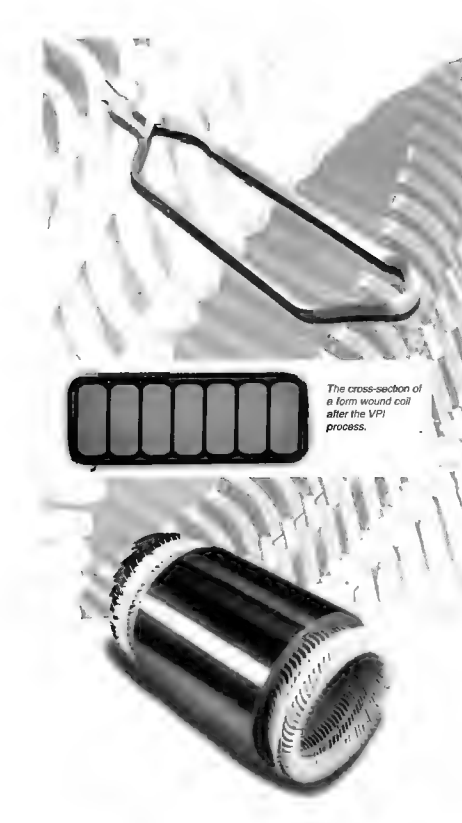


FIGURE 6.5 Open slot, low-voltage, single-stack stator winding (axial cooling). (Source: ABB.)

6.4 DEEP BAR CAGE ROTOR DESIGN

We will now resume here the design methodology in Section 6.2 with the deep bar cage-rotor design. More design specifications are needed for the deep bar cage.

$$\frac{\text{Breakdown torque}}{\text{Rated torque}} = \frac{T_{bk}}{T_{en}} = 2.7$$

$$\frac{\text{Starting current}}{\text{Rated current}} = \frac{I_{LR}}{I_n} \leq 6.1$$

$$\frac{\text{Starting torque}}{\text{Rated torque}} = \frac{T_{LR}}{T_{en}} = 1.2$$

The above data are merely an example.

As shown in Section 6.1, in order to size the deep bar cage, the stator leakage reactance X_{sl} is required. As the stator design is done, X_{sl} can be calculated.

6.4.1 STATOR LEAKAGE REACTANCE X_{sl}

As documented in Chapter 9, Vol. 1, the stator leakage reactance X_{sl} can be written as

$$X_{sl} = 15.8 \left(\frac{f_l}{100} \right) \left(\frac{W_l}{100} \right)^2 \frac{I_l}{p_l q_l} \sum \lambda_{is} \quad (6.37)$$

where $\sum \lambda_{is}$ is the sum of the leakage slot (λ_{ss}), differential (λ_{ds}), and end-connection (λ_{fs}) geometrical permeance coefficients.

$$\lambda_{ss} = \frac{(h_{s1} - h_{s3})}{3b_s} K_\beta + \frac{h_{s2}}{b_s} K_\beta + \frac{h_{s3}}{4b_s} K_\beta \quad (6.38)$$

$$K_\beta = \frac{1+3\beta}{4}; \quad \beta = \frac{y}{\tau} = \frac{5}{6}$$

In our case, (see Figure 6.6).

$$h_{s1} = n_s \cdot b_c + n_c \cdot 0.4 + 2 \cdot 2 + 1 = 18 \cdot 2 + 18 \cdot 0.4 + 2 \cdot 2 + 1 = 48.2 \text{ mm} \quad (6.39)$$

Also, $h_{s3} = 2 \cdot 2 + 1 = 5 \text{ mm}$, $h_{s2} = 1 \cdot 2 + 4 = 6 \text{ mm}$. From (6.37),

$$\lambda_{ss} = \left(\frac{48.2 - 5}{3 \cdot 10} + \frac{6}{10} \right) \left(\frac{1 + 3 \frac{5}{6}}{4} \right) + \frac{5}{4 \cdot 10} = 1.91$$

The differential geometrical permeance coefficient λ_{ds} is calculated as in Chapter 6, Vol. 1:

$$\lambda_{ds} = \frac{0.9 \cdot \tau_s (q_l K_{w1})^2 K_{01} \sigma_{dl}}{K_c g} \quad (6.40)$$

$$\text{with } K_{01} \approx 1 - 0.033 \frac{b_s^2}{g \tau_s} = 1 - 0.033 \frac{10^2}{1.5 \cdot 21.37} = 0.8975 \quad (6.41)$$

σ_{dl} is the ratio between the differential leakage and the main inductance, which is a function of coil chording (in slot pitch units) and q_s (slot/pole/phase) (Figure 6.7): $\sigma_{dl} = 0.3 \cdot 10^{-2}$.

The Carter coefficient K_c (as in Chapter 5, Vol. 1, Equations (5.53)–(5.56)) is

$$K_c = K_{c1} K_{c2} \quad (6.42)$$

K_{c2} is not known yet but, as the rotor slots are semiclosed, $K_{c2} < 1.1$ with $K_{c1} \gg K_{c2}$ due to the fact that the stator has open slots:

$$\gamma_1 = \frac{b_s^2}{5g + b_s} = \frac{10^2}{5 \cdot 1.5 + 10} = 5.714; \quad K_{c1} = \frac{\tau_s}{\tau_s - \gamma_1} = \frac{21.37}{21.37 - 5.71} = 1.365 \quad (6.43)$$

Consequently, $K_c \approx 1.365 \cdot 1.1 = 1.50$.

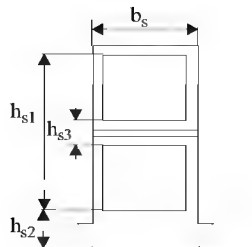


FIGURE 6.6 Stator slot geometry.

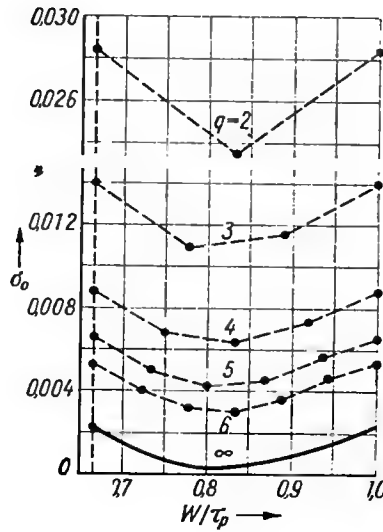


FIGURE 6.7 Differential leakage coefficient.

From (6.40),

$$\lambda_{ds} = \frac{0.9 \cdot 21.37 (6 \cdot 0.923)^2 \cdot 0.895 \cdot 0.3 \cdot 10^{-2}}{1.5 \cdot 1.5} = 0.6335$$

The end-connection permeance coefficient λ_{fs} is

$$\lambda_{fs} = 0.34 \frac{q_l}{l_i} (l_{fs} - 0.64 \beta \tau) \quad (6.44)$$

l_{fs} is the end-connection length (per one side of stator) and can be calculated based on the end-connection geometry in Figure 6.8.

$$\begin{aligned} l_{fs} &\approx 2(l_1 + l'_1) + \pi \gamma_1 = 2 \left(l_1 + \frac{\beta \tau}{\sin \alpha} \right) + \pi h_s \\ &= 2 \left(0.015 + \frac{5}{6} \frac{1}{2} \frac{0.314}{\sin 40^\circ} \right) + \pi \cdot 0.0562 = 0.548 \text{ m} \end{aligned} \quad (6.45)$$

So, from (6.44),

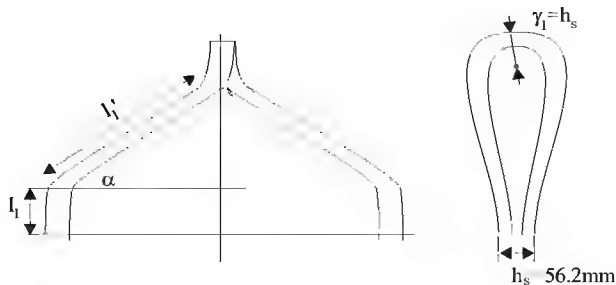


FIGURE 6.8 Stator end-connection coil geometry.

$$\lambda_{ts} = 0.34 \frac{6}{0.406} \left(0.548 - 0.64 \frac{5}{6} 0.314 \right) = 1.912$$

Finally, from (6.37), the stator leakage reactance X_{ls} (unaffected by leakage saturation) is

$$X_{ls} = 15.8 \left(\frac{60}{100} \right) \left(\frac{2 \cdot 6 \cdot 18}{100} \right)^2 \frac{0.406}{2 \cdot 6} (1.91 + 0.6335 + 1.912) = 6.667 \Omega$$

As the stator slots are open, leakage flux saturation does not occur even for $S = 1$ (standstill), at rated voltage. The leakage inductance of the field in the radial channels has been neglected.

The stator resistance R_s is

$$\begin{aligned} R_s &= K_R \rho_{Co80^\circ} \frac{W_l^2}{A_{Co}} (L_{geo} + l_{fs}) \\ &= 1 \cdot 1.8 \cdot 10^{-3} \left(1 + \frac{80 - 20}{272} \right) \cdot 12 \cdot 18 \cdot \frac{2 \cdot (0.451 + 0.548)}{11.048 \cdot 10^{-6}} = 0.8576 \Omega \end{aligned} \quad (6.46)$$

Although the rotor resistance at rated slip may be approximated from (6.4), it is easier to compare it to stator resistance:

$$(R_r)_{S=S_n} = (0.7 \div 0.8) R_s = 0.8 \cdot 0.8576 = 0.686 \Omega \quad (6.47)$$

for aluminium bar cage rotors and high-efficiency motors. The ratio of 0.7–0.8 in (6.47) is only orientative to produce a practical design start. Copper bars may be used when very high efficiency is targeted for single-stack axially-ventilated configurations.

6.4.2 THE ROTOR LEAKAGE INDUCTANCE L_{rl}

The rotor leakage inductance L_{rl} can be computed from the breakdown torque's expression (6.1):

$$L_{rl} = \frac{3p_l}{2T_{LR}} \left(\frac{V_{lph}}{\omega_l} \right)^2 - L_{sl}; \quad L_{sl} = \frac{X_{sl}}{\omega_l} = \frac{6.667}{2\pi 60} = 0.0177 \text{ H} \quad (6.48)$$

with

$$T_{LR} = t_{LR} T_{en} = t_{LR} \frac{P_n}{\omega_l} p_l \quad (6.49)$$

Now, L_{rl} is

$$L_{rl} = \frac{3p_l V_{lph}^2}{2t_{LR} P_n \omega_l} - L_{sl} = \frac{3 \cdot 4000^2}{2 \cdot 2.7 \cdot 736 \cdot 10^3 \cdot 2\pi 60} - 0.0177 = 0.01435 \text{ H} \quad (6.50)$$

From starting current and torque expressions (6.2) and (6.3),

$$(R_r)_{S=1} = \frac{\omega_l t_{LR} P_n \left(\frac{\omega_l}{p_l} \right)^{-1}}{3p_l K_{istart}^2 I_{LRphase}^2} = \frac{1.2 \cdot 736 \cdot 10^3}{3 \cdot 0.975^2 \cdot \left(5.6 \frac{120}{\sqrt{3}} \right)^2} = 2.0538 \Omega \quad (6.51)$$

$$\begin{aligned}
 (L_{rl})_{S=1} &= \frac{1}{\omega_1} \sqrt{\left(\frac{V_{tph}}{I_{LRphase}} \right)^2 - (R_s + (R_r)_{S=1})^2 - (L_{sl})_{S=1}} \\
 &= \frac{1}{2\pi 60} \left[\sqrt{\left(\frac{4000}{5.6 \frac{120}{\sqrt{3}}} \right)^2 - (1.083 + 2.0537)^2 - 6.667} \right] = 8.436 \cdot 10^{-3} \text{ H} \quad (6.52)
 \end{aligned}$$

Note that due to skin effect $(R_r)_{S=1} = 2.914(R_r)_{S=S_n}$ and due to both leakage saturation and skin effect, the rotor leakage inductance at stall is $(L_{rl})_{S=1} = 0.5878(L_{rl})_{S=S_n}$.

Using (6.8) and (6.10), the skin effect resistance ratio K_R is

$$K_R = \frac{(R_r)_{S=1}}{(R_r)_{S=S_n}} \cdot \frac{0.95}{0.8} = 2.999 \cdot \frac{0.95}{0.8} = 3.449 \quad (6.53)$$

The deep rotor bars are typically rectangular, but other shapes are also feasible. A modern aluminium rectangular bar insulated from core by a resin layer is shown in Figure 6.9. For a rectangular bar, the expression of K_R (when skin effect is notable) is (6.10):

$$K_R \approx \beta_{skin} h_r \quad (6.54)$$

$$\text{with } \beta_{skin} = \sqrt{\frac{\pi f_1 \mu_0}{\rho_{Al}}} = \sqrt{\frac{\pi \cdot 60 \cdot 1.256 \cdot 10^{-6}}{3.1 \cdot 10^{-8}}} = 87 \text{ m}^{-1} \quad (6.55)$$

The rotor bar height h_r is

$$h_r = \frac{3.449}{87} = 3.964 \cdot 10^{-2} \text{ m}$$

From (6.11),

$$\frac{(L_{rl})_{S=1}}{(L_{rl})_{S=S_n}} \cdot \frac{0.75}{0.85} = \frac{\frac{h_r}{3b_r} K_x + \frac{h_{or}}{b'_{or}}}{\frac{h_r}{3b_r} + \frac{h_{or}}{b_{or}}} = 0.5878 \cdot \frac{0.75}{0.85} = 0.51864 \quad (6.56)$$

$$K_x \approx \frac{3}{2\beta_{skin} h_r} = \frac{3}{2 \cdot 87 \cdot 3.977 \cdot 10^{-2}} = 0.43 \quad (6.57)$$

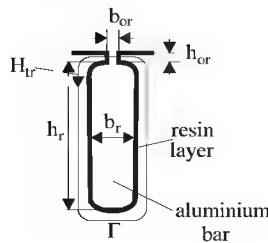


FIGURE 6.9 Insulated aluminium bar.

We have to choose the rotor slot neck $h_{or} = 1.0 \cdot 10^{-3}$ m from mechanical reasons. A value of b_{or} has to be chosen, say, $b_{or} = 2 \cdot 10^{-3}$ m. Now we have to check the saturation of the slot neck at start which modifies b_{or} into b'_{or} in (6.56). We use the approximate approach developed in Section 9.8, Vol. 1.

First, the bar current at start is

$$I_{bstart} = I_n \left(\frac{I_{start}}{I_n} \right) \cdot 0.95 \cdot K_{bs} \quad (6.58)$$

with $N_r = 64$ and straight rotor slots.

K_{bs} is the ratio between the reduced-to-stator and actual bar current.

$$K_{bs} = \frac{2mW_l K_{wl}}{N_r} = \frac{2 \cdot 3(12 \cdot 18)0.923}{64} = 18.69075 \quad (6.59)$$

$$I_{bstart} = 5.6 \cdot 0.95 \cdot \frac{120}{\sqrt{3}} \cdot 18.69 = 6896.9 \text{ A} \quad (6.60)$$

Using Ampere's law in the Γ contour shown in Figure 6.9 yields

$$\frac{B_{tr}}{\mu_{rel}\mu_0} [\tau_r - b_{or} + b_{os}\mu_{rel}(H_{tr})] = I_{bstart}\sqrt{2} \quad (6.61)$$

Iteratively, using the lamination magnetization curve (Table 5.4), with the rotor slot pitch τ_r ,

$$\tau_r = \frac{\pi(D_{is} - 2g)}{N_r} = \frac{\pi(0.49 - 2 \cdot 1.5 \cdot 10^{-3})}{64} = 23.893 \cdot 10^{-3} \text{ m} \quad (6.62)$$

the solution of (6.61) is $B_{tr} = 2.29$ T and $\mu_{rel} = 12$!

The new value of slot opening b'_{or} , to account for tooth top saturation at $S = 1$, is

$$b'_{or} = b_{or} + \frac{\tau_r - b_{or}}{\mu_{rel}} = 2 \cdot 10^{-3} + \frac{(23.893 - 2) \cdot 10^{-3}}{12} = 3.8244 \cdot 10^{-3} \text{ m} \quad (6.63)$$

Now, with $N_r = 64$ slots, the minimum rotor slot pitch (at slot bottom) is

$$\tau_{min} = \frac{\pi(D_{is} - 2g - 2h_r)}{N_r} = \frac{\pi(49 - 2 \cdot 1.5 - 2 \cdot 39.64) \cdot 10^{-3}}{64} = 20 \cdot 10^{-3} \text{ m} \quad (6.64)$$

With the maximum rotor tooth flux density $B_{tmax} = 1.45$ T, the maximum slot width b_{rmax} is

$$b_{rmax} = \tau_r - \frac{B_g}{B_{tmax}} \tau_r = 20 \cdot 10^{-3} \left(1 - \frac{0.7}{1.45} \right) = 10.58 \text{ mm} \quad (6.65)$$

The rated bar current density j_{Al} is

$$j_{Al} = \frac{I_b}{h_r b_r} = \frac{K_i I_n K_{bs}}{h_r b_r} = \frac{0.936 \cdot 120 \cdot 18.69}{39.64 \cdot 10 \sqrt{3}} = 3.061 \text{ A/mm}^2 \quad (6.66)$$

$$\text{with } K_i = 0.8 \cos \varphi_n + 0.2 = 0.8 \cdot 0.92 + 0.2 = 0.936 \quad (6.67)$$

We may now verify (6.56).

$$0.51864 \geq \frac{3 \cdot \frac{39.64}{10} \cdot 0.43 + \frac{1}{3.824}}{\frac{39.64}{3 \cdot 10} \cdot 1 + \frac{1}{2}} = \frac{0.829}{1.821} = 0.455!$$

The fact that approximately the large cage dimensions of $h_r = 39.64$ mm and $b_r = 10$ mm with $b_{or} = 2$ mm, $h_{or} = 1$ mm fulfilled the starting current, and starting and breakdown torques, for a rotor bar rated current density of only 3.06 A/mm², means that the design leaves room for further reduction of slot width.

We may now proceed, based on the rated bar current $I_b = 1213.38$ A (6.66), to the detailed design of the rotor slot (bar), end ring, rotor back iron.

Then, the teeth saturation coefficient K_{st} is calculated. If notably different from the initial value, the stator design may be redone from the beginning until acceptable convergence is obtained. Further on, the magnetization current equivalent circuit parameters, losses, rated efficiency and power factor, rated slip, torque, breakdown torque, starting torque, and starting current are all calculated.

Most of these calculations are to be done with the same expressions as in Chapter 5 which is why we do not repeat them here.

6.5 DOUBLE-CAGE ROTOR DESIGN

When a higher starting torque for lower starting current and good efficiency are all required, the double-cage rotor comes into play. However, the breakdown torque and the power factor tend to be slightly lower as the rotor cage leakage inductance at load is larger.

The main constraints are

$$\frac{T_{bk}}{T_{en}} = t_{ek} > 2.0$$

$$\frac{I_{LR}}{I_{In}} < 5.35$$

$$\frac{T_{LR}}{T_{en}} = t_{LR} \geq 1.5$$

Typical geometries of double-cage rotors are shown in Figure 6.10.

The upper cage is the starting cage as most of rotor current flows through it at start, mainly because the working cage leakage reactance is high, so its current at primary (f_1) frequency is small.

In contrast, at rated slip ($S < 1.5 \cdot 10^{-2}$), most of the rotor current flows through the working (lower) cage as its resistance is smaller and its reactance (at slip frequency) is smaller than for the starting cage resistance.

In principle, it is fair to say that there is always current in both cages, but, at high rotor frequency ($f_2 = f_1$), the upper cage is more important, whereas at rated rotor frequency ($f_2 = S_n f_1$), the lower cage takes more current.

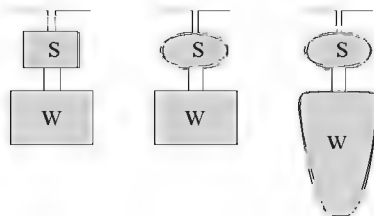


FIGURE 6.10 Typical rotor slot geometries for double-cage rotors.

The end ring may be common to both cages, but, when frequent starts are considered, separate end rings are preferred because of different thermal expansion (Figure 6.11). It is also possible to make the upper cage of brass and the lower cage of copper.

The equivalent circuit for the double cage has been introduced in Chapter 9 (Section 9.7), Vol. 1, and is inserted here only for easy reference (Figure 6.12).

For the common end-ring case, $R_{\text{ring}} = R_{\text{ring}'}$ (ring equivalent resistance, reduced-to-bar resistance), and R_{bs} and R_{bw} are the upper and lower bar resistances.

For separate end rings, $R_{\text{ring}} = 0$, but the rings resistances $R_{\text{bs}} \rightarrow R_{\text{bs}} + R_{\text{rings}}$, $R_{\text{bw}} \rightarrow R_{\text{bw}} + R_{\text{ringw}}$. L_{rl} contains the differential leakage inductance only for common end ring.

Also,

$$L_{\text{bs}} \approx \mu_0 (l_{\text{geo}} + l_s) \frac{h_{\text{rs}}}{b_{\text{rs}}}; \quad L_e = \mu_0 l_{\text{geo}} \frac{h_{\text{or}}}{b_{\text{or}}} \quad (6.68)$$

$$L_{\text{bw}} \approx \mu_0 (l_{\text{geo}} + l_w) \left(\frac{h_{\text{rw}}}{3b_{\text{rw}}} + \frac{h_n}{a_n} + \frac{h_{\text{rs}}}{b_{\text{rs}}} \right)$$

The mutual leakage inductance L_{ml} is

$$L_{\text{ml}} \approx \mu_0 l_{\text{geo}} \frac{h_{\text{rs}}}{2b_{\text{rs}}} \quad (6.69)$$

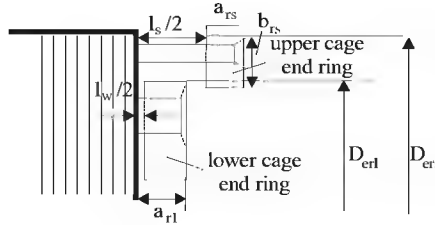


FIGURE 6.11 Separate end rings.

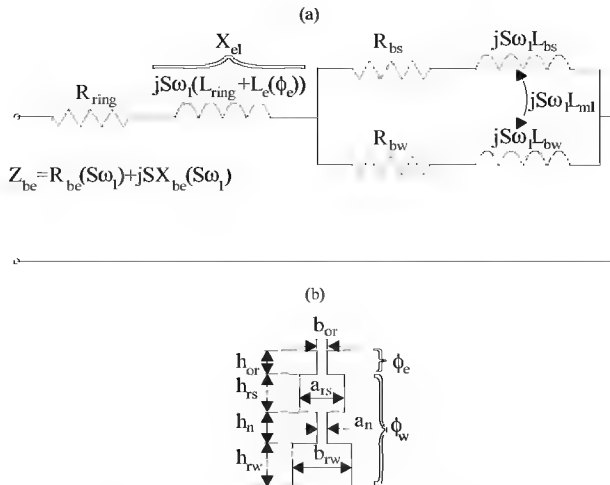


FIGURE 6.12 Equivalent circuit of Figure 6.11 double-cage (a) and slot geometrical parameters (b).

In reality, instead of l_{geo} , we should use l_i , but in this case, the leakage inductance of the rotor bar field in the radial channels is to be considered. The two phenomena are lumped into l_{geo} (the geometrical stack length).

The lengths of bars outside the stack are l_s and l_w , respectively.

First, we approach the starting cage, made of brass (in our case) with a resistivity $\rho_{\text{brass}} = 4\rho_{\text{Co}} = 4 \cdot 2.19 \cdot 10^{-8} = 8.76 \cdot 10^{-8} \text{ } (\Omega\text{m})$. We do this based on the fact that, at start, only the starting (upper) cage works.

$$T_{\text{LR}} = t_{\text{LR}} T_{\text{en}} \approx t_{\text{LR}} \frac{P_n}{\omega_1} p_l = 3 \frac{p_l}{\omega_1} (R_r)_{S=1} 0.95 I_{\text{LR}}^2 \quad (6.70)$$

$$I_{\text{LR}} = 5.35 I_{\text{In}} = 5.35 \frac{120}{\sqrt{3}} = 371.1 \text{ A} \quad (6.71)$$

From (6.70), the rotor resistance $((R_r)_{S=1})$ is

$$(R_r)_{S=1} = \frac{t_{\text{LR}} P_n}{3 \cdot 0.95 \cdot I_{\text{LR}}^2} = \frac{1.5 \cdot 736 \cdot 10^3}{3 \cdot 0.95 \cdot 371.1^2} = 2.8128 \text{ } \Omega \quad (6.72)$$

From the equivalent circuit at start, the rotor leakage inductance at $S = 1$, $(L_{rl})_{S=1}$, is

$$\begin{aligned} (L_{rl})_{S=1} &= \left(\sqrt{\left(\frac{V_{\text{ph}}}{I_{\text{LR}}} \right)^2 - (R_s + (R_r)_{S=1})^2} - L_{ls} \right) \frac{1}{\omega_1} \\ &= \left(\sqrt{\left(\frac{4000}{371.1} \right)^2 - (0.8576 + 2.8128)^2} - 6.667 \right) \cdot \frac{1}{2\pi 120} = 9.174 \cdot 10^{-3} \text{ H} \end{aligned} \quad (6.73)$$

If it were not for the rotor working cage large leakage in parallel with the starting cage, $(R_r)_{S=1}$ and $(L_{rl})_{S=1}$ would simply refer to the starting cage whose design would then have been straightforward.

To produce realistic results, we first have to design the working cage.

To do so, we again assume that the copper working cage (made of copper) resistance, referred to the stator, is

$$(R_r)_{S=S_n} = 0.8 R_s = 0.8 \cdot 0.8576 = 0.6861 \text{ } \Omega \quad (6.74)$$

This is the same as for the aluminium deep bar cage, although it is copper this time. The reason is to limit the slot area and depth in the rotor.

6.5.1 WORKING CAGE SIZING

The working cage bar approximate resistance R_{bc} is

$$R_{\text{bc}} \approx (0.7 \div 0.8) \frac{(R_r)_{S=S_n}}{K_{\text{bs}}} = \frac{0.75 \cdot 0.6861}{7452.67} = 0.6905 \cdot 10^{-4} \text{ } \Omega \quad (6.75)$$

From (6.6), K_{bs} is

$$K_{\text{bs}} = \frac{4m(W_l K_{w1})^2}{N_r} = \frac{4 \cdot 3 \cdot (12 \cdot 18 \cdot 0.923)^2}{64} = 7452.67 \quad (6.76)$$

The working cage cross section A_{bw} is

$$A_{bw} = \frac{\rho_{Co}(I_{geo} + I_w)}{R_{bw}} = 2.2 \cdot 10^{-8} \frac{0.451 + 0.01}{0.6905 \cdot 10^{-4}} = 1.468 \cdot 10^{-4} \text{ m}^2 \quad (6.77)$$

The rated bar current I_b (already calculated from (6.66)) is $I_b = 1213.38$ A. and thus. the rated current density in the copper bar j_{Cob} is

$$j_{Cob} = \frac{I_b}{A_{bw}} = \frac{1213.38}{1.468 \cdot 10^{-4}} = 8.26 \cdot 10^6 \text{ A/m}^2 \quad (6.78)$$

This is a value close to the maximum value acceptable for radial-axial air cooling.

A profiled bar $1 \cdot 10^{-2}$ m wide and $1.5 \cdot 10^{-2}$ m high is used: $b_{rw} = 1 \cdot 10^{-2}$ m, $h_{rw} = 1.5 \cdot 10^{-2}$ m.

The current density in the end ring has to be smaller than in the bar (about 0.7–0.8), and thus, the working (copper) end-ring cross section A_{rw} is

$$A_{rw} = \frac{A_{bw}}{0.75 \cdot 2 \sin \frac{\pi p_1}{N_r}} = \frac{1.468 \cdot 10^{-4}}{1.5 \sin \frac{\pi \cdot 2}{64}} = 9.9864 \cdot 10^{-4} \text{ m}^2 \quad (6.79)$$

So (from Figure 6.11).

$$a_{rl} b_{rl} = 9.9846 \cdot 10^{-4} \text{ m}^2 \quad (6.80)$$

We may choose $a_{rl} = 2 \cdot 10^{-2}$ m and $b_{rl} = 5.0 \cdot 10^{-2}$ m.

The slot neck dimensions will be considered as (Figure 6.12b) $b_{or} = 2.5 \cdot 10^{-3}$ m, $h_{or} = 3.2 \cdot 10^{-3}$ m (larger than in the former case) as we can afford a large slot neck permeance coefficient h_{or}/b_{or} because the working cage slot leakage permeance coefficient is already large.

Even for the deep cage, we could afford a larger h_{or} and b_{or} to stand the large mechanical centrifugal stresses occurring during full operation speed.

We go on to calculate the rotor differential geometrical permeance coefficient for the working cage (Chapter 5, Vol. 1, Equation (5.82)).

$$\lambda_{dr} = \frac{0.9 \cdot \tau_r \gamma_{dr}}{K_{cg}} \left(\frac{N_r}{6p_1} \right)^2 : \quad \gamma_{dr} = 9 \left(\frac{6p_1}{N_r} \right)^2 \cdot 10^{-2} \quad (6.81)$$

$$\gamma_{dr} = 9 \left(\frac{6 \cdot 2}{64} \right)^2 \cdot 10^{-2} = 0.3164 \cdot 10^{-2}$$

$$\lambda_{dr} = \frac{0.9 \cdot 23.893}{1.5 \cdot 1.5} \cdot 0.3164 \cdot 10^{-2} \cdot \left(\frac{64}{12} \right)^2 = 0.86$$

The saturated value of λ_{drl} takes into account the influence of tooth saturation coefficient K_{st} assumed to be $K_{st} = 0.25$.

$$\lambda_{drs} = \frac{\lambda_{drl}}{1 + K_{st}} = \frac{0.96}{1.25} = 0.688 \quad (6.82)$$

The working end-ring-specific geometric permeance λ_{erl} is (Chapter 5, Vol. 1, Equation (5.83)):

$$\begin{aligned}\lambda_{erl} &= \frac{2.3 \cdot (D_{erl} - b_{rl})}{N_r I_{geo} 4 \sin^2 \left(\frac{\pi p_l}{N_r} \right)} \log \frac{4.7 \cdot (D_{erl} - b_{rl})}{b_{rl} + 2a_{rl}} \\ &\approx \frac{2.3 \cdot (0.49 - 3 \cdot 10^{-3} - 4 \cdot 10^{-2} - 5 \cdot 10^{-2})}{64 \cdot 0.451 \cdot 4 \sin^2 \left(\frac{\pi \cdot 2}{64} \right)} \log \frac{397 \cdot 10^{-3}}{(50 + 2 \cdot 20) \cdot 10^{-3}} = 0.530\end{aligned}\quad (6.83)$$

The working end-ring leakage reactance X_{rel} (Chapter 5, Vol. 1, Equation 5.85) is

$$X_{rel} \approx \mu_0 2\pi f_l I_{geo} \cdot 10^{-8} \lambda_{erl} = 7.85 \cdot 60 \cdot 0.451 \cdot 10^{-6} \cdot 0.530 = 1.1258 \cdot 10^{-4} \Omega \quad (6.84)$$

The common (mutual) reactance (Figure 6.12) is made of the differential and slot neck components, unsaturated, for rated conditions and saturated at $S = 1$:

$$\begin{aligned}(X_{el})_{S=S_n} &= 2\pi f_l I_{geo} \mu_0 \left(\lambda_{dr} + \frac{h_{or}}{b_{or}} \right) \\ (X_{el})_{S=1} &= 2\pi f_l I_{geo} \mu_0 \left(\lambda_{drs} + \frac{h_{or}}{b'_{or}} \right)\end{aligned}\quad (6.85)$$

The influence of tooth top saturation at start is considered as before (in 6.60–6.63).

With $b'_{or} = 1.4b_{or}$, we obtain

$$\begin{aligned}(X_{el})_{S=S_n} &= 2\pi 60 \cdot 0.451 \cdot \left(0.86 + \frac{2.5}{3.2} \right) \cdot 1.256 \cdot 10^{-6} = 3.503 \cdot 10^{-4} \Omega \\ (X_{el})_{S=1} &= 2\pi 60 \cdot 0.451 \cdot \left(0.688 + \frac{2.5}{3.5 + 1.4} \right) \cdot 1.256 \cdot 10^{-6} = 2.6595 \cdot 10^{-4} \Omega\end{aligned}$$

Now, we can calculate the approximate values of starting cage resistance from the value of the equivalent resistance at start R_{start} .

$$R_{start} = \frac{(R_r)_{S=1}}{K_{bs}} = \frac{2.8128}{7452.67} = 3.7742 \cdot 10^{-4} \Omega \quad (6.86)$$

From Figure 6.12a, the starting cage resistance R_{bes} is

$$R_{bes} \approx \frac{R_{start}^2 + X_{el}^2}{R_{start}} = \frac{(3.7742 \cdot 10^{-4})^2 + (2.6595 \cdot 10^{-4})^2}{(3.7742 \cdot 10^{-4})} = 5.648 \cdot 10^{-4} \Omega \quad (6.87)$$

For the working cage reactance X_{rlw} , we also have

$$X_{rlw} = \frac{R_{start}^2 + X_{el}^2}{X_{el}} = \frac{(3.7742 \cdot 10^{-4})^2 + (2.6595 \cdot 10^{-4})^2}{2.6595 \cdot 10^{-4}} = 8.0156 \cdot 10^{-4} \Omega \quad (6.88)$$

The presence of common leakage X_{el} makes starting cage resistance R_{bes} larger than the equivalent starting resistance R_{start} . The difference is notable and affects the sizing of the starting cage.

The value of R_{bes} includes the influence of starting end ring. The starting cage bar resistance R_b is approximately

$$R_{bs} \approx R_{bes} (0.9 \div 0.95) = 0.9 \cdot 5.648 \cdot 10^{-4} = 5.083 \cdot 10^{-4} \Omega \quad (6.89)$$

The cross section of the starting bar A_{bs} is

$$A_{bs} = \rho_{brass} \frac{I_{geo} + I_s}{R_{bs}} = 4 \cdot 2.2 \cdot 10^{-8} \frac{0.451 + 0.05}{5.083 \cdot 10^{-4}} = 0.8673 \cdot 10^{-4} \text{ m}^2 \quad (6.90)$$

The utilization of brass has reduced drastically the starting cage bar cross section. The length of starting cage bar was prolonged by $l_s = 5 \cdot 10^{-2} \text{ m}$ (Figure 6.11) as only the working end-ring axial length is $a_{rl} = 2 \cdot 10^{-2} \text{ m}$ on each side of the stack.

We may adopt a rectangular bar again (Figure 6.12b) with $b_{rs} = 1 \cdot 10^{-2} \text{ m}$ and $h_{rs} = 0.86 \cdot 10^{-2} \text{ m}$.

The end-ring cross section A_{rs} (as in 6.79) is

$$A_{rs} = \frac{A_{bs}}{0.75 \cdot 2 \cdot \sin \frac{\pi p_l}{N_r}} = \frac{0.8673 \cdot 10^{-4}}{1.5 \cdot \sin \frac{\pi \cdot 2}{64}} = 5.898 \cdot 10^{-4} \text{ m}^2 \quad (6.91)$$

The dimensions of the starting cage ring are chosen to be the following: $a_{rs} \cdot b_{rs}$ (Figure 6.11) $= 2 \cdot 10^{-2} \cdot 3 \cdot 10^{-2} \text{ m}^2$.

Now, we calculate more precisely the starting cage bar equivalent resistance, R_{bes} , and that of the working cage, R_{bew} ,

$$R_{bes} = R_{bs} + \frac{R_{rs}}{2 \sin^2 \left(\frac{\pi p_l}{N_r} \right)}; \quad R_{rs} = \rho_{brass} \frac{I_{rs}}{A_{rs}} \quad (6.92)$$

$$R_{bew} = R_{bw} + \frac{R_{rw}}{2 \sin^2 \left(\frac{\pi p_l}{N_r} \right)}; \quad R_{rw} = \rho_{Co} \frac{I_{rw}}{A_{rw}}$$

The size of end rings is shown in Figure 6.13.

It is now straightforward to calculate R_{bes} and R_{bew} based on I_{rs} and I_{rw} , the end-ring segments length shown in Figure 6.13.

The only unknowns are (Figure 6.13) the stator middle neck dimensions a_n and h_n .

From the value of the working cage reactance (X_{rlw} (6.88)), if we subtract the working end-ring reactance (X_{rel} , (6.84)), we are left with the working cage slot reactance in rotor terms.

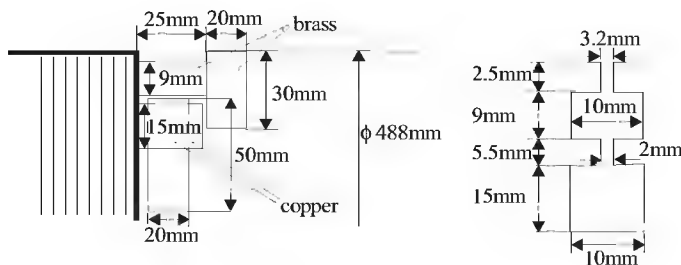


FIGURE 6.13 Double-cage design geometry.

$$\begin{aligned}
 X_{\text{rlslot}} &= X_{\text{rlw}} - X_{\text{rlcl}} \\
 X_{\text{rlslot}} &= 8.0156 \cdot 10^{-4} - 1.1258 \cdot 10^{-4} = 6.8898 \cdot 10^{-4} \, \Omega
 \end{aligned}
 \tag{6.93}$$

The slot geometrical permeance coefficient for the working cage λ_{rl} is

$$\lambda_{\text{rl}} = \frac{X_{\text{rlslot}}}{2\pi f_l \mu_o l_{\text{geo}}} = \frac{6.8898 \cdot 10^{-4}}{2\pi 60 \cdot 1.256 \cdot 10^{-6} \cdot 0.451} = 3.243!
 \tag{6.94}$$

Let us remember that λ_{rl} refers only to working cage body and the middle neck interval,

$$\lambda_{\text{rl}} = \frac{h_{\text{rw}}}{3b_{\text{rw}}} + \frac{h_n}{a_n}
 \tag{6.95}$$

Finally,

$$\frac{h_n}{a_n} = 3.243 - \frac{15}{3 \cdot 10} = 2.7435$$

$$\text{with } a_n = 2 \cdot 10^{-3} \text{ m; } h_n = 5.5 \cdot 10^{-3} \text{ m}$$

As for the deep bar cage rotor, we were able to determine all working and starting cage dimensions so that they meet (approximately) the starting torque, current, and rated efficiency and power factor.

Again, all the parameters and performance may now be calculated as for the design in Chapter 5.

Such an approach would drastically reduce the number of design iterations until satisfactory performance is obtained with all constraints observed. Also, having a workable initial sizing helps to approach the optimization design stage if so desired.

6.6 WOUND ROTOR DESIGN

For the stator as in previous paragraphs, we approach here the wound rotor design methodology. The rotor winding has diametrical coils and is placed in two layers. As the stator slots are open, to limit the airgap flux pulsations (and consequently, the tooth flux pulsation additional core losses), the rotor slots are to be at least half-closed (Figure 6.13). This leads to the solution with wave-shaped half-preformed coils made of a single bar with one or more (2) conductors in parallel.

The half-shaped uniform coils are introduced frontally in the slots, and then, the coil unfinished terminals are bent as required to form wave coils which suppose less material to connect the coils into phases.

The number of rotor slots N_r is now chosen to yield an integer q_2 (slots/pole/pitch) which is smaller than $q_1 = 6$. Choosing $q_2 = 5$, the value of N_r is

$$N_r = 2p_1 m q_2 = 2 \cdot 2 \cdot 3 \cdot 5 = 60
 \tag{6.96}$$

A smaller number of rotor than stator slots lead to low tooth flux pulsation core losses (Chapter 11, Vol. 1).

The winding factor is now the distribution factor.

$$K_{w2} = \frac{\sin \frac{\pi}{6}}{q_2 \sin \frac{\pi}{6q_2}} = \frac{0.5}{5 \sin \left(\frac{\pi}{6 \cdot 5} \right)} = 0.9566
 \tag{6.97}$$

The rotor tooth pitch τ_r is

$$\tau_r = \frac{\pi(D_{is} - 2g)}{N_r} = \frac{\pi(490 - 2 \cdot 1.5) \cdot 10^{-3}}{60} = 25.486 \cdot 10^{-3} \text{ m} \quad (6.98)$$

As we decided to use wave coils, there will be one conductor (turn) per coil and thus $n_r = 2$ conductor/slot.

The number of turns per phase, with a_2 current path in parallel, W_2 , is ($a_2 = 1$):

$$W_2 = \frac{N_r n_r}{2m_1 a_2} = \frac{60 \cdot 2}{2 \cdot 3 \cdot 1} = 20 \text{ turns/phase} \quad (6.99)$$

It is now straightforward to calculate the emf E_2 in the rotor phase based on $E_1 = K_E V_{ph}$ in the stator.

$$E_2 = E_1 \frac{W_2 K_{w2}}{W_1 K_{w1}} = \frac{0.97 \cdot 4000 \cdot 20 \cdot 0.9566}{12 \cdot 18 \cdot 0.923} = 372.33 \text{ V} \quad (6.100)$$

Usually, star connection is used and the inter-slip-ring (line) rotor voltage V_{2l} is

$$V_{2l} = E_2 \sqrt{3} = 372.33 \cdot 1.73 = 644 \text{ V} \quad (6.101)$$

We know by now the rated stator current and the ratio $K_i = 0.936$ (for $\cos \varphi_n = 0.92$) between rotor and stator current (6.67). Consequently, the rotor phase rated current is

$$I_{2n} = K_i \frac{W_1 K_{w1}}{W_2 K_{w2}} I_n = 0.936 \cdot \frac{12 \cdot 18 \cdot 0.923}{20 \cdot 0.9566} \cdot \frac{120}{\sqrt{3}} = 676.56 \text{ A} \quad (6.102)$$

The conductor area may be calculated after adopting the rated current density: $j_{Cor} > j_{Co} = 6.3 \text{ A/mm}^2$, by (5–15)%. We choose here $j_{Cor} = 7 \text{ A/mm}^2$.

$$A_{Cor} = \frac{I_{2n}}{j_{Cor}} = \frac{676.56}{7 \cdot 10^6} = 96.65 \cdot 10^{-6} \text{ m}^2 \quad (6.103)$$

As the voltage is rather low, the conductor insulation is only 0.5 mm thick/side, also 0.5 mm foil slot insulation per side and an interlayer foil insulation of 0.5 mm suffice.

The slot width b_r is

$$b_r < \tau_r \left(1 - \frac{B_g}{B_{trmin}} \right) = 28.486 \cdot 10^{-3} \left(1 - \frac{0.7}{1.3} \right) = 13.14 \cdot 10^{-3} \text{ m} \quad (6.104)$$

Let us choose $b_r = 10.5 \cdot 10^{-3} \text{ m}$

Considering the insulation system width (both side) of $2 \cdot 10^{-3} \text{ m}$, the conductor allowable width $a_{cond} \approx 8.5 \cdot 10^{-3} \text{ m}$. We choose two vertical conductors, so each has $a_c = 4 \cdot 10^{-3} \text{ m}$. The conductor height b_c is

$$b_c = \frac{A_{Cor}}{2a_c} = \frac{96.65}{2 \cdot 4.0} \approx 12 \cdot 10^{-3} \text{ m} \quad (6.105)$$

Rectangular conductor cross sections are standardized, so the sizes found in (6.105) may have to be slightly corrected.

Now the final slot size (with $1.5 \cdot 10^{-3} \text{ m}$ total insulation height (radially)) is shown in Figure 6.14.

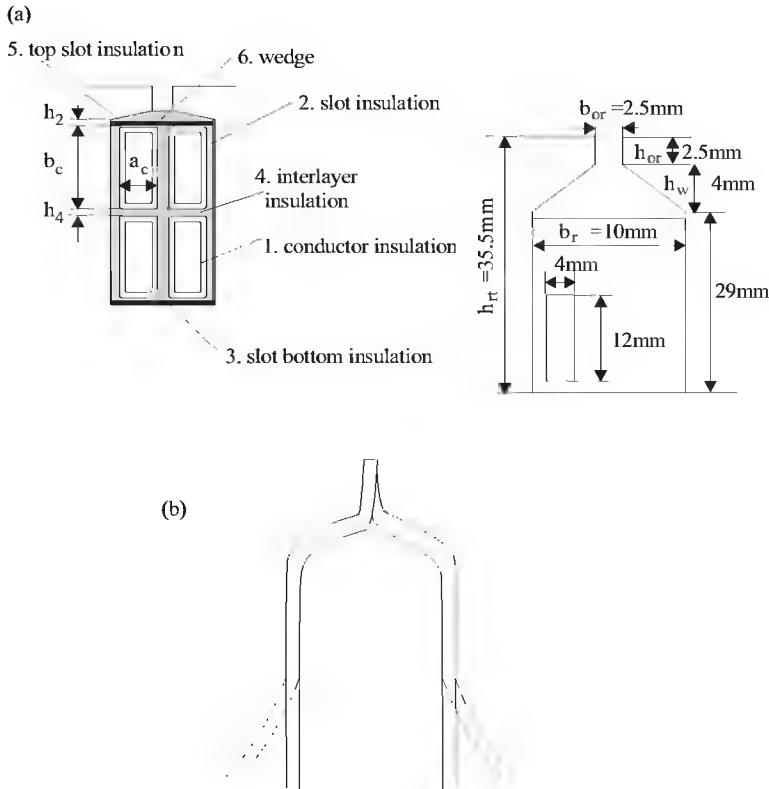


FIGURE 6.14 (a) Wound rotor slot and (b) half-preformed wave coils.

The maximum rotor tooth flux density B_{trmax} is

$$B_{trmax} = \left(\left(\pi(D_{is} - 2g) - 2h_{rt} \right) - N_r b_r \right)^{-1} \pi(D_{is} - 2g) B_g$$

$$= \frac{\pi(490 - 3) \cdot 0.8}{\left[\pi(490 - 3 - 2 \cdot 35.5) - 60 \cdot 10.5 \right]} = 1.582\text{T} \quad (6.106)$$

This is still an acceptable value.

The key aspects of wound rotor designs are by now solved. However, this time we will pursue the whole design methodology to prove the performance.

For the wound rotor, the efficiency, power factor ($\cos \varphi_n = 0.92$), and the breakdown p.u. torque $t_{bk} = 2.5$ (in our case) are the key performance parameters.

The starting performance is not so important as such a machine is to use either power electronics or a variable resistance for limited range speed control and for starting, respectively.

6.6.1 THE ROTOR BACK IRON HEIGHT

The rotor back iron has to flow half the pole flux, for a given maximum flux density $B_{cr} = (1.4\text{--}1.6)\text{ T}$. The rotor back core height h_{cr} is

$$h_{cr} = \frac{B_g}{B_{cr}} \frac{\tau}{\pi} \frac{1}{K_{Fe}} = \frac{0.7}{1.4} \frac{0.3925}{\pi \cdot 0.98} = 0.0638\text{m} \quad (6.107)$$

The interior lamination diameter D_{ir} is

$$D_{ir} = D_{is} - 2g - 2h_r - 2h_{cs} = (490 - 3 - 71 - 127.6) \cdot 10^{-3} \approx 0.290 \text{ m} \quad (6.108)$$

So there is enough radial room to accommodate the shaft.

6.7 IM WITH WOUND ROTOR-PERFORMANCE COMPUTATION

We start the performance computation with the magnetization current to continue with wound rotor parameters (stator parameters have already been computed); then losses and efficiency follow. The computation ends with the no-load current, short-circuit current, breakdown slip, rated slip, breakdown p.u. torque, and rated power factor.

6.7.1 MAGNETIZATION MMFS

The mmf per pole contains the airgap F_g , stator and rotor teeth (F_{st} and F_{rt}), stator and rotor back core (F_{cs} and F_{cr}) components (Figure 6.15):

$$F_{lm} = F_g + (F_{st} + F_{rt}) + \frac{F_{cs}}{2} + \frac{F_{cr}}{2} \quad (6.109)$$

6.7.2 THE AIRGAP F_g

The airgap F_g is

$$F_g = K_c g \frac{B_g}{\mu_v} \quad (6.110)$$

The Carter's coefficient K_c is

$$K_c = K_{c1} K_{c2}$$

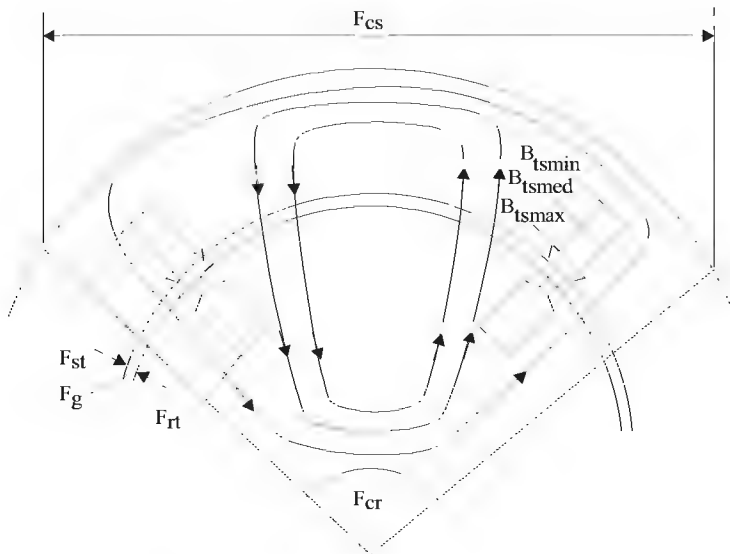


FIGURE 6.15 Main flux path and its mmfs.

$$\text{with } K_{c1,2} = \frac{\tau_{s,r}}{\tau_{s,r} - \gamma_{s,r} g}; \quad \gamma_{s,r} = \frac{\left(\frac{b_{os,r}}{g}\right)^2}{5 + \frac{b_{os,r}}{g}} \quad (6.111)$$

K_{c1} has been already calculated in (6.43): $K_{c1} = 1.365$. With $b_{or} = 5 \cdot 10^{-3} \text{ m}$, $g = 1.5 \cdot 10^{-3} \text{ m}$ and $\tau_r = 25.486 \cdot 10^{-3} \text{ m}$, γ_r is

$$\gamma_r = \frac{\left(\frac{5}{1.5}\right)^2}{5 + \frac{5}{1.5}} = 1.333; \quad K_{c2} = \frac{25.486}{25.486 - 1.5 \cdot 1.333} = 1.0852$$

Finally, $K_c = 1.365 \cdot 1.0852 = 1.4812 < 1.5$ (as assigned from start).

From (6.110), F_g is

$$F_g = 1.4812 \cdot 1.5 \cdot 10^{-3} \frac{0.7}{1.256 \cdot 10^{-6}} = 1188 \text{ Aturns}$$

6.7.3 THE STATOR TEETH MMF

With the maximum stator tooth flux density $B_{tmax} = 1.5 \text{ T}$, the minimum value B_{tmin} occurs at slot bottom diameter:

$$\begin{aligned} B_{tmin} &= (\pi(D_{is} + 2h_{st}) - N_s b_s)^{-1} B_g \pi D_{is} \\ &= (\pi(490 + 2 \cdot 57.2) - 72 \cdot 10)^{-1} \pi \cdot 490 \cdot 0.7 = 0.914 \text{ T} \end{aligned} \quad (6.112)$$

The average value B_{tsmed} is

$$B_{tsmed} = \frac{\pi D_{is} B_g}{\pi(D_{is} + h_{st}) - N_s b_s} = \frac{\pi \cdot 490 \cdot 0.7}{\pi(490 + 57.2) - 72 \cdot 10} = 1.0788 \text{ T} \quad (6.113)$$

For the three flux densities, the lamination magnetization curve (Chapter 5, Vol. 1, Table 5.4) yields:

$$H_{tsmax} = 1123 \text{ A/m}, \quad H_{tsmed} = 335 \text{ A/m}, \quad H_{tsmin} = 192 \text{ A/m}$$

The average value of H_s is

$$\begin{aligned} H_{ts} &= \frac{1}{6} (H_{tsmax} + 4H_{tsmed} + H_{tsmin}) \\ &= \frac{1}{6} (1123 + 4 \cdot 345 + 192) = 442.5 \text{ A/m} \end{aligned} \quad (6.114)$$

The stator tooth mmf F_{ts} becomes

$$F_{ts} = h_{st} H_{ts} = 57.2 \cdot 10^{-3} \cdot 442.5 = 25.31 \text{ Aturns} \quad (6.115)$$

6.7.4 ROTOR TOOTH MMF (F_{tr}) COMPUTATION

It is done as for the stator. The average tooth flux density B_{trmed} is

$$B_{trmed} = \frac{\pi D_{is} B_g}{\pi(D_{is} - h_{rt}) - N_r b_r} = \frac{\pi \cdot 490 \cdot 0.7}{\pi(490 - 35.52) - 60 \cdot 10.5} = 1.351 \text{ T} \quad (6.116)$$

The corresponding iron magnetic fields for $B_{trmin} = 1.3 \text{ T}$, $B_{trmed} = 1.351 \text{ T}$, and $B_{trmax} = 1.582 \text{ T}$ are

$$H_{trmax} = 9000 \text{ A/m}, \quad H_{trmed} = 1750 \text{ A/m}, \quad H_{trmin} = 760 \text{ A/m}$$

The average H_{tr} is

$$H_{tr} = \frac{1}{6}(H_{trmax} + 4H_{trmed} + H_{trmin}) = 2793.33 \text{ A/m} \quad (6.117)$$

The rotor tooth mmf F_{tr} is

$$F_{tr} = h_{rt} H_{tr} = 35.5 \cdot 10^{-3} \cdot 2793.33 = 99.163 \text{ Atums} \quad (6.118)$$

We are now in position to check the teeth saturation coefficient $1 + K_{st}$ (adopted as (1.25)):

$$1 + K_{st} = 1 + \frac{F_{ts} + F_{tr}}{F_g} = 1 + \frac{30.22 + 99.163}{1418} = 1.0912 < 1.25 \quad (6.119)$$

The value of $1 + K_{st}$ is a bit too small, indicating that the stator teeth are weakly saturated. Slightly wider stator slots would have been feasible. In case the stator winding losses are much larger than stator core loss, increasing the slot width by 10% could lead to higher conductor loss reduction. Another way to take advantage of this situation is illustrated in [3], Figure 6.16, where axial cooling is used, and the rotor diameter is reduced due to better cooling. For a two-pole 1.9 MW, 6600 V machine, the reduction of mechanical loss led to 1% efficiency rise (from 0.96 to 0.97) [3].

The stator back core mmf F_{cs} is

$$F_{cs} = l_{cs} H_{cs} \quad (6.120)$$

As both B_{cs} and l_{cs} vary within a pole-pitch span, a correct value of F_{cs} would warrant finite element modelling (FEM) calculations.

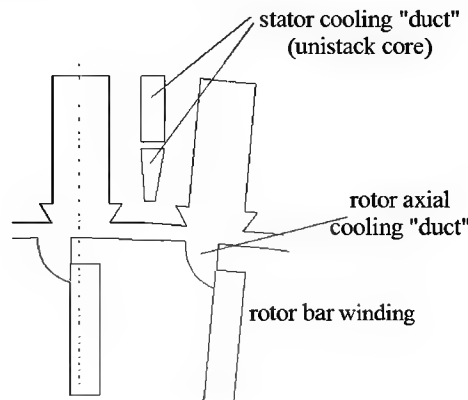


FIGURE 6.16 Tooth axial ventilation design (high-voltage stator).

We take here B_{csmax} as reference:

$$B_{\text{csmax}} = \frac{B_g \tau}{\pi h_{\text{cs}}} = \frac{0.7 \cdot 0.3925}{\pi \left(\frac{0.730 - 0.6044}{2} \right)} = 1.358 \text{ T} \quad (6.121)$$

$$H_{\text{csmax}} = 600 \text{ A/m}$$

The mean length l_{cs} is

$$l_{\text{cs}} = \frac{\pi(D_{\text{out}} - h_{\text{cs}})}{2P_1} K_{\text{cs}} = \frac{\pi(0.730 - 628 \cdot 10^{-3})}{2 \cdot 2} 0.4 = 0.2095 \text{ m} \quad (6.122)$$

K_{cs} is an amplification factor which essentially depends on the flux density level in the back iron.

Again B_{clmax} is a bit too small. Consequently, if the temperature rise allows, the outer stator diameter may be reduced, perhaps to 0.700 m (from 0.73 m).

$$F_{\text{cs}} = l_{\text{cs}} H_{\text{clmax}} = 0.2095 \cdot 600 = 125 \text{ Atturns}$$

6.7.5 ROTOR BACK IRON MMF F_{cr} (AS FOR THE STATOR)

$$F_{\text{cr}} = H_{\text{cr}} l_{\text{cr}} = 760 \cdot 0.1118 = 84.95 \text{ Atturns} \quad (6.123)$$

$$H_{\text{cr}}(B_{\text{cr}}) = H_{\text{cr}}(1.4) = 760 \text{ A/m}$$

$$l_{\text{cr}} = \frac{\pi(D_{\text{ir}} + h_{\text{cr}})}{2P_1} K_{\text{cr}} = \frac{\pi(0.296 - 0.06)}{2 \cdot 2} 0.4 = 0.1118 \text{ m} \quad (6.124)$$

From (6.109), the total mmf per pole F_{lm} is

$$F_{\text{lm}} = 1188 + 25.31 + 99.163 + \frac{125 + 84.95}{2} = 1417.45 \text{ Atturns}$$

The mmf per pole F_{lm} is

$$F_{\text{lm}} = \frac{3\sqrt{2} I_{\mu} W_l K_{wl}}{p_1 \pi} \quad (6.125)$$

So the magnetization current I_{μ} is

$$I_{\mu} = \frac{F_{\text{lm}} p_1 \pi}{3\sqrt{2} W_l K_{wl}} = \frac{1417.45 \cdot 2 \cdot \pi}{3\sqrt{2} \cdot 12 \cdot 18 \cdot 0.923} = 10.555 \text{ A} \quad (6.126)$$

The ratio between the magnetization and rated phase current I_{μ}/I_{ln} is

$$\frac{I_{\mu}}{I_{\text{ln}}} = \frac{10.55}{120/\sqrt{3}} = 0.152$$

Even at this power level and $2p_1 = 4$, because the ratio $\tau/g = 392/1.5 = 261.33$ (pole pitch/airgap) is rather large and the saturation level is low, the magnetization current is lower than 20% of rated current. The machine has slightly more iron than needed, or the airgap may be increased from $1.5 \cdot 10^{-3} \text{ m}$ to $(1.8-2)10^{-3} \text{ m}$. As a bonus, the additional surface and tooth flux pulsation core losses will be reduced. We may simply reduce the outer diameter such that to saturate notably the stator back iron. The gain will be less volume and weight.

6.7.6 THE ROTOR WINDING PARAMETERS

As the stator phase resistance R_s and leakage reactance (X_{sl}) have been already computed ($R_s = 0.8576 \Omega$, $X_{sl} = 6.667 \Omega$), only the rotor winding parameters R_r and X_{lr} have to be calculated.

The computation of R_r , X_{lr} requires the calculation of rotor coil end-connection length based on its geometry (Figure 6.17).

The end-connection length l_{fr} per rotor side is

$$\begin{aligned} l_{fr} &= \frac{y}{\cos \alpha_r} + \left(\frac{h_{rt}}{3} + h_{rt} \right) \cdot 2 \\ &= \frac{\left(0.314 - 35.5 \cdot 10^{-3} \cdot \frac{\pi}{4} \right)}{0.707} + \frac{4}{3} \cdot 2 \cdot 35.5 \cdot 10^{-3} = 0.5 \text{ m} \end{aligned} \quad (6.127)$$

So the phase resistance R_r^r (before reduction to stator) is

$$R_r^r = K_R \rho_{Co80^\circ} \frac{2(l_{fr} + l_{geo}) W_2}{A_{cor}} = 1.0 \cdot 2.2 \cdot 10^{-8} \frac{2(0.5 + 0.451) \cdot 20}{96 \cdot 10^{-6}} = 0.87 \cdot 10^{-2} \Omega \quad (6.128)$$

6.7.7 THE ROTOR SLOT LEAKAGE GEOMETRICAL PERMEANCE COEFFICIENT λ_{sr}

The rotor slot leakage geometrical permeance coefficient λ_{sr} is (Figure 6.14)

$$\begin{aligned} \lambda_{s,r} &\approx \frac{2b_c}{3b_r} + \frac{h_4}{4b_r} + \frac{h_2}{b_s} + \frac{h_{or}}{b_{or}} + \frac{2h_w}{(b_{or} + b_r)} \\ &= \frac{2 \cdot 12}{3 \cdot 10.5} + \frac{1.2}{4 \cdot 10.5} + \frac{1.8}{10.5} + \frac{2.5}{5} + \frac{2 \cdot 4}{5 + 10.5} = 1.978 \end{aligned} \quad (6.129)$$

The rotor differential leakage permeance coefficient λ_{dr} is (as in [4])

$$\lambda_{dr} = \frac{0.9 \cdot \tau_r (q_2 K_{w2})^2 K_{or} \sigma_{dr}}{K_c g} \quad (6.130)$$

$$K_{or} = 1.0 - 0.033 \frac{b_{or}^2}{g \tau_r} = 1.0 - 0.033 \frac{5^2}{1.5 \cdot 24.86} = 0.9778 \quad (6.131)$$

where σ_{dr} is the differential leakage coefficient as defined in Figure 6.3, Section 6.3, Vol. 1. For $q_2 = 5$ and diametrical coils $\sigma_{dr} = 0.64 \cdot 10^{-2}$,

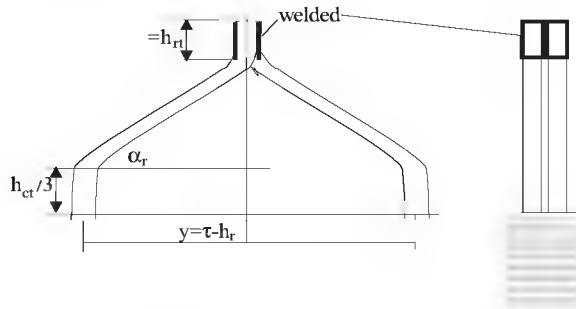


FIGURE 6.17 Rotor half-formed coil winding.

$$\lambda_{dr} = 0.9 \frac{24.86}{1.8 \cdot 1.5} (5 \cdot 0.9566)^2 \cdot 0.9778 \cdot 0.64 \cdot 10^{-2} = 1.4428$$

The end-connection permeance coefficient λ_{fr} (defined as for the stator) is

$$\lambda_{fr} = 0.34 \frac{q_2}{l_{ir}} (l_{fr} - 0.64\tau) = 0.34 \frac{5}{0.406} (0.5 - 0.64 \cdot 0.314) = 1.252 \quad (6.132)$$

The rotor phase leakage reactance X_{lr} (before reduction to stator) is

$$\begin{aligned} X_{lr}^r &= 4\pi f_l \mu_0 W_2^2 \frac{l_i}{P_l q_2} \sum \lambda_{ri} \\ &= 4 \cdot 3.14 \cdot 60 \cdot 1.256 \cdot 10^{-6} \cdot 20^2 \frac{0.406}{2 \cdot 5} (1.978 + 1.4428 + 1.252) = 7.1828 \cdot 10^{-2} \Omega \quad (6.133) \end{aligned}$$

After reduction to stator, the rotor resistance and leakage reactances R_r^r , X_{lr}^r become R_r and X_{lr} .

$$\begin{aligned} R_r &= R_r^r \left(\frac{W_1 K_{w1}}{W_2 K_{w2}} \right)^2 = 0.87 \cdot 10^{-2} \cdot \left(\frac{12 \cdot 18 \cdot 0.923}{20 \cdot 0.9566} \right)^2 = 0.94 \Omega \\ X_{lr} &= X_{lr}^r \left(\frac{W_1 K_{w1}}{W_2 K_{w2}} \right)^2 = 7.1828 \cdot 10^{-2} \cdot 108.6 = 7.8 \Omega \end{aligned} \quad (6.134)$$

The magnetization reactance X_m is

$$X_m \approx \frac{(V_{ph} - I_\mu X_{sl})}{I_\mu} = \frac{(4000 - 10.55 \cdot 6.667)}{10.55} = 372.48 \Omega \quad (6.135)$$

The p.u. parameters are

$$\begin{aligned} X_n &= \frac{V_{ph}}{I_{In}} = \frac{4000}{\frac{120}{\sqrt{3}}} = 57.66 \Omega \\ r_s &= \frac{R_s}{X_n} = \frac{0.8756}{57.66} = 0.01487 \text{ p.u.} \\ r_r &= \frac{R_r}{X_n} = \frac{0.94}{57.66} = 0.0163 \text{ p.u.} \\ x_{ls} &= \frac{X_{ls}}{X_n} = \frac{6.667}{57.66} = 0.1156 \text{ p.u.} \\ x_{lr} &= \frac{X_{lr}}{X_n} = \frac{7.8}{57.66} = 0.1352 \text{ p.u.} \\ x_m &= \frac{X_m}{X_n} = \frac{372.48}{57.66} = 6.45 \text{ p.u.} > (3.2 - 4) \text{ p.u.} \end{aligned} \quad (6.136)$$

Again, x_m is visibly higher than usual but, as explained earlier, this may be the starting point for machine size further reduction. A simple reduction of the outer stator diameter to saturate the stator back iron would bring x_m into the standard interval of (3.5–4)p.u. for four-pole IMs of 700 kW

power range with wound rotors. An increase in airgap will produce similar effects, with additional core loss reduction.

We leave here x_m as it is in (6.136) as a reminder that the IM design is an iterative procedure.

6.7.8 LOSSES AND EFFICIENCY

The losses in IMs may be classified as

- Conductor (winding or electric) losses
- Core losses
- Mechanical/ventilation losses

The stator winding rated losses $p_{\cos}$ are

$$P_{\cos} = 3R_s I_{1n}^2 = 3 \cdot 0.8576 \cdot 69.36^2 = 12.378 \cdot 10^3 \text{ W} \quad (6.137)$$

For the rotor winding:

$$p_{\text{cor}} = 3R_r I_{1n}^2 = 3R_r K_i^2 I_{1n}^2 = 3 \cdot 0.94 \cdot (0.936 \cdot 69.36)^2 = 11.885 \cdot 10^3 \text{ W} \quad (6.138)$$

Considering a voltage drop, $V_{ss} = 0.75 \text{ V}$, in the slip ring and brushes, the slip-ring brush losses p_{sr} (with I_{2n} from (6.102)) is

$$P_{\text{sr}} = 3V_{\text{sr}} I_{2n} = 3 \cdot 0.75 \cdot 676.56 = 1.522 \cdot 10^3 \text{ W}$$

Additional winding losses due to skin and proximity effects (Chapter 11, Vol. 1) may be neglected for this case due to proper conductor sizing.

The core losses have three components (Chapter 11, Vol. 1):

- Fundamental core losses
- Additional no-load core losses
- Additional load (stray) core losses.

The fundamental core losses are calculated by empirical formulas (FEM may also be used) as in Chapter 5, Vol. 1.

The stator teeth fundamental core losses P_{iront} is (5.98)

$$P_{\text{iront}} = K_t p_{10} \left(\frac{f_l}{50} \right)^{1.3} B_{\text{ts}}^{1.7} G_{\text{ts}} \quad (6.139)$$

$K_t = 1.6\text{--}1.8$, takes into account the mechanical machining influence on core losses; $p_{10} (1 \text{ T}, 50 \text{ Hz}) = (2\text{--}3) \text{ W/Kg}$ for 0.5 mm thick laminations.

The value of B_t varies along the tooth height so the average value $B_{\text{ts}} (H_{\text{ts}})$, Equation (6.114), is $B_{\text{ts}} (528 \text{ A/m}) = 1.32 \text{ T}$.

The stator teeth weight G_{ts} is

$$\begin{aligned} G_{\text{ts}} &= \left[\frac{\pi}{4} \left((D_{\text{is}} + 2h_{\text{st}})^2 - D_{\text{is}}^2 \right) - h_{\text{ts}} b_s N_s \right] l_i \gamma_{\text{iron}} \\ &= \left[\frac{\pi}{4} \left((0.490 + 2 \cdot 56.2 \cdot 10^{-3})^2 - 0.490^2 \right) - 56.2 \cdot 10^{-3} \cdot 10 \cdot 10^{-3} \cdot 72 \right] \cdot 0.406 \cdot 7800 \\ &= 175.87 \text{ Kg} \end{aligned} \quad (6.140)$$

From (6.139),

$$P_{\text{iron}} = 1.8 \cdot 2.4 \left(\frac{60}{50} \right)^{1.3} 1.32^{1.7} \cdot 175.87 = 1.543 \cdot 10^3 \text{ W}$$

Similarly, the stator back iron (yoke) fundamental losses P_{iron}^y are

$$P_{\text{iron}}^y = K_y p_{10} \left(\frac{f_1}{50} \right)^{1.3} B_{\text{cs}}^{1.7} G_{\text{ys}} \quad (6.141)$$

$K_y = 1.2\text{--}1.4$ takes care of the influence of mechanical machining on yoke losses (for lower power machines this coefficient is larger).

The stator yoke weight G_{ys} is

$$\begin{aligned} G_{\text{ys}} &= \frac{\pi (D_{\text{out}}^2 - (D_{\text{is}} + 2h_{\text{st}})^2)}{4} l_i \gamma_{\text{iron}} \\ &= \frac{\pi}{4} (0.93^2 - (0.49 + 2 \cdot 0.0562)^2) \cdot 0.406 \cdot 7800 = 422.6 \text{ Kg} \end{aligned} \quad (6.142)$$

$$p_{\text{iron}}^y = 1.35 \cdot 2.4 \cdot \left(\frac{60}{50} \right)^{1.7} \cdot 1.274^{1.7} \cdot 422.6 = 2.619 \cdot 10^3 \text{ W}$$

Stator and rotor surface core losses, due to slot openings (Chapter 11, Vol. 1), are approximately:

$$P_{\text{siron}}^s = 2p_1 \tau \frac{(\tau_s - b_{\text{os}})}{\tau_s} l_i K_{\text{Fe}} p_{\text{siron}}^s \quad (6.143)$$

with the specific surface stator core losses (in W/m^2) p_{siron}^s as

$$p_{\text{siron}}^s \approx 5 \cdot 10^5 B_{\text{os}}^2 \tau_r^2 K_0 \left(\frac{N_r 60 f_1}{P_1 \cdot 10,000} \right)^{1.5} \quad (6.144)$$

B_{os} is the airgap flux pulsation due to rotor slot openings

$$B_{\text{os}} = \beta_s K_c B_g = 0.23 \cdot 1.48 \cdot 0.7 = 0.2368 \text{ T} \quad (6.145)$$

β_s can be taken from Figure 5.4 (Chapter 5, Vol. 1). With $b_{\text{or}}/g = 5/1.5$, $\beta_s \approx 0.23$. K_0 again depends on mechanical factors: $K_0 = 1.5$, in general.

$$p_{\text{siron}}^s = 5 \cdot 10^5 \cdot 0.2368^2 \cdot 0.02486^2 \cdot 1.5 \cdot \left(\frac{60 \cdot 60 \cdot 60}{2 \cdot 10,000} \right)^{1.5} = 922.49 \text{ W/m}^2$$

From (6.143):

$$P_{\text{siron}}^s = 2 \cdot 2 \cdot 0.314 \frac{(21.37 - 10)}{21.37} \cdot 0.406 \cdot 0.95 \cdot 922.49 = 0.237 \cdot 10^3 \text{ W}$$

As the rotor slot opening b_{or} per gap g ratio is only $5/1.5$, the stator surface core losses are expected to be small.

The rotor surface core losses p_{siron}^r are

$$p_{siron}^r = 2P_l \frac{(\tau_r - b_{or})}{\tau_s} \tau_l K_{Fe} p_{siron}^r \quad (6.146)$$

$$\text{with } p_{siron}^r = 5 \cdot 10^5 B_{or}^2 \tau_s^2 K_0 \left(\frac{N_r 60 f_l}{P_l \cdot 10,000} \right)^{1.5} ; \quad B_{or} = \beta_r K_c B_g$$

The coefficient β_r for $b_{os}/g = 10/1.5$ is, Figure 5.4, Chapter 5, Vol. 1, $\beta_r = 0.41$.
Consequently,

$$B_{or} = 0.41 \cdot 1.48 \cdot 0.7 = 0.426 \text{ T}$$

$$p_{siron}^r = 5 \cdot 10^5 \cdot 0.426^2 \cdot 0.0214^2 \cdot 2.0 \cdot \left(\frac{72 \cdot 60 \cdot 60}{2 \cdot 10,000} \right)^{1.5} = 3877 \text{ W/m}^2$$

$K_0 = 2.0$ as the rotor is machined to the rated airgap.

$$P_{siron}^r = 2 \cdot 2 \frac{(24.86 - 5)}{24.86} \cdot 0.314 \cdot 0.406 \cdot 0.95 \cdot 3877 = 1.5 \cdot 10^3 \text{ W}$$

The tooth flux pulsation core losses are still to be determined.

$$P_{puls} = P_{puls}^s + P_{puls}^r \approx K'_0 0.5 \cdot 10^{-4} \left[\left(N_r \frac{f_l}{p_l} \beta_{ps} \right)^2 G_{ts} + \left(N_s \frac{f_l}{p_l} \beta_{pr} \right)^2 G_{tr} \right] \quad (6.147)$$

$$\beta_{ps} \approx (K_{c2} - 1) B_g = (1.085 - 1) \cdot 0.7 = 0.060 \text{ T}$$

$$\beta_{pr} \approx (K_{c1} - 1) B_g = (1.36 - 1) \cdot 0.7 = 0.252 \text{ T}$$

$K'_0 > 1$ is a technological factor to account for pulsation loss increases due to saturation and type of lamination manufacturing technology (cold or hot stripping). We take here $K'_0 = 2.0$.

The rotor teeth weight G_{tr} is

$$\begin{aligned} G_{tr} &= \left[\frac{\pi \left((D_{is} - 2g)^2 - (D_{is} - 2g - 2h_r)^2 \right)}{4} - N_r h_r b_r \right] l_i \gamma_{iron} \\ &= \left[\frac{\pi \left((0.49 - 3 \cdot 10^{-3})^2 - (0.487 - 20.0355)^2 \right)}{4} - 60 \cdot 0.0355 \cdot 105 \cdot 10^{-2} \right] \cdot 0.406 \cdot 7800 = 88.55 \text{ Kg} \end{aligned} \quad (6.148)$$

From (6.147),

$$\begin{aligned} P_{puls} &= 2 \cdot 0.5 \cdot 10^{-4} \left[\left(60 \cdot \frac{60}{2} \cdot 0.060 \right)^2 \cdot 175.87 + \left(72 \cdot \frac{60}{2} \cdot 0.252 \right)^2 \cdot 88.55 \right] \\ &= 2.828 \cdot 10^3 \text{ W} \end{aligned}$$

The mechanical losses due to friction (brush-slip-ring friction and ventilator on-shaft mechanical input) depend essentially on the type of cooling machine system design.

We take them all here as being $P_{mec} = 0.01P_n$. For two-pole or high-speed machines, $P_{mec} > 0.01P_n$ and should be, in general, calculated as precisely as feasible in a thorough mechano-thermal design.

6.7.9 THE MACHINE RATED EFFICIENCY η_n

The machine rated efficiency η_n is

$$\begin{aligned}\eta_n &= \frac{P_n}{P_n + p_{cos} + P_{cor} + P_{sr} + P_{iron}^I + P_{iron}^y + P_{siron}^s + P_{siron}^f + P_{puls} + P_{mec}} \\ &= \frac{736 \cdot 10^3}{10^3 (736 + 12.378 + 11.885 + 1.522 + 1.543 + 2.619 + 0.237 + 1.5 + 2.828 + 7.36)} \\ &= 0.946\end{aligned}\quad (6.149)$$

As the efficiency is notably below the target value (0.96), efforts to improve the design are in order. Additional losses are to be reduced. Here is where optimization comes into play.

6.7.10 THE RATED SLIP S_n (WITH SHORT-CIRCUITED SLIP RINGS)

The rated slip S_n (with short-circuited slip rings) can be calculated as

$$S_n \approx \frac{R_r K_i I_{In}}{K_e V_{ph}} = \frac{0.94 \cdot 0.936 \cdot 69.36}{0.97 \cdot 4000} = 0.0157 \quad (6.150)$$

Alternatively, accounting for all additional core losses in the stator,

$$S_n = \frac{P_{cor}}{P_n + p_{cor} + p_{mec}} = \frac{11.885 \cdot 10^3}{10^3 \cdot (736 + 11.885 + 7.36)} = 0.015736 \quad (6.151)$$

The rather large value of rated slip is an indication of not so high efficiency. A reduction of current density in both the stator and rotor windings seems appropriate to further increase efficiency.

The rather low teeth flux densities allow for slot area increases to cause winding loss reductions.

6.7.11 THE BREAKDOWN TORQUE

The approximate formula for breakdown torque is

$$T_{bk} \approx \frac{3p_l}{2\omega_l} \frac{V_{ph}^2}{X_{ls} + X_{lr}} = \frac{3 \cdot 2}{2 \cdot 2\pi \cdot 60} \frac{4000^2}{(6.67 + 7.8)} = 8.8035 \cdot 10^3 \text{ Nm} \quad (6.152)$$

The rated torque T_{en} is

$$T_{en} = \frac{p_l P_n}{\omega_l (1 - S_n)} = \frac{2 \cdot 736 \cdot 10^3}{2\pi \cdot 60 \cdot (1 - 0.015)} = 3.966 \cdot 10^3 \text{ Nm} \quad (6.153)$$

The breakdown torque ratio t_{ek} is

$$t_{ek} = \frac{T_{bk}}{T_{en}} = \frac{8.8035}{3.966} \approx 2.2 \quad (6.154)$$

This is an acceptable value.

6.8 SUMMARY

- Above 100 kW, induction motors are built for low voltage (≤ 690 V) and for medium voltage (between 1 and 12 kV).
- Low-voltage IMs are designed and built lately up to more than 2 MW power per unit, in the eventuality of their use with PWM converter supplies.
- For constant voltage and frequency supply, besides efficiency and initial costs, the starting torque and current and the peak torque are important design factors.
- The stator design is based on targeted efficiency and power factor, whereas rotor design is based on the starting performance and breakdown torque.
- Deep bar cage rotors are used when moderate p.u. starting torque is required ($t_{LR} < 1.2$). The bar depth and width are determined based on breakdown torque, starting torque, and starting current constraints. This methodology reduces the number of design iterations drastically.
- The stator design is dependent on the cooling system: axial cooling (unistack core configuration) and radial–axial cooling (multistack core with radial ventilation channels, respectively).
- For medium-voltage stator design, open rectangular slots with form-wound chorded (rigid) coils, built into two-layer windings, are used. Practically only rectangular cross-sectional conductors are used [5.6].
- The rather large insulation total thickness makes the stator slot fill in high-voltage windings rather low, with slot aspect ratios $h_{st}/b_s > 5$. Large slot leakage reactance values are expected.
- In low-voltage stator designs, above 100 kW, semiclosed-slot, round-wire coils are used only up to 200–250 kW for $2p_1 = 2, 4$; also, for larger powers and large numbers of poles ($2p_1 = 8, 10$), quite a few current paths in parallel ($a_1 = p_1$) may be used.
- For low-voltage stator designs, prepared also for PWM converter use, only rectangular shape conductors in form-wound coils are used, to secure good insulation as required for PWM converter supplies. To reduce skin effect, the conductors are divided into a few elementary conductors in parallel. Stranding is performed twice, at the end-connection zone (Chapter 9, Vol. 1).
- Deep bar design handles large breakdown torques, $t_{bk} \approx 2.5$ – 2.7 and more, with moderate starting torques $t_{LR} < 1.2$ and starting currents $i_{LR} < 6.1$. Based on breakdown torque, starting torque, and current values, with stator already designed (and thus stator resistance and leakage reactance values known), the rotor resistance and leakage reactance values (with and without skin effect and leakage saturation consideration) are calculated. This leads directly to the adequate rectangular deep bar geometry. If other bar geometries are used, the skin effect coefficients are calculated by the refined multilayer approach (Chapter 9, Vol. 1).
- Double-cage rotor designs lead to lower starting currents $i_{LR} < 5.5$, larger starting torque $t_{LR} > (1.4$ – $1.5)$ and good efficiency but moderate power factor.
- A similar methodology, based on starting and rated power performance leads to the starting and working cage sizing, respectively. Thus, a first solution meeting the main constraints is obtained. Refinements should follow, eventually through design optimization methods.
- For limited variable speed range, or for heavy and frequent starting (up to breakdown torque at start), wound rotor designs are used.
- The two-layer wound rotor windings are, in general, low-voltage type, with half-formed diametrical wave uniform coils in semiclosed or semiopen slots (to reduce surface and tooth flux pulsation core loss). A few elementary conductors in parallel may be needed to handle the large rotor currents as the power goes up.

- The wound rotor design is similar to stator design, being based only on rated targeted efficiency (and power factor) and on the assigned rotor current density.
- This chapter presents design methodologies using a single motor (low- and medium-voltage stator 736 kW (1000 HP) design with deep bar cage, double-cage and wound rotor). Only for the wound rotor, all parameters (losses, efficiency, rated slip, breakdown torque, and rated power factor) are calculated. Its design for large slip and converter-fed in the rotor can be found (as doubly fed induction generator or DFIG) in [7].
- Ways to improve the resultant performance are indicated. Temperature rise calculations may be performed as in Chapter 5, Vol. 2, but with more complex mathematics. If temperature rise is not met (insulation class F and temperature rise class B) and it is too high, a new design with a larger geometry or lower current densities or (and) flux densities has to be tried.
- Chapters 4–6, laid out the machine models (analysis) and a way to size the machine for given performance and constraints (preliminary synthesis).
- The mathematics in these chapters may be included in a design optimization software. Design optimization will be treated in a separate chapter. FEM is to be used for refinements.
- The above methodologies are by no means unique to the scope, as IM design is both a science and an art [8–10].

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7 Induction Machine Design for Variable Speed

7.1 INTRODUCTION

Variable speed drives with induction motors are by now a mature technology with strong and dynamic markets for applications in all industries. Based on the load torque/speed envelope, three main types of applications are distinguished:

- Servodrives: no constant power speed range (CPSR)
- General drives: moderate CPSR ($\omega_{\max}/\omega_b \leq 2$)
- Constant power drives: large CPSR ($\omega_{\max}/\omega_b \geq 2$).

Servodrives for robots, machine tools, are characterized, in general, by constant torque versus speed up to base speed ω_b . The base speed ω_b is the speed for which the motor can produce (for continuous service) for rated voltage and rated temperature rise, the rated (base) power P_b , and rated torque T_{eb} .

Servodrives are characterized by fast torque and speed response, and thus for short time, during transients, the motor has to provide a much higher torque T_{ek} than T_{eb} . The higher the torque, the better the speed response. Also servodrives are characterized by sustained very low-speed and up to rated (base) torque operation, for speed or position control. In such conditions, low torque pulsations and limited temperature rise are imperative.

Temperature rise has to be limited to avoid both winding insulation failure and mechanical deformation of the shaft which would introduce errors in position control.

In general, servodrives have a constant speed (separate shaft) power, grid fed, ventilator attached to the induction machine (IM) at the non-driving end. The finned stator frame is thus axially cooled through the ventilator's action. Alternatively, liquid cooling of the stator may be provided.

Even from such a brief introduction, it becomes clear that the design performance indexes of IMs for servodrives need special treatment. However, fast torque and speed response and low torque pulsations are paramount. Efficiency and power factor are second-order performance indexes as the inverter KVA rating is designed for the low duration peak torque (speed) transients requirements.

General drives, which cover the bulk of variable speed applications, are represented by fans, pumps, compressors, etc.

General drives are characterized by a limited speed control range, in general, from $0.1\omega_b$ to $2\omega_b$. Above base speed ω_b , constant power is provided. A limited CPSR $\omega_{\max}/\omega_b = 2.0$ is sufficient for most cases. Above base speed, the voltage stays constant.

Based on the stator voltage circuit equation at steady state,

$$\bar{V}_s = \bar{I}_s R_s + j\omega_l \bar{\Psi}_s \quad (7.1)$$

with $R_s \approx 0$

$$\bar{\Psi}_s \approx \frac{\bar{V}_s}{\omega_l} \quad (7.2)$$

Above base speed (ω_b), the frequency ω_1 increases for constant voltage. Consequently, the stator flux level Ψ_s decreases. Flux weakening occurs. We might say that general drives have a 2/1-flux weakening speed range.

As expected, there is some torque reserve for fast acceleration and braking at any speed. About 150%–200%, overloading is typical.

General drives use IMs with on-the-shaft ventilators. More sophisticated radial–axial cooling systems with a second cooling agent in the stator may be used.

General drives may use high-efficiency IM designs as in this case efficiency is important.

Made with class F insulated preformed coils and insulated bearings for powers above 100kW and up to 2000kW, and at low voltage (maximum 690 V), such motors are used in both constant and variable speed applications. While designing IMs on purpose for general variable speed drives is possible, it may seem more practical to have a single design for both constant and variable speed: the high-efficiency induction motor.

In constant power variable-speed applications such as spindles or hybrid (or electric) car propulsion and generator systems, the main objective is a large flux weakening speed range $\omega_{\max}/\omega_b > 2$, in general more than 3–4, and even 6–7 in special cases. Designing an IM for a wide CPSR is very challenging because the breakdown torque T_{bk} is in p.u. limited: $t_{bk} < 3$ in general.

$$T_{eK} \approx \frac{3p_l}{2} \left(\frac{V_{ph}}{\omega_1} \right)^2 \cdot \frac{1}{L_{sc}} \quad (7.3)$$

Increasing the breakdown torque as the base speed (frequency) increases could be done by

- Increasing the pole number $2p_l$.
- Increasing the phase voltage.
- Decreasing the leakage inductance L_{sc} (by increased motor size, winding tapping, phase connection changing, special slot (winding) designs to reduce L_{sc}). Each of these solutions has impact on both IM and static power converter costs. The global cost of the drive and the capitalized cost of its losses are solid criteria for appropriate designs. Such applications are most challenging. Yet another category of variable speed applications is represented by super-high-speed drives.
- For fast machine tools, vacuum pumps etc., speeds which imply fundamental frequencies above 300Hz (say above 18,000rpm) are considered here for up to 100kW powers and above 150 Hz (9000rpm) for higher powers.

As the peripheral speed goes up, above (60–80) m/s, the mechanical constraints become predominant and thus novel rotor configurations become necessary. Solid rotors with copper bars are among the solutions considered in such applications. Also, as the size of IM increases with torque, high-speed machines tend to be small (in volume/power) and thus heat removal becomes a problem. In many cases, forced liquid cooling of the stator is mandatory.

Despite worldwide efforts in the last decade, the design of IMs for variable speed, by analytical and numerical methods, did not crystallize in widely accepted methodologies.

What follows should be considered a small step towards such a daring goal.

As basically the design expressions and algorithms developed for constant V/f (speed) are applicable to variable speed design, we will concentrate only on what distinguishes this latter enterprise.

- In the end, a rather detailed design case study is presented. Among the main issues in IM design for variable speed, we treat here
- Power and voltage derating
- Reducing skin effect

- Reducing torque pulsations
- Increasing efficiency
- Approaches to leakage inductance reduction
- Design for wide constant power wide speed range
- Design for variable very high speed.

7.2 POWER AND VOLTAGE DERATING

An induction motor is only a part of a variable speed drive assembly (Figure 7.1).

As such, the IM is fed from the power electronics converter (PEC) directly, but indirectly, in most cases, from the industrial power grid.

There are a few cases where the PEC is fed from a D.C. source (battery).

The PEC inflicts on the motor voltage harmonics (in a voltage source type) or current harmonics (in a current source type). In this case, voltage and current harmonics, whose frequency and amplitude are dependent on the PWM (control) strategy and power level, are imposed on the induction motor.

Additionally, high-frequency common voltage mode currents may occur in the stator phases in high-frequency PWM voltage source converter IM drives. All modern PECs have incorporated filtering methods to reduce the additional current and voltage (flux) harmonics in the IMs, as they produce additional losses.

Analytical and even finite element methods have been proposed to cater to these time harmonics losses (see Chapter 11, Vol. I). Still, these additional time harmonics core and winding losses depend not only on machine geometry and materials, but also on PWM, switching frequency, and load level [1,2].

On top of this, for given power grid voltage, the maximum fundamental voltage at motor terminals depends on the type of PEC (single or double stage, type of power electronics switches (PES)) and PWM (control) strategy.

Though each type of PEC has its own harmonics and voltage drop signature, the general rule is that lately both these indexes have decreased. The matrix converter is a notable exception in the sense that its voltage derating (drop) is larger (up to 20%) in general.

Voltage derating – <10%, in general 5% – means that the motor design is performed at a rated voltage V_m which is smaller than the ac power grid voltage V_g :

$$V_m = V_g (1 - v_{\text{derat}}); v_{\text{derat}} < 0.1 \quad (7.4)$$

Power derating comes into play in the design when we choose the value of Esson's constant C_0 (W/m^3), as defined by past experience for sinusoidal power supply, and reduce it to C'_0 for variable V/f supply:

$$C'_0 = C_0 (1 - p_{\text{derat}}); p_{\text{derat}} \approx (0.08 - 0.12) \quad (7.5)$$

It may be argued that this way of handling the PEC-supplied IM design is quite empirical. True, but this is done only to initiate the design (sizing) process. After the sizing is finished, the voltage drops in the PEC and the time harmonics core and winding losses may be calculated (see Chapter 11, Vol. I).

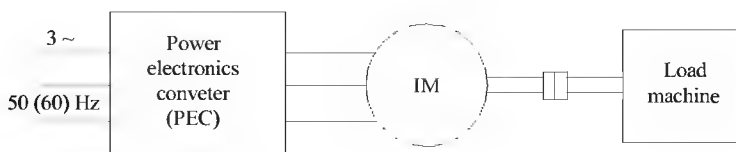


FIGURE 7.1 IM in a variable speed drive system.

Design refinements are then done. Alternatively, if prototyping is feasible, test results are used to validate (or correct) the loss computation methodologies.

There are two main cases: one when the motor exists, as designed for sinusoidal power supply, and the other when a new motor is to be designed for the scope.

The derating concepts deal with both these cases the same way.

However, the power derating concept is of little use where no solid past experience exists, such as in wide CPSR drives or in super-high-speed drives. In such cases, the tangential specific force (N/cm^2), Chapter 4, Vol. 2, with limited current sheet (or current density) and flux densities, seems to be the right guidelines for practical solutions. Finally, the temperature rise and performance (constraints) checks may lead to design iterations. As already mentioned in Chapter 4, the rated (base) tangential specific force (σ_t) for sinusoidal power supply is

$$\sigma_t^{\sin} \approx (0.3 - 4.0) \text{ N}/\text{cm}^2 \quad (7.6)$$

Derating may now be applied to $\sigma_t^{\sin}$, to get σ_t^{PEC}

$$\sigma_t^{\text{PEC}} = \sigma_t^{\sin} (1 - p_{\text{derat}}) \quad (7.7)$$

for the same rated (base) torque and speed.

The value of σ_t^{PEC} increases with rated (base) torque and decreases with base speed.

7.3 REDUCING THE SKIN EFFECT IN WINDINGS

In variable speed drives, variables V and f are used. Starting torque and current constraints are not anymore relevant in designing the IM. However, for fast torque (speed) response during variable frequency and voltage starting or loading or for constant power wide speed range applications, the breakdown torque has to be large.

Unfortunately, increasing the breakdown torque without enlarging the machine geometry is not an easy task.

On the other hand, the rotor skin effect that limits the starting current and produces larger starting torque, based on a larger rotor resistance, is no longer necessary.

Reducing skin effect is thus now mandatory to reduce additional time harmonics winding losses.

Skin effect in winding losses depends on frequency, conductor size, and position in slots. First, the rotor and stator skin effect at fundamental frequency is to be reduced. Second, the rotor and stator skin effect has to be checked and limited at PEC switching frequency. The amplitude of currents is larger for the fundamental than for time harmonics. Still, the time harmonics conductor losses at large switching frequencies are notable. In super-high-speed IMs, the fundamental frequency is already large, (300–3(5)000) Hz. In this case, the fundamental frequency skin effects are to be severely checked and kept under control for any practical design as the slip frequency may reach tenth of Hz (up to 50–60 Hz).

As the skin effect tends to be larger in the rotor cage, we will start with this problem.

7.3.1 ROTOR BAR SKIN EFFECT REDUCTION

The skin effect is a direct function of the parameter:

$$\zeta = h \sqrt{\frac{S \omega_1 \sigma_{\text{cor}} \mu_0}{2}} \quad (7.8)$$

The slot shape also counts. But if the slot is rectangular or circular, only the slot diameter and the slot height count.

Rounded trapezoidal slots may also be used to secure constant tooth flux density and further reduce the skin effects (Figure 7.2).

For given rotor slot (bar) area A_b (Figure 7.2a–c), we have

$$\begin{aligned}
 A_b &= \frac{\pi d_r^2}{4} \\
 &= h_r \cdot \frac{2d_r}{3} + \frac{h_{r1}}{2} (b_{or} + d_r) \\
 &= h_r \frac{2}{3} d_1 + \frac{\pi 5}{36} d_r^2
 \end{aligned} \tag{7.9}$$

For the rectangular slot, the skin effect coefficients K_R and K_X have the standard formulas:

$$\begin{aligned}
 K_R &= \zeta \frac{\sinh 2\zeta + \sin 2\zeta}{\cosh 2\zeta - \cos 2\zeta} \\
 K_X &= \frac{3}{2\zeta} \frac{\sinh 2\zeta - \sin 2\zeta}{\cosh 2\zeta - \cos 2\zeta}
 \end{aligned} \tag{7.10}$$

In contrast, for round or trapezoidal-round slots, the multiple-layer approach, discussed in Chapter 9, Vol. 1, has to be used.

A few remarks are in order:

- As expected, for given geometry and slip frequency, skin effects are more important in copper than in aluminium bars
- For given rotor slot area, the round bar has limited skin effect
- As the bar area (bar current or motor torque) increases, the maximum slip frequency $f_{sr} = Sf_1$ for which $K_R < 1.1$ diminishes
- Peak slip frequency f_{srk} varies from 2 to 10 Hz
- The smaller values correspond to larger (MW) machines and larger values to sub-KW machines designed for base frequencies of 50 (60) Hz. For f_{srk} , $K_R < 1.1$ has to be fulfilled if rotor additional losses are to be limited. Consequently, the maximum slot depth depends heavily on motor peak torque requirements
- For super-high-speed machines, f_{srk} may reach even 50 (60) Hz, so extreme care in designing the rotor bars is to be exercised (in the sense of severe limitation of slot depth, if possible).
- Maintaining reduced skin effect at f_{srk} means, apparently, less deep slots and thus, for given stator bore diameter, longer lamination stacks. As shown in the next paragraph, this leads to slightly lower leakage inductances and thus to larger breakdown torque. That is, a beneficial effect.

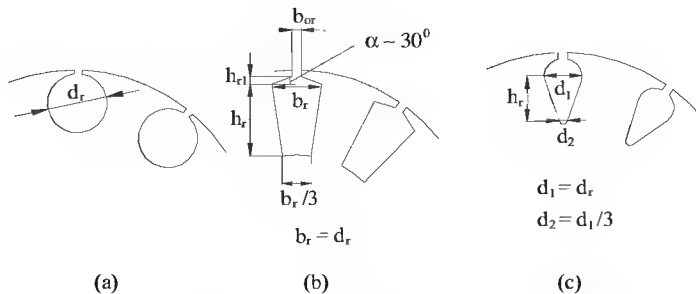


FIGURE 7.2 Rotor bar slots with low skin effect: (a) round shape, (b) rectangular, and (c) pair shape.

- When the rotor skin effect for f_{srK} may not be limited by reducing the slot depth, we have to go so far as to suggest the usage of a wound rotor with short-circuited phases and mechanically enforced end connections against centrifugal forces
- To reduce the skin effect in the end rings, they should not be placed very close to the laminated stack, although their heat transmission and mechanical resilience are a bit compromised
- Using copper instead of aluminium leads to a notable reduction of rotor bar resistance for the same bar cross section although the skin effect is larger. A smaller copper bar cross section is allowed, for the same resistance as aluminium, but for less deep slots and thus smaller slot leakage inductance. Again, larger breakdown torque may be obtained. The extra cost of copper may prove well worthwhile due to lower losses in the machine.
- As the skin effect is maintained low, the slot-body geometrical specific permeance λ_{sr} for the three cases mentioned earlier (Figure 7.2) is

$$\begin{aligned}
 \lambda_{sr}^{\text{round}} &\approx 0.666 \\
 \lambda_{sr}^{\text{rect}} &\approx \frac{h_r}{3b_r} + \frac{2}{3\sqrt{3}} \\
 b_r &= d_r \\
 \lambda_{sr}^{\text{trap}} &\approx \frac{h_r}{2d_r} + 0.4
 \end{aligned} \tag{7.11}$$

Equations (7.11) suggest that, in order to provide for identical slot geometrical specific permeance λ_{sr} , $h_r/b_r \leq 1.5$ for the rectangular slot and $h_r/d_r < 0.5$ for the trapezoidal slot. As the round part of slot area is not negligible, this might be feasible ($h_r/d_r \approx \pi/8 < 0.5$), especially for low torque machines.

Also for the rectangular slot with $b_r = d_r$, $h_r = (\pi/4) d_r \ll 1.5$, so the rectangular slot may produce:

$$\lambda_{st}^{\text{rect}}(\pi/4) = \pi/12 + 2/3\sqrt{3} = 0.67 \approx \lambda_{sr}^{\text{round}}$$

In reality, as the rated torque gets larger, the round bar is difficult to adopt as it would lead to a very small number of rotor slots or a very larger rotor diameter. In general, a slot aspect ratio $h_r/b_r \leq 3$ may be considered acceptable for many practical cases.

- The skin effect in the stator windings, at least for fundamental frequencies less than 100(120) Hz, is negligible in well-designed IMs for all power levels. For high-powers, elementary rectangular cross-sectional conductors in parallel are used. They are eventually stranded in the end-connection zone. The skin effect and circulating current additional losses have to be limited in large motors
- In super-high-speed IMs, for fundamental frequencies above 300 Hz (up to 3 kHz or more), stator skin effect has to be carefully investigated and suppressed by additional methods such as Litz wire or even by using thin wall pipe conductors with direct liquid cooling when needed
- Skin effect stator and rotor winding losses at PWM inverter carrier frequency are to be calculated as shown in Chapter 11, Vol. 1.

7.4 TORQUE PULSATIONS REDUCTION

Torque pulsations are produced both by airgap magnetic permeance space harmonics in interaction with stator (rotor) mmf space harmonics and by voltage (current) time harmonics produced by the PEC which supplies the IM to produce variable speed.

As torque time harmonics pulsations depend mainly on the PEC type and power level, we will not treat them here. The space harmonic torque pulsations are produced by the so-called parasitic torques (see Chapter 10, Vol. 1). They are of two categories, asynchronous and synchronous, and depend on the number of rotor and stator slots, slot opening/airgap ratios and airgap/pole pitch ratio, and the degree of saturation of stator (rotor) core. They all however occur at rather large values of slip: $S > 0.7$ in general.

This fact seems to suggest that for pump-/fan-type applications, where the minimum speed hardly goes below 30% base speed, the parasitic torques occur only during starting.

Even so, they should be considered, and the same rules apply, in choosing stator/rotor slot number combinations, as for constant V and f design (Chapter 5, Table 5.5).

- As shown in Chapter 5, slot openings tend to amplify the parasitic synchronous torques for $N_r > N_s$ (N_r – rotor slot count and N_s – stator slot count). Consequently, $N_r < N_s$ appears to be a general design rule for variables V and f , even without rotor slot skewing (for series-connected stator windings).
- Adequate stator coil throw chording (5/6) will reduce drastically asynchronous parasitic torque.
- Carefully chosen slot openings to mitigate between low parasitic torques and acceptable slot leakage inductances are also essential.
- Parasitic torque reduction is all the more important in servodrives applications with sustained low (even very low)-speed operation. In such cases, additional measures such as skewed resin insulated rotor bars and eventually closed rotor slots and semiclosed stator slots are necessary. Finite element modelling (FEM) investigation of parasitic torques may become necessary to secure sound results.

7.5 INCREASING EFFICIENCY

Increasing efficiency is related to loss reduction. There are fundamental core and winding losses and additional ones due to space and time harmonics.

Lower values of current and flux densities lead to a larger but more efficient motor. This is why high-efficiency motors are considered suitable for variables V and f .

Additional core losses and winding losses are discussed in detail in Chapter 11, Vol. 1.

Here, we only point out that the rules to reduce additional losses, presented in Chapter 11, Vol. 1, still hold. They are reproduced here and extended for convenience and further discussion.

- Large number of slots/pole/phase in order to increase the order of the first slot space harmonic.
- Insulated or uninsulated high bar-slot wall contact resistance rotor bars in long stack skewed rotors, to reduce inter-bar current losses.
- Skewing is not adequate for low bar-slot wall contact resistance as it does not reduce the harmonics (stray) cage losses while it does increase inter-bar current losses.
- $0.8N_s < N_r < N_s$ – to reduce the differential leakage coefficient of the first slot harmonics ($N_s \pm p_1$) and thus reduce the inter-bar current losses.
- For $N_r < N_s$, skewing may be altogether eliminated after parasitic torque levels are checked. For $q = 1, 2$ skewing seems mandatory, though.
- Usage of thin special laminations (0.1–0.2 mm) above $f_{in} = 300\text{ Hz}$ is recommended to reduce core loss in super-high-speed IM drives.
- Chorded coils ($y/\tau \approx 5/6$) reduce the asynchronous parasitic torque produced by the first phase belt harmonic ($v = 5$).
- With delta connection of stator phases: $(N_s - N_r) \neq 2p_1, 4p_1, 8p_1$.
- With parallel paths stator windings, the stator interpath circulating currents produced by rotor bar current mmf harmonics have to be avoided by observing a certain symmetry of stator winding paths.

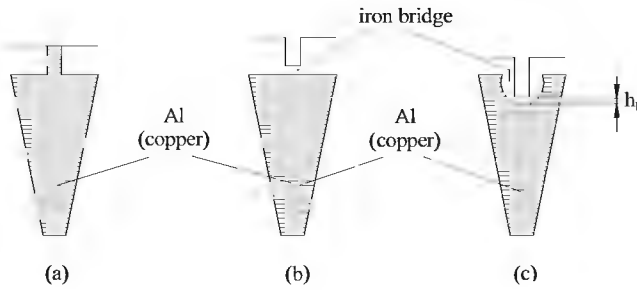


FIGURE 7.3 Rotor slot designs: (a) conventional, (b) straight bridge closed slot, (c) u-bridge close slot, and (d) asymmetric slot.

- Small stator (rotor) slot openings lead to smaller surface and tooth flux pulsation additional core losses, but they tend to increase the leakage inductances and thus reduce the break-down torque
- Carefully increase the airgap to reduce additional core and cage losses without compromising too much the power factor and efficiency.
- Use sharp tools and annealed laminations to reduce surface core losses.
- Return rotor surface to prevent rotor lamination short circuits which would lead to increased rotor surface core losses.
- Use only recommended N_s and N_r combinations and check for parasitic torque and stray load levels, and avoid large radial local forces.
- To reduce the time and space harmonics losses in the rotor cage, U-shaped bridge rotor slots have been proposed (Figure 7.3) [3].

In essence in conventional rotor slots, the airgap flux density harmonics induce voltages which produce eddy currents in the aluminium situated in the slot necks. By providing a slit in the rotor laminations (Figure 7.3b and c), the rotor conductor is moved further away from the airgap and thus, the additional cage losses are reduced.

However, this advantage comes with three secondary effects.

First, the eddy currents in the aluminium cage close to airgap damp the airgap flux density variation on the rotor surface and in the rotor tooth. This, in turn, limits the rotor core surface and tooth flux pulsation core losses.

In our new situation, this no longer occurs. Skewed rotor slots seem appropriate to keep the rotor surface and tooth flux pulsation core losses under control.

Second, the iron bridge height h_b above the slot, even when saturated, leads to a notable additional slot leakage geometrical permeance coefficient: λ_b .

Consequently, the value of L_{sc} is slightly increased leading to a breakdown torque reduction.

Third, the mechanical resilience of the rotor structure is somewhat reduced which might prevent the usage of this solution to super-high-speed IMs.

7.6 INCREASING THE BREAKDOWN TORQUE

As already inferred, a large breakdown torque is desirable either for high transient torque reserve or for widening the CPSR. Increasing the breakdown torque boils down to **leakage inductance decreasing**, when the base speed and stator voltage are given (7.3).

The total leakage inductance of the IM contains a few terms as shown in Chapter 6, Vol. 1.

The stator and rotor leakage inductances are (Chapter 6, Vol. 1).

$$L_{sl} = 2\mu_0 L_i \cdot n_s^2 p_i q_l (\lambda_{ss} + \lambda_{zs} + \lambda_{ds} + \lambda_{end}) \quad (7.12)$$

n_s – conductors slot

$$L_{rl} = 4m_1 \frac{(W_1 K_{w1})^2}{N_r} \cdot 2\mu_0 L_i (\lambda_b + \lambda_{er} + \lambda_{zr} + \lambda_{dr} + \lambda_{skew}) \quad (7.13)$$

m_1 – number of phases

L_i – stack length

q_1 – slots/pole/phase

λ_{vs} – stator slot permeance coefficient

λ_{vzs} – stator zig-zag permeance coefficient

λ_{vds} – stator differential permeance coefficient

λ_{vend} – stator end-coil permeance coefficient

λ_b – rotor slot permeance coefficient

λ_{er} – end ring permeance coefficient

λ_{vzr} – rotor zig-zag permeance coefficient

λ_{dr} – rotor differential permeance coefficient

λ_{vskew} – rotor skew leakage coefficient.

The two general expressions are valid for open and semiclosed slots. For closed rotor slots λ_b has the slot iron bridge term as rotor current dependent.

With so many terms, out of which very few may be neglected in any realistic analysis, it becomes clear that an in-depth sensitivity analysis of L_{sl} and L_{rl} to various machine geometrical variables is not easy to wage.

The main geometrical variables which influence $L_{sc} = L_{sl} + L_{rl}$ are

- Pole number: $2p_1$
- Stack length/pole pitch ratio: L_i/τ
- Slot/tooth width ratio: b_{slr}/b_{tslr}
- Stator bore diameter
- Stator slots/pole/phase q_1
- Rotor slots/pole pair N_r/p_1
- Stator (rotor) slot aspect ratio h_{ssl}/b_{slr}
- Airgap flux density level B_g
- Stator (rotor) base torque (design) current density.

Simplified sensitivity analysis [4] of j_{cos} , j_{Al} to t_{sc} (or t_{bk} – breakdown torque in p.u.) has revealed that two, four, and six poles are the main pole counts to consider except for very low direct speed drives – conveyor drives – where even $2p_1 = 12$ is to be considered, only to eliminate the mechanical transmission.

Globally, when efficiency, breakdown torque, and motor volume are all considered, the four-pole motor seems most desirable.

Reducing the number of poles to $2p_1 = 2$, for given speed $n_1 = f_1/p_1$ (rps), means lower frequency, but, for the same stator bore diameter, it means larger pole pitch and longer end connections and thus larger λ_{end} in (7.12).

Now with q_1 larger, for $2p_1 = 2$, the differential leakage coefficient λ_{ds} is reduced. So, unless the stack length is not small at design start (pancake shape), the stator leakage inductance (7.12) decreases by a ratio between 1 and 2 when the pole count increases from two to four poles, for the same current and flux density. Considering the two rotors identical, with the same stack length, L_{rl} in (7.13) would not be much different. This leads us to the peak torque formula (7.3).

$$T_{ek} \approx \frac{3p_1}{2} \frac{(\Psi_{sph})^2}{2L_{sc}}; \quad \Psi_{sph} = \frac{V_{ph}}{\omega_1} \quad (7.14)$$

For the same number of stator slots for $2p_1 = 2$ and 4, conductors per slot n_s , same airgap flux density, and same stator bore diameter, the phase flux linkage ratio in the two cases is

$$\frac{(\Psi_{\text{sph}})_{2p_1=2}}{(\Psi_{\text{sph}})_{2p_1=4}} = \frac{2}{1} \quad (7.15)$$

as the frequency is doubled for $2p_1 = 4$, in comparison with $2p_1 = 2$, for the same no-load speed (f_1/p_1).

Consequently,

$$\frac{(L_{\text{sc}})_{2p_1=2}}{(L_{\text{sc}})_{2p_1=4}} = 1.5 - 1.8 \quad (7.16)$$

Thus, for the same bore diameter, stack length, and slot geometry,

$$\frac{(T_{\text{ek}})_{2p_1=2}}{(T_{\text{ek}})_{2p_1=4}} \approx \frac{2}{1.5 - 1.8} \quad (7.17)$$

From this simplified analysis, we may draw the conclusion that the $2p_1 = 2$ pole motor is better. If we consider the power factor and efficiency, we might end up with a notably better solution.

For super-high-speed motors, $2p_1 = 2$ seems a good choice ($f_{1n} > 300 \text{ Hz}$).

For given total stator slot area (same current density and turns/phase) and the same stator bore diameter, increasing q_1 (number of slot/pole/phase) does not essentially influence L_{sl} (7.12) – the stator slot leakage and end-connection leakage inductance components – as $n_s p_1 q_1 = W_1 = ct$ and slot depth remains constant, whereas the slot width decreases to the extent q_1 increases, and so does λ_{end} :

$$\lambda_{\text{end}} \approx \frac{0.34}{L_1} (l_{\text{end}} - 0.64 \cdot y) \cdot q_1 \quad (7.18)$$

where l_{end} – end-connection length, y – coil throw, and L_1 – stack length.

However, λ_{ds} decreases and apparently so does λ_{zs} . In general, with larger q_1 , the total stator leakage inductance will decrease slightly. In addition, the stray losses have been proved to decrease with q_1 increasing.

A similar rationale is valid for the rotor leakage inductance L_{rl} (7.13) where the number of rotor slots increases. It is well understood that the condition $0.8N_s < N_r < N_s$ is to be observed.

A safe way to reduce the leakage reactance is to reduce the slot aspect ratio $h_{\text{ss,r}}/b_{\text{ss,r}} < 3.0 - 3.5$. For given current density, this would lead to lower q_1 (or N_s) for a larger bore diameter, that is, a larger machine volume.

However, if the design current density is allowed to increase (sacrificing to some extent the efficiency) with a better cooling system, the slot aspect ratio could be kept low to reduce the leakage inductance L_{sc} .

- A low leakage (transient) inductance L_{sc} is also required for current source inverter IM drives [4].

So far, we have considered the same current and flux densities, stator bore diameter, stack length, but the stator and yoke radial height for $2p_1 = 2$ is doubled with respect to the four-pole machine.

$$\frac{(h_{\text{cs,r}})_{2p_1=2}}{(h_{\text{cs,r}})_{2p_1=4}} \approx \frac{(1.5 - 2)}{1} \quad (7.19)$$

Even if we oversaturated the stator and rotor yokes, more so for the two-pole machine, the outer stator diameter will still be larger in the latter case. It is true that this leads to a larger heat exchange area with the environment, but still the machine size is larger.

So, when the machine size is crucial, $2p_1 = 4$ [5] even $2p_1 = 6$ is chosen (urban transportation traction motors).

Whether to use long or short stack motors is another choice to make in the pursuit of smaller leakage inductance L_{sc} . Long stator stacks may allow smaller stator bore diameters, smaller pole pitches, and thus smaller stator end connections.

Slightly smaller L_{sc} values are expected. However, a lower stator (rotor) diameter does imply deeper slots for the same current density. An increase in slot leakage occurs.

Finally, increasing the stack length leads to limited breakdown torque increase.

When low inertia is needed, the stack length is increased, whereas the stator bore diameter is reduced. The efficiency will vary little, but the power factor will likely decrease. Consequently, the PEC KVA rating has to be slightly increased. The kVA ratings for two-pole machines with the same external stator diameter and stack length, torque, and speed, is smaller than those for a four-pole machine because of higher power factor. So when the inverter KVA is to be limited, the two-pole machine might prevail.

A further way to decrease the stator leakage inductance may be to use four layers (instead of two) and chorded coils to produce some cancelling of mutual leakage fluxes between them. The technological complication seems to render such approaches as less practical.

7.7 WIDE CONSTANT POWER SPEED RANGE VIA VOLTAGE MANAGEMENT

CPSR varies for many applications from 2 to 1 to 5(6) to 1 or more.

The obvious way to handle such requirements is to use an IM capable to produce, at base speed ω_b , a breakdown torque T_{bk} :

$$\frac{T_{bk}}{T_{en}} = \frac{\omega_{max}}{\omega_b} = C_\omega \quad (7.20)$$

- A larger motor

In general, IMs may not develop a peak to rated torque higher than 2.5 (3) to 1 (Figure 7.4a).

In the case when a large CPSR C_ω is required, it is only intuitive to use a larger IM (Figure 7.4b).

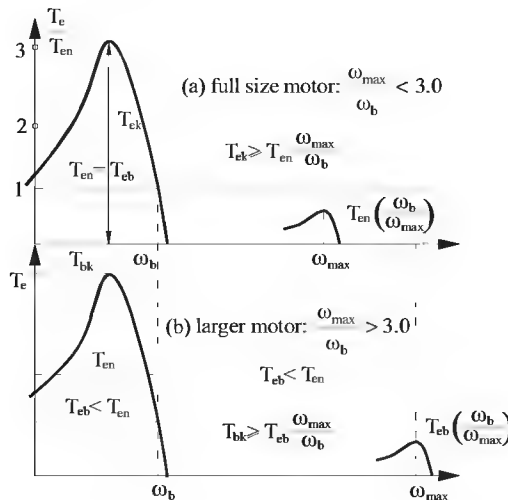


FIGURE 7.4 Torque–speed envelope for constant power: (a) full size motor and (b) larger motor.

Adopting a larger motor, to have enough torque reserve up to maximum speed for constant power may be done with either an IM with $2p_1 = 2, 4$ of higher rating or a larger number of pole motor with the same power. While such a solution is obvious for wide CPSR ($C_\omega > 2.0\text{--}3.0$), it is not always acceptable as the machine size is notably increased.

- Higher voltage/phase

The typical torque/speed, voltage/speed, and current/speed envelopes for moderate CPSR are shown in Figure 7.5.

The voltage is considered constant above base speed. The slip frequency f_{sr} is rather constant up to base speed and then increases up to maximum speed. Its maximum value $f_{sr\max}$ should be less or equal to the critical value (that corresponding to breakdown torque):

$$f_{sr} \leq f_{sr\max} = \frac{R_r}{2\pi L_{sc}} \quad (7.21)$$

$$(T_{bk})_{\omega_{\max}} = \frac{3}{2} p_1 \left(\frac{V_{ph}}{\omega_{r\max}} \right)^2 \cdot \frac{1}{L_{sc}} \geq T_{eb} \cdot \frac{\omega_b}{\omega_{\max}} \quad (7.22)$$

If

$$(T_{bk})_{\omega_{\max}} < T_{eb} \frac{\omega_b}{\omega_{\max}}, \quad (7.23)$$

it means that the motor has to be oversized to produce enough torque at maximum speed.

If a torque (power reserve) is to be secured for fast transients, a larger torque motor is required.

Alternatively, the phase voltage may be increased during the entire constant power range (Figure 7.6).

To provide a certain constant overloading over the entire speed range, the phase voltage has to increase over the entire CPSR. This means that for base speed ω_b , the motor will be designed for lower voltage and thus larger current. The inverter current loading (and costs) will be increased.

Let us calculate the torque T_e , and stator current I_s from the basic equivalent circuit.

$$T_e \approx \frac{3p_1}{\omega_1} \cdot \frac{V_{ph}^2 \cdot R_r / s}{\left(R_s + C_1 \frac{R_r}{s} \right)^2 + \omega_1^2 L_{sc}^2}; \quad (7.24)$$

$$L'_{sc} = L_{sl} + C_1 L_{rl}; C_1 = 1 + \frac{L_{sl}}{L_m} \approx 1.02\text{--}1.08$$

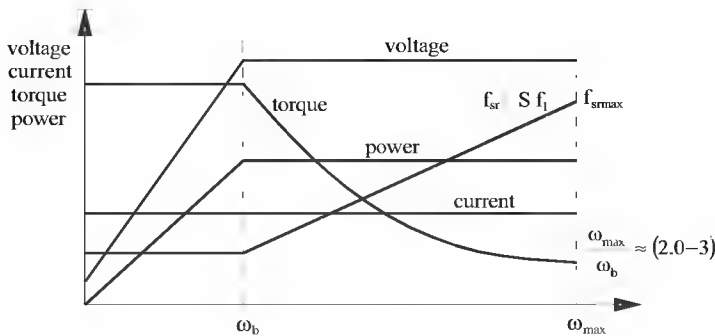


FIGURE 7.5 Torque, voltage, current versus frequency.

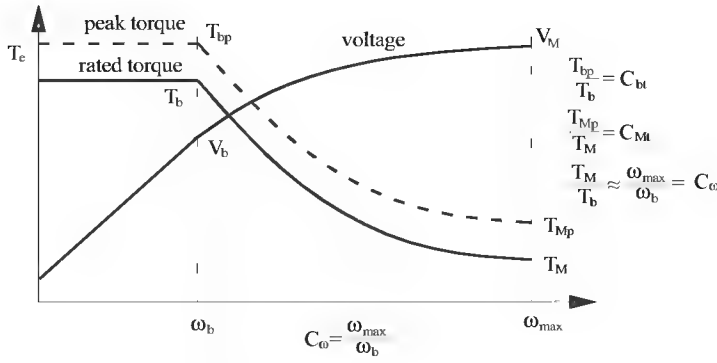


FIGURE 7.6 Raising voltage for the CPSR.

$$I_s \approx V_{ph} \sqrt{\left(\frac{1}{\omega_l L_m}\right)^2 + \frac{1}{\left(R_s + C_l \frac{R_r}{s}\right)^2 + \omega_l^2 L_{sc}^2}} \quad (7.25)$$

At maximum speed, the machine has to develop the peak torque T_{Mp} at maximum voltage V_M .

Now if for both T_{bp} and T_{Mp} , the breakdown torque conditions are met

$$\begin{aligned} T_{bp} &= \frac{3p_l}{2} \left(\frac{V_b}{\omega_b}\right)^2 \cdot \frac{1}{2L_{sc}} \\ T_{Mp} &= \frac{3p_l}{2} \left(\frac{V_M}{\omega_{max}}\right)^2 \cdot \frac{1}{2L_{sc}}. \end{aligned} \quad (7.26)$$

the voltage ratio V_M/V_b is (as in [6])

$$\left(\frac{V_M}{V_b}\right)^2 = \left(\frac{\omega_{max}}{\omega_b}\right)^2 \frac{T_{Mp}}{T_{bp}} = \frac{\omega_{max}}{\omega_b} \cdot \frac{\left(\frac{T_{Mp}}{T_M}\right)}{\left(\frac{T_{bp}}{T_b}\right)} = C_\omega \frac{C_{Mt}}{C_{bt}} \quad (7.27)$$

For $\omega_{max}/\omega_b = 4$, $C_{Mt} = 1.5$, $C_{bt} = 2.5$, we find from (7.17) that $V_M/V_b = 1.55$. So the current rating of the inverter (motor) has to be increased by 55%.

Such a solution looks extravagant in terms of converter extra costs, but it does not suppose IM overrating (in terms of power). Only a special design for lower voltage V_b at base speed ω_b is required. Moreover, 55% overvoltage design in the PEC is rather usual.

For

$$S = S_{K,b,m} = \frac{C_l R_r}{\sqrt{R_s^2 + \omega_{b,M}^2 L_{sc}^2}} \quad (7.28)$$

and $\omega_l = \omega_b$ and, respectively, $\omega_l = \omega_M$ from (7.25), the corresponding base and peak currents values I_b and I_M may be obtained.

These currents are to be used in the motor (I_b) and converter ($I_M \sqrt{2}$) design.

Now, based on the above rationale, the various cases, from constant overloading $C_{MT} = C_{bt}$ to zero overloading at maximum speed $C_{MT} = 1$, may be treated in terms of finding V_M/V_b and I_M/I_b ratios and thus prepare for motor and converter design (or selection) [6].

As already mentioned, reducing the slot leakage inductance components through lower slot aspect ratios may help to increase the peak torque by as much as (50%–60%) in some cases, when forced air cooling with a constant speed fan is used [6].

- High CPSR: $\omega_{\max}/\omega_b > 4$

For a large CPSR, the above methods are not generally sufficient. Changing the phase voltage by switching motor phase connection from star to delta (Y to Δ) leads to a sudden increase in phase voltage by $\sqrt{3}$. A notably larger CPSR is obtained this way (Figure 7.7).

The Y to Δ connection switching has to be done through a magnetic switch with the PEC control temporarily inhibited and then so smoothly reconnected that no notable torque transients occurred. This voltage change is thus done “on the fly”.

An even larger CPSR extension may be obtained by using a winding tap (Figure 7.8) to switch from a large to a small number of turns per phase (from W_1 to W'_1 , $W_1/W'_1 = C_{W1} > 1$).

A double single-throw magnetic switch suffices to switch from high (W_1) to low number of turns (W_1/C_{W1}) for high speed.

Once the winding is designed with the same current density for both winding sections, the switching from W_1 to W_1/C_{W1} turns leads to stator and rotor resistance reduction by $C_{W1} > 1$ times and a total leakage inductance L_{sc} reduction by C_{W1}^2 .

In terms of peak (breakdown torque), it means a strong increase by C_{W1}^2 . The CPSR is extended by

$$\frac{\omega_{\max H}}{\omega_{\max L}} = C_{W1}^2 \quad (7.29)$$

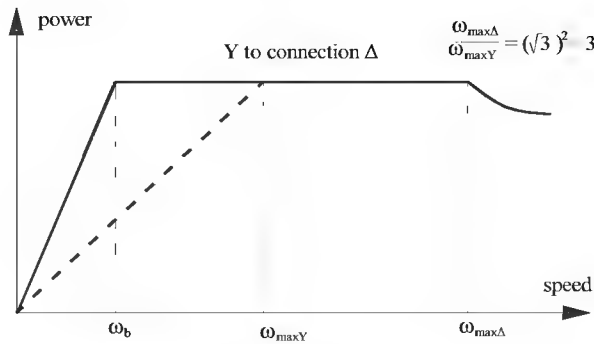


FIGURE 7.7 Extended CPSR by Y to Δ stator connection switching.

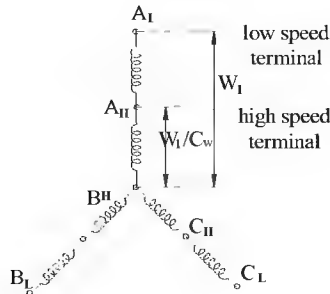


FIGURE 7.8 Tapped stator winding.

The result is similar to that obtained with Y to Δ connection switching, but this time, C_{w_i} may be made larger than $\sqrt{3}$ and thus a larger extension of CPSR is obtained without oversizing the motor. The price to be paid is the magnetic (or thyristor-soft) switch. The current ratios for the two winding situations for the peak torque are

$$\frac{I_{s \max H}}{I_{s \max L}} \approx C_{w_i} \quad (7.30)$$

The leakage inductance L_{sc} decreases with C_w^2 , whereas the frequency increases C_w times. Consequently, the impedance decreases, at peak torque conditions, C_w times. This is why the maximum current increases C_{w_i} times at peak speed $\omega_{\max H}$ with respect to its value at $\omega_{\max L}$. Again, the inverter rating has to be increased accordingly while the motor cooling has to be adequate for these high demands in winding losses. The core losses are much smaller, but they are generally smaller than copper losses unless super-high speed (or high-power) is considered. As for how to build the stator winding, to remain symmetric in the high-speed connection, it seems feasible to use two unequal coils per layer and make two windings, with W_l/C_{w_i} and $W_l(1 - C_{w_i}^{-1})$ turns per phase, respectively, and connect them in series. The slot filling factor will be slightly reduced but, with the lower turns coil on top of each slot layer, a further reduction of slot leakage inductance for high-speed connection is obtained as a bonus.

• Inverter pole count switching

It is basically possible to reduce the number of poles in the ratio of 2 to 1 using two twin half-rating inverters and thus increase the CPSR by a factor of two [7].

No important oversizing of the converter seems needed. In applications when PEC redundancy for better feasibility is a must, such a solution may prove adequate.

It is also possible to use a single PEC and two simple three-phase soft switches (with thyristors) to connect it to the 2 three-phase terminals corresponding to two different pole count windings (Figure 7.9).

Such windings with good winding factors are presented in Chapter 4, Vol. 1.

As can be seen from Figure 7.9, the current remains rather constant over the extended CPSR, as does the converter rating. Apart from the two magnetic (or static) switches, only a modest additional motor overcost due to dual pole count winding is to be considered the price to pay for speed range extension at constant power. For cost-sensitive applications, this solution might be practical.

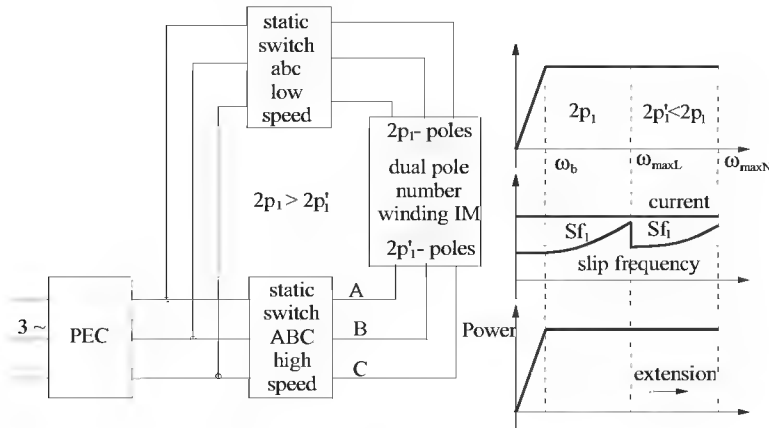


FIGURE 7.9 Dual static switch for dual pole count winding IM.

7.8 DESIGN FOR HIGH- AND SUPER-HIGH-SPEED APPLICATIONS

As we already mentioned, super-high-speed range start at about 18 krpm (300 Hz in two-pole IMs). Between 3 krpm (50 Hz) and 18 krpm (300 Hz), the interval of high speeds is located (Figure 7.10).

Below 300 Hz, where standard PWM-PEC is available, up to tens of kW (recently up to MW/unit) but above 50 Hz, the design is very much similar to that for speeds below 50 Hz. Two-pole motors are favoured, with lower flux densities to limit core losses, forced stator cooling, and carefully limited mechanical losses. Laminations and conductors are of the standard type with close watch on skin effect containment.

Above 300 Hz, in the super-high-speed applications, the increase in mechanical stresses, mechanical and core loss, and skin effects has triggered specialized worldwide research efforts. We will characterize the most representative of them next.

7.8.1 ELECTROMAGNETIC LIMITATIONS

The Esson's constant C_0 still holds (Chapter 4):

$$C_0 \approx \frac{\pi^2}{\sqrt{2}} K_{w1} A_1 B_g \quad (7.31)$$

The linear current density A_1 (Aturns/m) is limited to $(5-20)10^3$ A/m for most super-high-speed IMs and in general increases with stator bore diameter D_{is} .

The stator bore diameter D_{is} is related to motor power P_n as (Chapter 4).

$$D_{is} = \sqrt{\frac{K_E P_n}{\eta \cos \phi} \cdot \frac{p_1}{f_1} \cdot \frac{1}{C_0} \cdot \frac{1}{L}}; K_E \approx 0.98 - p_1 \cdot 5 \cdot 10^{-3} \quad (7.32)$$

where f_1 – frequency, p_1 – pole pairs, L – stack length, η – efficiency, and K_E – emf coefficient.

As the no-load speed $n_1 = f_1/p$ is large, for given stack length L , the stator bore diameter D_{is} may be obtained for given C_0 , from (7.32).

For super-high-speed motors, even with thin laminations (0.1 mm thick), the airgap flux density is lowered to $B_g < (0.5-0.6)$ T to reduce the core losses which tend to be large as frequency increases. Longer motors (L – larger) tend to require, as expected, low stator bore diameters D_{is} .

7.8.2 ROTOR COOLING LIMITATIONS

Due to the unistack rotor construction, made either from solid iron or from laminations, the heat due to rotor losses – total conductor, core, and air friction losses – has to be transferred to the cooling agent almost entirely through the rotor surface. Thus, the rotor diameter $D_{er} \approx D_{is}$ is limited by

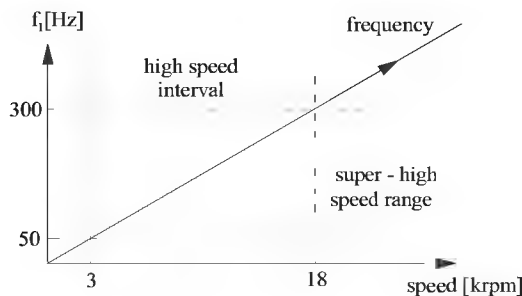


FIGURE 7.10 High- and super-high-speed division.

$$D_{is} \geq \frac{\text{Rotor_losses}}{\pi L \cdot \alpha \Delta \theta_{ra}} \quad (7.33)$$

where α – heat transfer factor $\text{W/m}^2\text{K}$ and $\Delta \theta_{ra}$ – rotor to internal air temperature differential.

Among rotor losses, besides fundamental cage and core losses, space and time harmonics losses in the core and rotor cage are notable.

7.8.3 ROTOR MECHANICAL STRENGTH

The centrifugal stress on the rotor increases with speed squared. At the laminated rotor bore $D_{er} \approx D_{is}$, the critical stress $\sigma_K (\text{N/m}^2)$ should not be surpassed [6].

$$D_{is} < \sqrt{\frac{16\sigma_K}{\gamma\omega_r^2(3+\mu)} - \frac{(1-\mu)}{3+\mu}} \cdot D_{int}^2 \quad (7.34)$$

where γ – rotor material density (kg/m^3), ω_r – rotor angular speed (rad/s), μ – Poisson's ratio, and $D_{int}(\text{m})$ – interior diameter of rotor laminations ($D_{int} = 0$ for solid rotor).

For 100kW IMs with $L/D_{is} = 3$ (long stack motors) and $C_0 = 60 \cdot 10^3 \text{ J/m}^3$, the limiting curves given by (7.32)–(7.34) are shown in Figure 7.12 [8].

Results in Figure 7.11 lead to remarks such that

- To increase the speed, improved (eventually liquid) rotor cooling may be required.
- When speed and the Esson's constant increase, the rotor losses per rotor volume increase and thus thermal limitations become the main problem.
- The centrifugal stress in the laminated rotor restricts the speed range (for 100kW).

7.8.4 THE SOLID IRON ROTOR

As the laminated rotor shows marked limitations, extending the speed range for given power leads, inevitably, to the solid rotor configuration. The absence of the central hole and the solid structure produce a more rugged configuration.

The solid iron rotor is also better in terms of heat transmission, as it allows for good axial heat exchange by thermal conductivity.

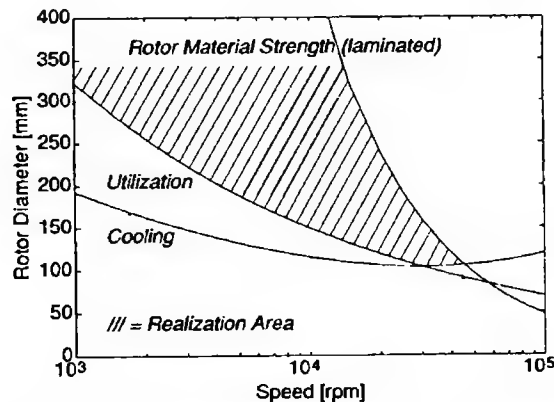


FIGURE 7.11 Stator bore diameter limiting curves for 100kW machines at very high speed. (After Ref. [8].)

Unfortunately, the smooth solid iron rotor is characterized by a large equivalent resistance R_r and a rather limited magnetization inductance. This is so because the depth of field penetration in the rotor δ_{iron} is [9]

$$\delta_i \approx \left| \operatorname{Re} \left(\sqrt{\frac{1}{(\pi/\tau)^2 + j2\pi f_1 S \frac{\sigma_{iron}}{K_T} \cdot \mu_0 \cdot \mu_{rel}}} \right) \right| \quad (7.35)$$

where τ – pole pitch, μ_{rel} – relative iron permeability, and K_T – conductivity correction factor to be explained later in this section:

$$a = (\pi/\tau)^2 \quad (7.36)$$

$$b = 2\pi f_1 S \frac{\sigma_{iron}}{K_T} \cdot \mu_0 \mu_{rel}$$

$$\delta_i = \frac{\sqrt{a + \sqrt{a^2 + b^2}}}{\sqrt{2(a^2 + b^2)}} \quad (7.37)$$

For $\tau = 0.1$ m, $\mu_{rel} = 18$, $S f_1 = 5$ Hz, $\sigma_{iron} = 10^{-1}$ $\sigma_{co} = 5 \cdot 10^6$ (Ωm)⁻¹, from (7.36) to (7.37), $\delta_i = 1.316 \cdot 10^{-2}$ m.

To provide reasonable airgap flux density $B_g \approx 0.5$ – 0.6 T, the field penetration depth in iron δ_i for a highly saturated iron, with say $\mu_{rel} = 36$ (for $S f_1 = 5$ Hz) and $B_{iron} = 2.3$ T ($H_{iron} = 101733.6$ A/m), must be

$$\delta_{iron} \geq \frac{B_g \cdot \tau / \pi}{B_{iron}} = \frac{0.55}{\pi \cdot 2.3} \cdot \tau = 7.6 \cdot 10^{-2} \cdot \tau [\text{m}] \quad (7.38)$$

For $\delta_{iron} = 0.01316$ m, the pole pitch τ should be $\tau < 0.173$ m. As the pole pitch was supposed to be $\tau = 0.1$ m, the iron permeability may be higher.

For a $2p_1 = 2$ pole machine, typical for super-high-speed IMs, the stator bore diameter D_{is}

$$D_{is} < \frac{2p_1 \tau}{\pi} = \frac{2 \cdot 1 \cdot 0.173}{\pi} = 0.110 \text{ m!} \quad (7.39)$$

This is a severe limitation in terms of stator bore diameter (see Figure 7.11). For higher diameters D_{is} , smaller relative permeabilities have to be allowed for. The consequence of the heavily saturated rotor iron is a large magnetization mmf contribution which in relative values to airgap mmf is

$$\frac{F'_{iron}}{F_{airgap}} \approx \frac{\frac{1}{3} \tau \cdot H_{iron} \cdot \mu_0}{B_g \cdot 2g} \approx \frac{\frac{1}{3} 0.1 \cdot 101,733 \cdot 1.256 \cdot 10^{-6}}{0.55 \cdot 2 \cdot 1.0 \cdot 10^{-3}} = 3.872! \quad (7.40)$$

A rather low power factor is expected. The rotor leakage reactance to resistance ratio may be approximately considered as 1.0 (for constant, even low permeability)

$$\frac{\omega_l L_{rl}}{R_r} \approx 1 \quad (7.41)$$

From [8], the rotor resistance reduced to the stator is

$$R_r = \frac{6L \cdot K_T}{\delta_i \tau p_i \sigma_{\text{iron}}} \cdot (K_{w1} W_1)^2 \quad (7.42)$$

In (7.42), the rotor length was considered equal to stator stack length, and K_T is a transverse edge coefficient which accounts for the fact that the rotor current paths have a circumferential component (Figure 7.12):

From [8],

$$K_T \approx \frac{1}{1 - \frac{\tanh(\pi L / 2\tau \cdot n)}{(\pi L / 2\tau \cdot K)}} > 1 \quad (7.43)$$

The longer the stack length/pole pitch ratio L/τ , the smaller the coefficient.

Here, $n = 1$. While K_T reduces the equivalent apparent conductivity of iron, it does increase the rotor resistance as $\sqrt{K_T}$, lowering the torque for given slip frequency Sf_1 .

This is why the rotor structure may be slitted with the rotor longer than the stack to get lower transverse edge effect coefficient K_T . With copper end rings, we may consider $K_T = 1$ in (7.43) and in (7.35). (since the end rings resistance is much smaller than the rotor iron resistance (Figure 7.13)).

How deep the rotor slits should be is a matter of optimal design as the main flux paths have to close below their bottom. So definitely $h_{\text{slit}} > \delta_i$. Notably larger output has been demonstrated for given stator with slitted rotor and copper end rings [10].

The radial slots may be made wider and filled with copper bars (Figure 7.14a); copper end rings are added. As expected, still larger output for better efficiency and power factor has been obtained [10].

Another solution would be closed rotor slots with copper bars and circumferential deep slits in the rotor (Figure 7.14b).

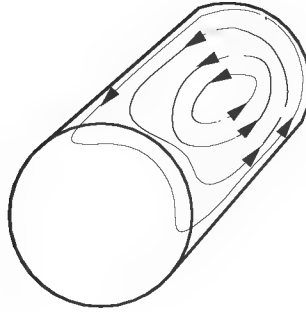


FIGURE 7.12 Smooth solid rotor induced current paths.

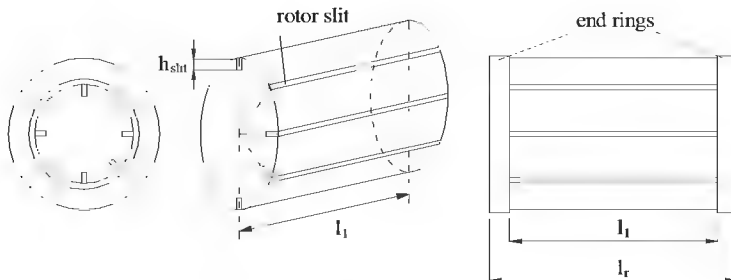


FIGURE 7.13 Radially slitted rotor with end rings.

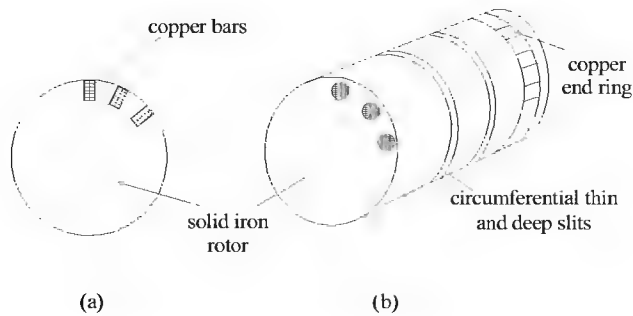


FIGURE 7.14 Solid iron rotor with copper bars: (a) copper bars in radial open slots (slits) and (b) copper bars in closed slots.

This time, before the copper bars are inserted in slots, the rotor is provided with thin and deep circumferential slits such that to increase the transverse edge effect coefficient K_T (7.43) where n is the number of stack axial sections produced by slits.

This time they serve to destroy the solid iron eddy currents as the copper bars produce more efficiently torque, that is, for lower rotor resistance. The value of n may be determined such that K_T is so large that depth of penetration $\delta_{\text{iron}} \geq 2d_{\text{co}}$ (d_{co} – copper bar diameter). Three to six such circumferential slits generally suffice.

The closed rotor slots serve to keep the copper bar tight, while the slitted iron serves as a rugged rotor and better core while the space harmonics losses in the rotor iron are kept reasonably small.

A thorough analysis of the solid rotor with or without copper cage with and without radial or circumferential slits may require a full-fledged 3D FEM approach. This is time-consuming, and thus, full-scale careful testing may not be avoided.

7.8.5 21 kW, 47,000 rpm, 94% EFFICIENCY WITH LAMINATED ROTOR

As shown in Figure 7.11, at 50,000 rpm, a stator bore diameter D_{is} of up to 80 mm is acceptable from the rotor material strength point of view [11].

Reference [11] reports a good performance 21 kW, 47,000 rpm ($f_1 = 800$ Hz) IM with a laminated rotor which is very carefully designed and tested for mechanical and thermal compliance.

High resistivity silicon 3.1% laminations (0.36 mm thick) were specially thermally treated to reduce core loss at 800 Hz to less than 30 W/kg at 1 T, and to avoid brittleness, while still holding an yield strength above 500 MPa. Closed rotor slots are used, and the rotor lamination stress in the slot bridge zone was carefully investigated by FEM.

Al 25 (85% copper conductivity) was used for the rotor bars, whereas the more rugged Al 60 (75% copper conductivity) was used to make the end rings. The airgap was $g = 1.27$ mm for a rotor diameter of 51 mm, a stack length $L = 102$ mm, and $2p_1 = 2$ poles. The stator/rotor slot numbers are $N_s/N_r = 24/17$, rated voltage 420 V, and current 40 A. A rather thick shaft is provided (30 mm in diameter) for mechanical reasons. However, in this case part of it is used as rotor yoke for the main flux. As the rated slip frequency $Sf_1 = 5$ Hz, the flux penetrates enough in the shaft to make the latter a good part of the rotor yoke. The motor is used as a direct drive for a high-speed centrifugal compressor system. Shaft balancing is essential. Cooling of the motor is carried out in the stator by using a liquid cold refrigerant. The refrigerant also flows through the end connections and through the airgap zone in the gaseous form. To reduce the pressure drop, a rather large airgap ($g = 1.27$ mm) was adopted. The motor is fed from an IGBT voltage source PWM converter switched at 15 kHz.

The solution proves to be the least expensive by a notable margin when compared with PM-brushless and switched reluctance motor drives of equivalent performance.

This is a clear example of how, by carefully pushing forward existing technologies, new boundaries are reached [11,12]

7.9 SAMPLE DESIGN APPROACH FOR WIDE CONSTANT POWER SPEED RANGE

Design specifications are

Base power: $P_n = 7.5 \text{ kW}$

Base speed: $n_b = 500 \text{ rpm}$

Maximum speed $n_{\max} = 6000 \text{ rpm}$

CPSR: 500–6000 rpm

Power supply line voltage: 400 V, 50 Hz, 3 phase

7.9.1 SOLUTION CHARACTERIZATION

The design specifications indicate an unusually large CPSR $n_{\max}/n_b = 12:1$. Among the solutions for this case, it seems that a combination of winding connection switching from Y to Δ and high peak torque design with lower than rated voltage at base speed might do it.

The Y to Δ connection produces an increase of constant power speed zone by

$$\frac{n_{\max \Delta}}{n_{\max Y}} = \left[\frac{(V_{ph})_{\Delta}}{(V_{ph})_Y} \right]^2 = \frac{3}{1} \quad (7.44)$$

It remains a 4/1 ratio CPSR for the Y (low-speed) connection of windings.

As the peak/rated torque seldom goes above 2.5–2.7, the voltage at base speed has to be reduced notably.

To leave some torque reserve up to maximum speed, even in the Δ (high-speed) connection, a voltage reserve has to be left.

First, from (7.27), we may calculate the $V_{MY/\Delta}/V_{\text{rated}}$ from the high-speed zone,

$$\left(\frac{V_{MY/\Delta}}{V_{\text{rated}}} \right)^{-2} = \frac{\omega_{\max \Delta}}{\omega_{\max Y}} \cdot \frac{C_{Mt}}{C_{bt}} \quad (7.45)$$

C_{MT} is the overload capacity at maximum speed. Let us consider $(C_{MT}) = 1$ for $\omega_{\max \Delta}$ (6000 rpm). C_{bt} is the ratio between peak torque and base torque, a matter of motor design. Let us take $C_{bt} = 2.7$. In this case from (7.45) (point B),

$$\left(\frac{V_{MY/\Delta}}{V_{\text{rated}}} \right) = \sqrt{\frac{2.7}{3 \cdot 1}} = 0.95 \quad (7.46)$$

Now for point A in Figure 7.15, we may apply the same formula:

$$\left(\frac{V_b}{V_{\text{rated}}} \right)^{-2} = \left(\frac{V_{MY/\Delta}}{V_{\text{rated}}} \right)^{-2} \cdot \frac{\omega_{\max Y}}{\omega_b} \cdot \frac{C_{Mt}}{C_{bt}} \quad (7.46')$$

As for point A $V < V_{\text{rated}}$, there is a torque reserve, so we may consider again $C_{Mt} = 1.0$:

$$\frac{V_b}{V_{\text{rated}}} = 0.95 \sqrt{\frac{1}{4} \cdot \frac{2.7}{1.0}} = 0.78 \quad (7.47)$$

Admitting a voltage derating of 5% ($V_{\text{derat}} = 0.05$) due to the PEC, the base line voltage at motor terminals, which should produce rated power at $n_b = 500 \text{ rpm}$, is

- General drives have a moderate CPSR of $\omega_{\max}/\omega_b < 2$, for constant (ceiling) voltage from the PEC. Increases in the ratio $\omega_{\max}/\omega_b$ are obtainable basically only by reducing the total leakage inductance of the machine. Various parameters such as stack length, bore diameter, or slot aspect ratio may be used to increase the “natural” CPSR up to $\omega_{\max}/\omega_b \leq T_{bk}/T_b < (2.2\text{--}2.7)$. No overrating of the PEC or of the IM is required.
- Constant power drives are characterized by $\omega_{\max}/\omega_b > T_{bk}/T_b$. There are quite a few ways to go around this challenging task. Among them are designing the motor at base speed for a lower voltage and allowing the voltage to increase steadily and reach its peak value at maximum speed.
- The constant power speed zone may be extended notably this way but at the price of higher rated (base) current in the motor and PEC. While the motor does not have to be oversized, the PEC does. This method has to be used with caution.
- Using a winding tap, and some static (or magnetic) switch, is another very efficient approach to extend the constant power speed zone because the leakage inductance is reduced by the number of turns reduction ratio for high speeds. A much higher peak torque is obtained. This explains the widening of CPSR. The current raise is proportional to the turns reduction ratio. So PEC oversizing is inevitable. Also, only a part of the winding is used at high speeds.
- Winding connection switching from Y to Δ is capable of increasing the speed range at constant power by as much as three times without overrating the motor or the converter. A dedicated magnetic (or static) switch with PEC short time shutdown is required during the connection switching. Using a Z-PWM converter to increase phase voltage might also be feasible.
- Pole count changing from higher to lower values also extends the constant power speed zone by p_i/p'_i times. For $p_i/p'_i = 2/1$, a twin half rated power PEC may be used to do the switching of pole count. Again, no oversizing of motor or total converter power is required. Alternatively, a single full-power PEC may be used to supply a motor with dual pole count winding terminals by using two conventional static (or magnetic) three-phase power switches.
- The design for variable speed has to reduce the skin effect which has a strong impact on rotor cage sizing.
- The starting torque and current constraints typical for constant V/f design no longer exist.
- High-efficiency IMs with insulated bearings designed for constant V/f may be (in general) safely supplied by PEC to produce variable speed drives.
- Super-high-speed drives (centrifugal compressors, vacuum pumps, spindles) are considered here for speeds above 18,000 rpm (or 300 Hz fundamental frequency). This classification reflects the actual standard PWM-PEC performance limitations.
- For super-high speeds, the torque and size are reduced and thus the rotor thermal and mechanical limitations become even more evident. Centrifugal force-produced mechanical stress might impose, at certain peripheral rotor speed, the usage of a solid iron rotor.
- While careful design of laminated rotor IMs has reached 21 kW at 47,000 rpm (800 Hz), for larger powers and equivalent or higher peripheral speed, apparently, solid iron rotors may be mandatory.
- To reduce iron equivalent conductivity and thus increase field penetration depth, the solid iron rotor with circumferential deep (and thin) slits and copper bars in slots seems a practical solution.
- The most challenging design requirements occur for wide CPSR applications ($\omega_{\max}/\omega_b > \text{peak_torque}/\text{base_torque}$). The electromagnetic design is performed for base speed and base power for lower than maximum PEC voltage $V_{s\max}$. The thermal design has to consider the loss evolution with increasing speed.
- Once the design specifications and the most demanding performance are identified, the design (sizing) of IM for variable speed may follow a similar path as for constant V and f operation (Chapters 4–6, Vol. 2).

- Still new rotor slot shaping was recently proved to reduce rotor cage harmonics losses to boost efficiency by 1% in a 1 MW variable speed traction IM [13], signalling the use of surrogate FEM IM model (only in optimal design attempts of the future) regularly.
- Using FEM allowed the reduction of torque pulsations notably in IMs (from 44% to 33% for square wave voltages and down to 10% for sinusoidal currents); those remaining are mainly due to voltage time harmonics currents [14].
- New attempts in better cage rotor design and manufacturing for super-high-speed applications (4 kW, 120 rpm), with promising results, keep surfacing [15].

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8 Optimization Design Issues

8.1 INTRODUCTION

As we have seen in previous chapters, the design of an induction motor means determining the induction machine (IM) geometry and all data required for manufacturing so as to satisfy a vector of performance variables, together with a set of constraints.

As IMs are now a mature technology, there is a wealth of practical knowledge, validated in industry, on the relationship between performance constraints and the physical aspects of the IM itself.

Also, mathematical modelling of IMs by circuit, field, or hybrid models provides formulas of performance and constraint variables as functions of design variables.

The path from given design variables to performance and constraints, is called *analysis*, whereas the reverse path is called *synthesis*.

Optimization design refers to ways of doing efficiently synthesis by repeated analysis such that some single (or multiple) objective (performance) function is maximized (minimized) while all constraints (or part of them) are fulfilled (Figure 8.1).

Typical single objective (optimization) functions for IMs are

- Efficiency, η
- Cost of active materials, c_{am}
- Motor weight, W_m
- Global cost (C_{am} + cost of manufacturing and selling + loss capitalized cost + maintenance cost).

While single objective function optimization is rather common, multiobjective optimization methods have been recently introduced [1].

The IM is a rather complex artefact, and thus, there are many design variables that describe it rather completely. A typical design variable set (vector) of limited length is given here.

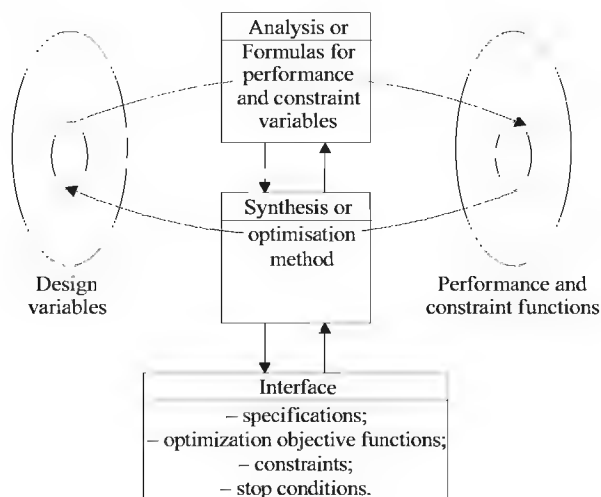


FIGURE 8.1 Optimization design process.

- Number of conductors per stator slot
- Stator wire gauge
- Stator core (stack) length
- Stator bore diameter
- Stator outer diameter
- Stator slot height
- Airgap length
- Rotor slot height
- Rotor slot width
- Rotor cage end-ring width.

The number of design variables may be increased or reduced depending on the number of adopted *constraint functions*. Typical constraint functions are

- Starting/rated current
- Starting/rated torque
- Breakdown/rated torque
- Rated power factor
- Rated stator temperature
- Stator slot filling factor
- Rated stator current density
- Rated rotor current density
- Stator and rotor tooth flux density
- Stator and rotor back iron flux density.

The performance and constraint functions may change attributes in the sense that any of them may switch roles. With efficiency as the only objective function, the other possible objective functions may become constraints.

Also, breakdown torque may become an objective function, for some special applications, such as variable speed drives. It may be even possible to turn one (or more) design variables into a constraint. For example, the stator outer diameter or even the entire stator slotted lamination cross section may be fixed in order to reduce manufacturing costs of the newly designed IM.

The constraints may be equalities or inequalities. Equality constraints are easy to handle when their assigned value is used directly in the analysis and thus the number of design variables is reduced.

Not so with an equality constraint such as starting torque/rated torque, or starting current/rated current as they are calculated making use, in general, of all design variables.

Inequality constraints are somewhat easier to handle as they are not so tight restrictions.

The optimization design's main issue is the computation time (effort) until convergence towards a global optimum is reached.

The problem is that, with such a complex nonlinear model with lots of restrictions (constraints), the optimization design method may, in some cases, converge too slowly or not converge at all.

Another implicit problem with convergence is that the objective function may have multiple maximal (minimal) and the optimization method gets trapped in a local rather than the global optimum (Figure 8.2).

It is only intuitive that, in order to reduce the computation time and increase the probability of reaching a global optimum, the search in the subspace of design variables has to be thorough.

This process gets simplified if the number of design variables is reduced. This may be done by intelligently using the constraints in the process. In other words, the analysis model has to be wisely manipulated to reduce the number of variables.

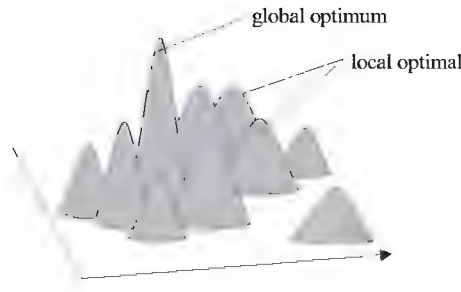


FIGURE 8.2 Multiple maxima objective function for two design variables.

It is also possible to start the optimization design with a few different sets of design variable vectors, within their existing domain. If the final objective function value is the same for the same final design variables and constraint violation rate, then the optimization method was able to find the global optimum.

But there is no guarantee that such a successful ending will take place for other IM with different specifications, investigated with the same optimization method.

These challenges have led to numerous optimization method proposals for the design of electrical machines, IMs in particular.

8.2 ESSENTIAL OPTIMIZATION DESIGN METHODS

Most optimization design techniques employ nonlinear programming (NLP) methods. A typical form for a uni-objective NLP problem can be expressed in the form:

$$\text{Minimize } F(x) \quad (8.1)$$

$$\text{Subject to: } g_j(x) = 0; \quad j = 1, \dots, m_e \quad (8.2)$$

$$g_j(x) \geq 0; \quad j = m_e + 1, \dots, m \quad (8.3)$$

$$X_{\text{low}} \leq X \leq X_{\text{high}} \quad (8.4)$$

where

$$X = \{x_1, x_2, \dots, x_n\} \quad (8.5)$$

is the design variable vector, $F(x)$ is the objective function, and $g_j(x)$ are the equality and inequality constraints. The design variable vector X is bounded by lower (X_{low}) and upper (X_{high}) limits.

The NLP problems may be solved by direct methods (DMs) and indirect methods (IDMs). The DM converts the constrained problem into a simpler, unconstrained problem by integrating the constraints into an augmented objective function.

Among the DMs, the complex method [2] stands out as an extension of the simplex method [3]. It is basically a stochastic approach. From the numerous IDMs, we mention here first the sequential quadratic programming (SQP) [4,5]. In essence, the optimum is sought by successively solving quadratic programming (QP) subproblems which are produced by quadratic approximations of the Lagrangian function.

The QP is used to find the search direction as part of line search procedure. Under the name of “augmented Lagrangian multiplies method” (ALMM) [6], it has been adopted for inequality constraints. The objective function and constraints gradients must be calculated.

The Hooke–Jeeves [7,8] direct search method may be applied in conjunction with sequential unconstrained minimization technique (SUMT) [8] or without it. No gradients are required. Given a large number of design variables and the problem nonlinearity, the multitude of constraints has ruled out many other general optimization techniques such as grid search, mapping linearization, and simulated annealing, when optimization design of the IM is concerned.

Among the stochastic (evolutionary) practical methods for IM optimization design, the genetic algorithm (GA) method [9] and the Monte Carlo approach [10] have gained most attention.

Finally, a fuzzy artificial experience-based approach to optimization design of double-cage IM is mentioned here [11].

Evolutionary methods start with a few vectors of design variables (initial population) and use genetics inspired operations such as selection (reproduction) crossover and mutation to approach the highest fitness chromosomes by the survival of the fittest principle.

Such optimization approaches tend to find the global optimum but for a larger computation time (slower convergence). They do not need the computation of the gradients of the fitness function and constraints. Nor do they require an already good initial design variable set as most nongradient deterministic methods do.

No single optimization method has gained absolute dominance so far and stochastic and deterministic methods have complimentary merits. So it seems that the combination of the two is the way of the future. First, the GA is used to yield in a few generations a rough global optimization. After that, ALMM or Hooke–Jeeves methods may be used to secure faster convergence and larger precision in constraints meeting.

The DM called the complex (random search) method is also claimed to produce good results [12]. A feasible initial set of design variables is necessary, but no penalty (wall) functions are required as the stochastic search principle is used. The method is less probable to land on a local optimum due to the random search approach applied.

8.3 THE AUGMENTED LAGRANGIAN MULTIPLIER METHOD (ALMM)

To account for constraints, in ALMM, the augmented objective function $L(x,r,h)$ takes the form

$$L(X,r,h) = F(X) + r \sum_{i=1}^m \min[0, g_i(X) + h_{(i)}/r]^2 \quad (8.6)$$

where X is the design variable vector, $g_i(s)$ is the constraint vector (8.2)–(8.3), $h_{(i)}$ is the multiplier vector having components for all m constraints, and r is the penalty factor with an adjustable value along the optimization cycle.

An initial value of design variables set (vector X_0) and that of penalty factor r are required. The initial values of the multiplier vector h_0 components are all considered as zero.

As the process advances, r is increased.

$$r_{k+1} = C \cdot r_k; \quad C = 2 \div 4 \quad (8.7)$$

Also a large initial value of the maximum constraint error is set. With these initial settings, based on an optimization method, a new vector of design variables X_k which minimizes $L(X,r,h)$ is found. A maximum constraint error δ_k is found for the most negative constraint function $g_i(X)$.

$$\delta_k = \max_{1 \leq i \leq m} |\min[0, g_i(X_k)]| \quad (8.8)$$

The large value of δ_0 was chosen such that $\delta_l < \delta_0$. With the same value of the multiplier, $h_{(i)k}$ is set to

$$h_{(i)k} = \min_{l \leq l \leq m} [0, r_k g_i(X_k) + h_{(i)k-1}] \quad (8.9)$$

to obtain $h_{i(1)}$. The minimization process is then repeated.

The multiplier vector is reset as long as the iterative process yields a 4/1 reduction of the error δ_k . If δ_k fails to decrease the penalty factor, r_k is increased. It is claimed that ALMM converges well and that even an infeasible initial X_0 is acceptable. Several starting (initial) X_0 sets are to be used to check the reaching of the global (and not a local) optimum.

8.4 SEQUENTIAL UNCONSTRAINED MINIMIZATION

In general, the induction motor design contains not only real but also integer (slot number, conductors/coil) variables. The problem can be treated as a multivariable NLP problem if the integer variables are taken as continuously variable quantities. At the end of optimization process, they are rounded off to their closest integer feasible values. SQP is a gradient method [4,5]. In SQP, SQ subproblems are successively solved based on quadratic approximations of Lagrangian function. Thus, a search direction (for one variable) is found as part of the line search procedure. SQP has some distinctive merits.

- It does not require a feasible initial design variable vector.
- Analytical expressions for the gradients of the objective functions or constraints are not needed. The quadratic approximations of the Lagrangian function along each variable direction provide for easy gradient calculations.

To terminate the optimization process, there are quite a few procedures:

- Limited changes in the objective function with successive iterations.
- Maximum acceptable constraint violation.
- Limited change in design variables with successive iterations.
- A given maximum number of iterations is specified.

One or more of them may, in fact, be applied to terminate the optimization process.

The objective function is also augmented to include the constraints as

$$\Gamma(X) = f(X) + \gamma \sum_{i=1}^m \left| \langle g_i(X) \rangle \right|^2 \quad (8.10)$$

where γ is again the penalty factor and

$$\langle g_i(X) \rangle = \begin{cases} g_i(X) & \text{if } g_i(X) \geq 0 \\ 0 & \text{if } g_i(X) < 0 \end{cases} \quad (8.11)$$

As in (8.7), the penalty factor increases when the iterative process advances.

The minimizing point of $\Gamma(X)$ may be found by using the univariate method of minimizing steps [13]. The design variables change in each iteration as

$$X_{j+1} = X_j + \alpha_j S_j \quad (8.12)$$

where S_j are unit vectors with one nonzero element. $S_1 = (1, 0, \dots, 0)$; $S_2 = (0, 1, \dots, 0)$, etc.

The coefficient α_j is chosen such that

$$f'(X_{j+1}) < f'(X_j) \quad (8.13)$$

To find the best α , we may use a quadratic equation in each point.

$$f'(X + \alpha) = H(\alpha) = a + b\alpha + c\alpha^2 \quad (8.14)$$

$H(\alpha)$ is calculated for three values of α :

$$\alpha_1 = 0, \quad \alpha_2 = d, \quad \alpha_3 = 2d, \quad (d \text{ is arbitrary})$$

$$H(0) = t_1 = a$$

$$H(d) = t_2 = a + bd + cd^2 \quad (8.15)$$

$$H(2d) = t_3 = a + 2bd + 4cd^2$$

From (8.15), a , b , and c are calculated. But from

$$\frac{\partial H}{\partial \alpha} = 0; \quad \alpha_{\text{opt}} = \frac{-b}{2c} \quad (8.16)$$

$$\alpha_{\text{opt}} = \frac{4t_2 - 3t_1 - t_3}{4t_2 - 3t_3 - 2t_1} d \quad (8.17)$$

To be sure that the extreme is a minimum,

$$\left(\frac{\partial^2 H}{\partial \alpha^2} \right) = c > 0; \quad t_3 + t_1 > 2d \quad (8.18)$$

These simple calculations have to be done for each iteration and along each design variable direction.

8.5 MODIFIED HOOKE–JEEVES METHOD

A direct search method may be used in conjunction with the pattern search of Hooke–Jeeves [7]. Pattern search relies on evaluating the objective function for a sequence of points (within the feasible region). By comparisons, the optimum value is chosen. A point in a pattern search is accepted as a new point if the objective function has a better value than in the previous point.

Let us denote

$X^{(k-1)}$ – Previous base point

$X^{(k)}$ – Current base exploratory point

$X^{(k+1)}$ – Pattern point (after the pattern move)

The process includes exploratory and pattern moves. In an exploratory move, for a given step size (which may vary during the search), the exploration starts from $X^{(k-1)}$ along each coordinate (variable) direction.

Both positive and negative directions are explored. From these three points, the best $X^{(k)}$ is chosen. When all n variables (coordinates) are explored, the exploratory move is completed. The resulting point is called the current base point $X^{(k)}$.

A pattern move refers to a move along the direction from the previous to the current base point. A new pattern point is calculated:

$$X^{(k+1)} = X^{(k)} + a(X^{(k)} - X^{(k-1)}) \quad (8.19)$$

where a is an accelerating factor.

A second pattern move is initiated.

$$X^{(k+2)} = X^{(k+1)} + a(X^{(k+1)} - X^{(k)}) \quad (8.20)$$

The success of this second pattern move $X^{(k+2)}$ is checked. If the result of this pattern move is better than that of point $X^{(k+1)}$, then $X^{(k+2)}$ is accepted as the new base point. If not, then $X^{(k+1)}$ constitutes the new current base point.

A new exploratory-pattern cycle begins but with a smaller step search and the process stops when the step size becomes sufficiently small.

The search algorithm may be summarized as

Step 1: Define the starting point $X^{(k-1)}$ in the feasible region and start with a large step size.

Step 2: Perform exploratory moves in all coordinates to find the current base point $X^{(k)}$.

Step 3: Perform a pattern move: $X^{(k+1)} = X^{(k)} + a(X^{(k)} - X^{(k-1)})$ with $a < 1$.

Step 4: Set $X^{(k-1)} = X^{(k)}$.

Step 5: Perform tests to check if an improvement took place. Is $X^{(k+1)}$ a better point?

If "YES" set $X^{(k)} = X^{(k+1)}$ and go to step 3.

If "NO" continue.

Step 6: Is the current step size the smallest?

If "YES", stop with $X^{(k)}$ as the optimal vector of variables.

If "NO" reduce the step size and go to step 2.

To account for the constraints, the augmented objective function $f^*(X)$, (8.10)–(8.11), is used. This way, the optimization problem becomes an unconstrained one. In all nonevolutionary methods presented so far, it is necessary to do a few runs for different initial variable vectors to make pretty sure that a global optimum is obtained. It is necessary to have a feasible initial variable vector. This requires some experience from the designer. Comparisons between the above methods reveal that the sequential unconstrained minimization method (Han and Powell) is a very powerful but time-consuming tool, whereas the modified Hooke–Jeeves method is much less time-consuming [14–16].

8.6 GENETIC ALGORITHMS

GAs are computational models which emulate biological evolutionary theories to solve optimization problems. The design variables are grouped in finite length strings called chromosomes. GA maps the problem to a set of strings (chromosomes) called population. An initial population is adopted by way of a number of chromosomes. Each string (chromosome) may constitute a potential solution to the optimization problem.

The string (chromosome) can be constituted with an orderly alignment of binary or real coded variables of the system. The chromosome – the set of design variables – is composed of genes which may take a number of values called alleles. The choice of the coding type, binary or real, depends on the number and type of variables (real or integer) and the required precision. Each design variable (gene) is allowed a range of feasible values called search space. In GA, the objective function is called fitness value. Each string (chromosome) of population of generation i is characterized by a fitness value.

The GA manipulates upon the population of strings in each generation to help the fittest survive and thus, in a limited number of generations, obtain the optimal solution (string or set of design variables). This genetic manipulation involves copying the fittest string (elitism) and swapping genes in some other strings of variables (genes).

Simplicity of operation and the power of effect are the essential merits of GA. On top of that, they do not need any calculation of gradients (of fitness function) and provide more probably the global rather than a local optimum.

They do so because they start with a random population – a number of strings of variables – and not only with a single set of variables as nonevolutionary methods do.

However, their convergence tends to be slow, and their precision is moderate. Handling the constraints may be done as for nonevolutionary methods through an augmented fitness function.

Finally, multiobjective optimization may be handled mainly by defining a comprehensive fitness function incorporating as linear combinations, for example, the individual fitness functions.

Although the original GAs make use of binary coding of variables, real coded variables seem more practical for induction motor optimization as most variables are continuous. Also, in a hybrid optimization method, mixing GAs with a nonevolutionary method for better convergence, precision, and less computation time requires real coded variables.

For simplicity, we will refer here to binary coding of variables. That is, we describe first a basic GA algorithm.

A simple GA uses three genetic operations:

- Reproduction (evolution and selection)
- Crossover
- Mutation.

8.6.1 REPRODUCTION (EVOLUTION AND SELECTION)

Reproduction is a process in which individual strings (chromosomes) are copied into a new generation according to their fitness (or scaled fitness) value. Again, the fitness function is the objective function (value).

Strings with higher fitness value have a higher probability of contributing one or more offsprings in the new generation. As expected, the reproduction rate of strings may be established in many ways.

A typical method emulates the *biased roulette wheel* where each string has a roulette slot size proportional to its fitness value.

Let us consider as an example 4 five-digit binary numbers whose fitness value is the decimal number value (Table 8.1).

The percentage given in Table 8.1 may be used to draw the corresponding biased roulette wheel (Figure 8.3).

TABLE 8.1
Fitness Values

String Number	String	Fitness Value	% of Total Fitness Value
1	01000	64	5.5
2	01101	469	14.4
3	10011	361	30.9
4	11000	576	49.2
	Total	1170	100

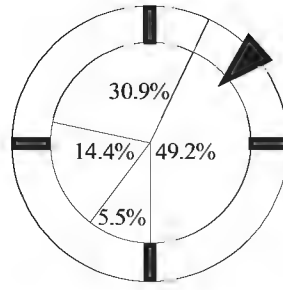


FIGURE 8.3 The biased roulette wheel.

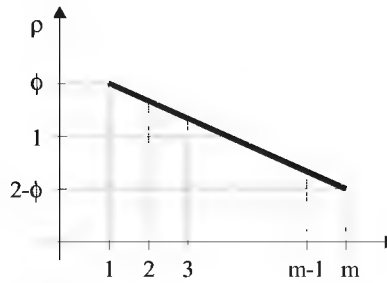


FIGURE 8.4 Selection by arrangement.

Each time a new offspring is required, a simple spin of the biased roulette produces the reproduction candidate. Once a string has been selected for reproduction, an exact replica is made and introduced into the mating pool for the purpose of creating a new population (generation) of strings with better performance.

The biased roulette rule of reproduction might not be fair enough in reproducing strings with very high fitness value. This is why other methods of selection may be used. The selection by the arrangement method, for example, takes into consideration the diversity of individuals (strings) in a population (generation). First, the m individuals are arranged in the decreasing order of their fitness in m rows.

Then, the probability of selection p_i as offspring of one individual situated in row i is [1]

$$p_i = \frac{[\phi - (r_i - 1)(2\phi - 2)] / (m - 1)]}{m} \quad (8.21)$$

ϕ – pressure of selection $\phi = (1-2)$

m – population size (number of strings)

r_i – row of i th individual (there are m rows)

p_i – probability of selection of i th row (individual).

Figure 8.4 shows the average number of offsprings versus the row of individuals. The pressure of selection ϕ is the average number of offspring of best individual. For the worst, it will be $2 - \phi$. As expected, an integer number of offspring is adopted.

By the pressure of selection ϕ value, the survival chance of best individuals may be increased as desired.

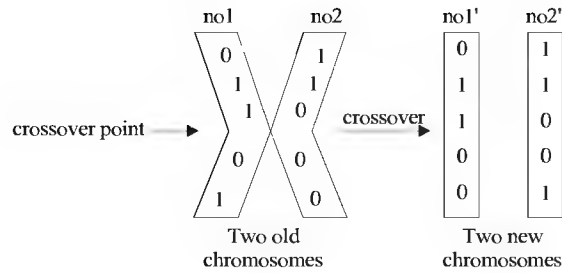


FIGURE 8.5 Simple crossover operation.

8.6.2 CROSSOVER

After the reproduction process is done, *crossover* may proceed. A simple crossover contains two steps:

- Choosing randomly two individuals (strings) for mating
- Mating by selection of a crossover point K along the chromosome length l . Two new strings are created by swapping all characters in positions $K + 1$ to l (Figure 8.5).

Besides simple (random) crossover, at the other end of scale, completely continued crossover may be used. Let us consider two individuals of the t^{th} generation $A(t)$ and $B(t)$, whose genes are real variables $a_1, \dots, a_n$ and $b_1, \dots, b_n$,

$$A(t) = [a_1, a_2, \dots, a_n] \quad (8.22)$$

$$B(t) = [b_1, b_2, \dots, b_n] \quad (8.23)$$

The two new offspring $A(t + 1)$, $B(t + 1)$ may be produced by linear combination of parents $A(t)$ and $B(t)$.

$$A(t + 1) = [\rho_1 a_1 + (1 - \rho_1) b_1, \dots, \rho_n a_n + (1 - \rho_n) b_n] \quad (8.24)$$

$$B(t + 1) = [\rho_1 b_1 + (1 - \rho_1) a_1, \dots, \rho_n b_n + (1 - \rho_n) a_n] \quad (8.25)$$

where $\rho_1, \dots, \rho_n \in [0, 1]$ are random values (uniform probability distribution).

8.6.3 MUTATION

Mutation is required because reproduction and crossover tend to become overzealous and thus lose some potentially good “genetic material”.

In simple GAs, mutation is a low probability (occasional) random alteration of one or more genes in one or more offspring chromosomes. In binary coding GAs, a zero is changed to a 1. The mutation plays a secondary role, and its frequency is really low.

Table 8.2 gives a summary of simple GAs.

In real coded GAs, the mutation changes the parameters of selected individuals by a random change in predefined zones. If $A(t)$ is an individual of t^{th} generation, subject to mutation, each gene will undergo important changes in first generations. Gradually, the rate of change diminishes. For the t^{th} generation, let us define two numbers (p) and (r) which are randomly used with equal probabilities:

TABLE 8.2
GA Stages

Stage	Chromosomes	Fitness Value
Initial population: size 3, with 8 genes and its fitness values	P ₁ : 11010110 P ₂ : 10010111 P ₃ : 01001001	F(P ₁) = 6% F(P ₂) = 60% F(P ₃) = 30%
Reproduction: based on fitness value a number of chromosomes survive. There will be two P ₂ and one P ₃	P ₂ : 10010111 P ₂ : 10010111 ←1→ P ₃ : 01001001	No need to recalculate fitness as P ₂ and P ₃ will mate
Crossover: some portion of P ₂ and P ₃ will be swapped	P ₂ : 10010111 P ₂ : 10010001 P ₃ : 01001111	No need of it here
Mutation: some binary genes of some chromosomes are inverted	P ₂ : 10010111 P ₂ : 10011001 P ₃ : 01001111	A new generation of individuals has been formed; a new cycle starts

$p = +1$ – Positive alteration

$p = -1$ – Negative alteration (8.26)

$r \in [0, 1]$ – Uniform distribution

The factor r selected for uniform distribution determines the amplitude of change. The mutated (K^{th}) parameter (gene) is given by [17]

$$a'_k = a_k + (a_{kmax} - a_k) \left(1 - r \left(1 - \frac{t}{T} \right)^5 \right); \text{ for } p = +1$$

$$a'_k = a_k - (a_k - a_{kmin}) \left(1 - r \left(1 - \frac{t}{T} \right)^5 \right); \text{ for } p = -1$$
(8.27)

where a_{kmax} and a_{kmin} are the maximum and minimum feasible values of a_k parameter (gene).

T is the generation index when mutation is cancelled. Figure 8.6 shows the mutation relative amplitude for various generations (t/T varies) as a function of the random number r , based on (8.27).

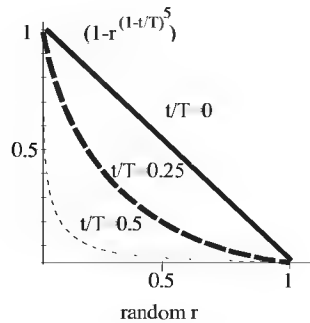


FIGURE 8.6 Mutation amplitude for various generations (t/T).

8.6.4 GA PERFORMANCE INDICES

As expected, GAs are supposed to produce a global optimum with a limited number (%) of all feasible chromosomes (variable sets) searched. It is not easy to assess the minimum number of generations (T) or the population size (m) required to secure a sound optimal selection. The problem is still open, but a few attempts to solve it have been made.

A schema, as defined by Holland, is a similarity template describing a subset of strings with similarities at certain string positions.

If to a binary alphabet {0,1} we add a new symbol * which may be indifferent, a ternary alphabet is created {1,0,*}.

A subset of 4 members of a schema like *111* is shown in (8.28):

$$\{11111,11110,01111,01110\} \quad (8.28)$$

The length of the string $n_1 = 5$, and thus, the number of possible schemata is 3^5 . In general, if the ordinality of the alphabet is K (K = 2 in our case), there are $(K + 1)^n$ schemata.

The exact number of unique schemata in a given population is not countable because we do not know all the strings in this particular population. But a bound for this is feasible.

A particular binary string contains 2^{n_1} schemata as each position may take its value; thus, a population of size m contains between 2^{n_1} and $m \cdot 2^{n_1}$ schemata. How many of them are usefully processed in a GA? To answer this question, let us introduce two concepts:

- The order o of a schema H ($\delta(H)$): the number of fixed positions in the template. For 011**1**, the order o(H) is 4.
- The length of a schema H, ($\delta(H)$): the distance between the first and the last specified string position. In 011**1**, $\delta(H) = 6 - 1 = 5$.

The key issue now is to estimate the effect of reproduction, crossover, and mutation on the number of schemata processed in a simple GA.

It has been shown [9] that short, low-order, above-average fitness schemata receive exponentially increasing chances of survival in subsequent generations. This is the fundamental theorem of GAs.

Despite the disruption of long, high-order schemata by crossover and mutation, the GA implicitly process a large number of schemata ($\%n_1^3$), whereas in fact explicitly they process a relatively small number of strings (m in each population). So, despite the processing of n_1 structures, each generation GAs process approximately n_1^3 schemata in parallel with no memory of bookkeeping. This property is called *implicit parallelism* of computation power in GAs.

To be efficient, GAs must find solutions by exploring only a few combinations in the variable search space. Those solutions must be found with a better probability of success than with random search algorithms. The traditional performance criterion of GA is the evolution of average fitness of the population through subsequent generations.

It is almost evident that such a criterion is not complete as it may concentrate the point around a local, rather than global optimum.

A more complete performance criterion could be the ratio between the number of success runs per total given number of runs (1000, for example, [18], $P_{ag}(A)$):

$$P_{ag}(A) = \frac{\text{Number of success runs}}{1000} \quad (8.29)$$

For a pure random search algorithm, it may be proved that the theoretical probability to find p optima in a searching space of M possible combinations, after making A% calls of the objective functions, is [18]

$$P_{\text{rand}}(A) = \sum_{i=1}^K \frac{P}{M} \left(1 - \frac{P}{M}\right)^{i-1}; \quad A = \frac{K}{M} \cdot 1000 \quad (8.30)$$

The initial population size M in GA is generally $M > 50$.

We should note that when the searching space is discretized, the number of optima may be different from the case of continuous searching space.

In general, the crossover probability $P_c = 0.7$ and the mutation probability $P_m \leq 0.005$ in practical GAs. For a good GA algorithm, searching less than $A = 10\%$ of the total searching space should produce a probability of success $P_{\text{ag}}(A) > P_{\text{rand}}(A)$ with $P_{\text{ag}}(A) > 0.8$ for $A = 10\%$ and $P_{\text{ag}}(A) > 0.98$ for $A = 20\%$.

Special selection mechanisms, different from the biased roulette wheel selection, are required. Stochastic remainder techniques [19] assign offsprings to strings based on the integer part of the expected number of offspring. Even such solutions are not capable to produce performance as stated above.

However if the elitist strategy is used to make sure that the best strings (chromosomes) survive intact in the next generation, together with selection by stochastic remainder technique and real coding, the probability of success may be drastically increased [18,20].

Surrogate FEM model-based optimal IM design codes are still due to appear. However, most of optimal design code of electric machines are proprietary, and even if not, they require extended space for a usable presentation. Consequently, we suggest you Ref. [21], Chapter 15, and its companion computer codes (one by Hooke–Jeeves and the other by G.A.) for two IM optimal design source codes, presented with case studies.

Recently, the GAs and some deterministic optimization methods have been compared in IM design [22–24]. Mixed conclusions have resulted.

8.7 SUMMARY

- GAs offer some advantages over deterministic methods.
- GAs reduce the risk of being trapped in local optima.
- GAs do not need a good starting point as they start with a population of possible solutions. However, a good initial population reduces the computation time.
- GAs are more time-consuming although using real coding, elitist strategy, and stochastic remainder selection techniques increases the probability of success to more than 90% with only 10% of searching space investigated.
- GAs do not need to calculate the gradient of the fitness function; the constraints may be introduced in an augmented fitness function as penalty functions as done with deterministic methods. Many deterministic methods do not require computation of gradients.
- GAs show in general lower precision and slower convergence than deterministic methods.
- Other refinements such as scaling the fitness function might help in the selection process of GAs and thus reduce the computation time for convergence.
- It seems that hybrid approaches, which start with GAs (with real coding), to avoid trapping in local optima, and continue with deterministic (even gradient) methods for fast and accurate convergence, might become the way of the future.
- If a good initial design is available, optimization methods produce only incremental improvements; however, for new designs (high-speed IMs, for example), they may prove indispensable.
- Once optimization design is performed based on nonlinear analytical IM models, with frequency effects approximately considered, the final solution (design) performance may be calculated more precisely by FEM as explained in previous chapters.
- For IM with involved rotor slot configurations – deep bars, closed slots, and double-cage – it is also possible to leave the stator unchanged after optimization and change only the

main rotor slot dimensions (variables) and explore by FEM their effect on performance and constraints (starting torque and starting current) until a practical optimum is reached.

- Any optimization approach may be suited to match the FEM. In [14], a successful attempt with Fuzzy Logic optimizer and FEM assistance is presented for a double-cage IM. Only a few tens of 2DFEM runs are used, and thus, the computation time remains practical.
- Notable advances in matching FEM with optimization IM design methods are still expected as the processing power of PCs is increasing continuously. The FEM-based optimal design codes developed recently for permanent magnet synchronous motors (PMSMs) [25] may be a way to go for IMs too.
- A cost pattern value method for local search algorithm has been applied recently to optimal FEA-based design of IMs for high torque density [22,26,27] with a case study at 260 kW.

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9 Single-Phase IM Design

9.1 INTRODUCTION

Designing involves “dimensioning”, that is the finding of a suitable geometry and manufacturing data and performance indexes for given specifications. Designing, then, involves first dimensioning (or synthesis), sizing, and then assessing the performance (analysis). Finally, if the specifications are not met, the process is repeated according to an adopted strategy until satisfactory performance is obtained.

On top of this, optimization is performed according to one (or more) objective functions, as detailed in Chapter 8, in relation to three-phase induction machines (IMs). Typical specifications (with a case study) are

- Rated power $P_n = 186.5 \text{ W}$ (1/4 HP)
- Rated voltage $V_{sn} = 115 \text{ V}$
- Rated frequency $f_{ln} = 60 \text{ Hz}$; 4 poles
- Rated power factor $\cos \varphi_n = 0.98$ lagging; service: continuous or short duty
- Breakdown p.u. torque 1.3–2.5
- Starting p.u. torque 0.5–3.5
- Starting p.u. current 5–6.5
- Capacitor p.u. maximum voltage: 0.6–1.6.

The breakdown torque p.u. may go as high as 4.0 for the dual capacitor configuration and special-service motors.

Also, starting torques above 1.5 p.u. are obtained only with a starting capacitor.

The split-phase IM is also capable of high starting and breakdown torques in p.u. as during starting, both windings are active, at the expense of rather high resistance, both in the rotor and in the auxiliary stator windings.

For two (three)-speed operation, the 2 (3) speed levels in % of ideal synchronous speed have to be specified. They are to be obtained with tapped windings. In such a case, it has to be verified that for each speed, there is some torque reserve up to the breaking torque of that tapping.

Also, for multispeed motors, the locked rotor torque on low-speed mode has to be less than the load torque at the desired low speed. For a fan load, at 50% as the low speed, the torque is 25% and thus the locked rotor torque has to be <25%.

We start with the sizing of the magnetic circuit, move on to the selection of stator windings, and continue with rotor slotting and cage sizing. The starting and (or) permanent capacitors are defined. Further on, the parameter expressions are given, and steady-state performance is calculated. When optimization design is performed, objective (penalty) functions are calculated and constraints are verified. If their demands are not met, the whole process is repeated, according to a deterministic or stochastic optimization mathematical method, until sufficient convergence is reached.

9.2 SIZING THE STATOR MAGNETIC CIRCUIT

As already discussed in Chapter 4, when dealing with design principles of three-phase IMs, there are basically two design initiation constants, based on past experience:

- The machine utilization factor $C_u: D_o^2 L$
- The rotor tangential stress f_t in N/cm^2 or N/m^2 .

D_o is here the outer stator diameter and L the stator stack length.

As design optimization methods advance and better materials are produced. C_u and f_t tend to increase slowly. Also, low service duty allows for larger C_u and f_t .

However, in general, better efficiency requires larger C_u and lower f_t .

Figure 9.1 [1] presents standard data on C_u in cubic inches (1 inch = 25.4 mm):

$$C_u = D_o^2 L \quad (9.1)$$

for the three-phase small-power IMs.

Concerning the rated tangential stress, there is not yet a history of its use, but it is known that it increases with the stator interior (bore) diameter D_i , with values of around $f_t = 0.20 \text{ N/cm}^2$ for $D_i = 30 \text{ mm}$ to $f_t = 1 \text{ N/cm}^2$ for $D_i = 70 \text{ mm}$ or so, and more.

In general, any motor company could calculate f_t (D_i) for the two, four, six, eight single-phase IMs fabricated so far and then produce its own f_t database.

The rather small range of f_t variation may be exploited best, in our era of computers, for optimization design.

The ratio between stator interior (bore) diameter D_i and the external diameter D_o depends on the number of poles, on D_o , and on magnetic (flux densities) and electric (current density) loadings.

Figure 9.2 presents standard data [1] from three sources: T. C. Lloyd, P. M. Trickey, and Ref. [2]. In Ref. [2], the ratio D_i/D_o is obtained for maximum airgap flux density in the airgap per given stator magnetization mmf in three-phase IMs ($D_i/D_o = 0.58$ for $2p_1 = 2$, 0.65 for $2p_1 = 4$, 0.69 for $2p_1 = 6$, 0.72 for $2p_1 = 8$).

The D_i/D_o values of Ref. [2] are slightly larger than those of P. M. Trickey, as they are obtained from a contemporary optimization design method for three-phase IMs.

In our case study, from Figure 9.1, for 186.5 W (1/4 HP), $2p_1 = 4$ poles, we choose

$$C_u = D_o^2 L = 3.5615 \cdot 10^{-3} \text{ m}^3 \quad (9.2)$$

with $L/D_o = 0.380$, $D_o = 0.137 \text{ m}$, $L = 0.053 \text{ m}$.

The outer stator punching diameter D_o might not be free to choose, as the frames for single-phase IMs come into standardized sizes [3].

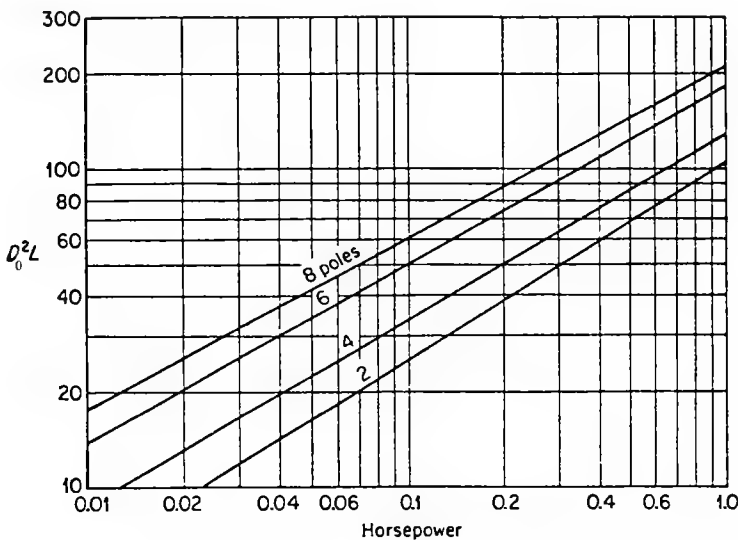


FIGURE 9.1 Machine utilization factor $C_u = D_o^2 L$ (cubic inches) for fractional/horsepower three-phase IMs.

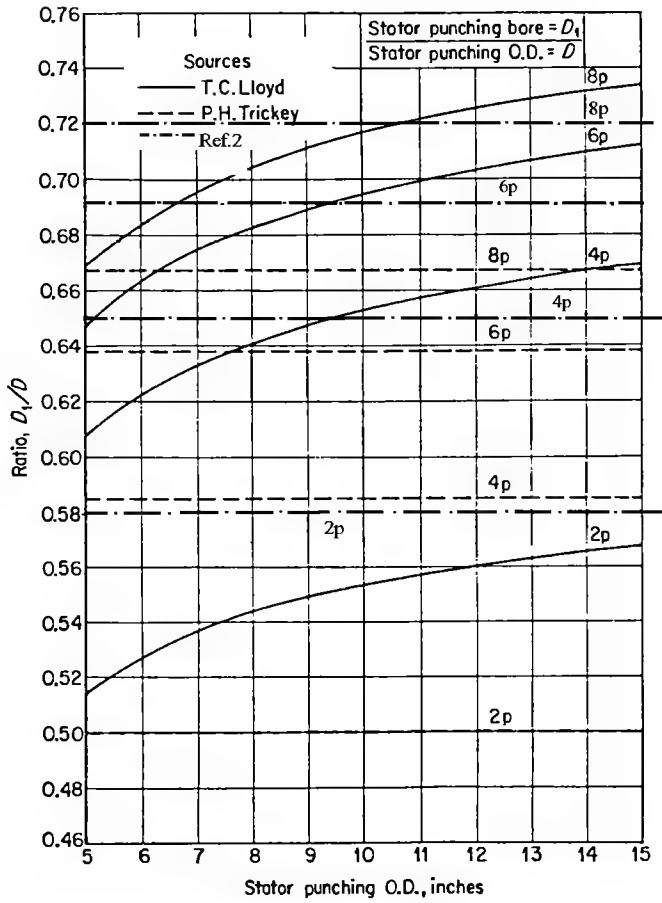


FIGURE 9.2 Interior/outer diameter ratio D_i/D_o versus D_o .

For four poles, we choose from Figure 9.2 a kind of average value of the three sets of data: $D_i/D_o = 0.60$. Consequently, the stator bore diameter $D_i = 0.6 \cdot 0.137 = 82.8 \cdot 10^{-3} \text{ m}$.

The airgap $g = 0.3 \text{ mm}$ and thus the rotor external diameter $D_{or} = D_i - 2g = (82.8 - 2 \cdot 0.3)10^{-3} = 82.2 \cdot 10^{-3} \text{ m}$.

The number of slots of stator N_s is chosen as for three-phase IMs (the rules for most adequate combinations N_s and N_r established for three-phase IMs hold in general also for single-phase IMs see Chapter 10, Vol. 1). Let us consider $N_s = 36$ and $N_r = 30$.

The theoretical peak airgap flux density $B_g = 0.6\text{--}0.75 \text{ T}$. Let us consider $B_g = 0.705 \text{ T}$. Due to saturation, it will be somewhat lower (flattened). Consequently, the flux per pole in the main winding Φ_m is

$$\Phi_m \approx \frac{2}{\pi} \cdot K_{dis} \cdot B_g \cdot \tau \cdot L \quad (9.3)$$

The pole pitch

$$\tau = \pi \cdot \frac{D_i}{2p_1} = \frac{\pi}{4} \cdot 82.2 \cdot 10^{-3} \approx 64.57 \cdot 10^{-3} \text{ m} \quad (9.4a)$$

With $K_{dis} = 0.9$

$$\Phi_m = \frac{2}{\pi} 0.9 \cdot 0.705 \cdot 64.57 \cdot 10^{-3} \cdot 0.053 \approx 1.382 \cdot 10^{-3} \text{ Wb}$$

Once we choose the design stator back iron flux density $B_{cs} = 1.3\text{--}1.7 \text{ T}$, the back iron height h_{cs} can be calculated as

$$h_{cs} = \frac{\Phi_m}{2 \cdot B_{cs} \cdot L} = \frac{1.382 \cdot 10^{-3}}{2 \cdot 1.5 \cdot 0.053} = 8.69 \cdot 10^{-3} \approx 9 \cdot 10^{-3} \text{ m} \quad (9.4b)$$

The stator slot geometry is shown in Figure 9.3.

The number of slots per pole is $N_s/2p_1 = 36/(2 \cdot 2) = 9$. So the tooth width b_{ts} is

$$b_{ts} = \frac{\Phi_m \cdot 2p_1}{N_s \cdot B_{ts} \cdot L} \quad (9.5)$$

With the tooth flux density $B_{ts} \cong (0.8\text{--}1.0) B_{cs}$

$$b_{ts} = \frac{1.382 \cdot 10^{-3} \cdot 4}{36 \cdot 1.3 \cdot 0.053} \approx 2.55 \cdot 10^{-3} \text{ m} \quad (9.6)$$

This value is close to the lowest limit in terms of punching capabilities.

Let us consider $h_{os} = 1 \cdot 10^{-3} \text{ m}$, $w_{os} = 6g = 6 \cdot 0.3 \cdot 10^{-3} = 1.8 \cdot 10^{-3} \text{ m}$.

Now the lower and upper slot width w_{1s} and w_{2s} are

$$w_{1s} = \frac{\pi(D_i + 2(h_{os} + h_{ws}))}{N_s} - b_{ts} = \left(\frac{\pi(82.8 + 2(1 + 1))}{36} - 2.55 \right) 10^{-3} \approx 5.00 \cdot 10^{-3} \text{ m} \quad (9.7)$$

$$w_{2s} = \frac{\pi(D_o - 2h_{cs})}{N_s} - b_{ts} = \left(\frac{\pi(137 - 2 \times 9)}{36} - 2.5 \right) 10^{-3} \approx 8.00 \cdot 10^{-3} \text{ m} \quad (9.8)$$

The useful slot height

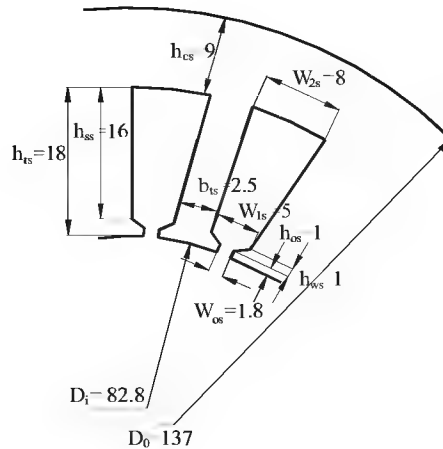


FIGURE 9.3 Stator slot geometry in millimetre.

$$h_{ls} = \frac{D_o}{2} - h_{cs} - h_{ws} - h_{os} - \frac{D_i}{2} = \left(\frac{137}{2} - 9 - 2 - \frac{82.8}{2} \right) 10^{-3} \approx 16.0 \cdot 10^{-3} \text{ m} \quad (9.9a)$$

So the “active” stator slot area A_s is

$$A_s = \frac{(W_{ls} + W_{2s})}{2} h_{ss} = \frac{(5.00 + 8.00)}{2} 16 \cdot 10^{-6} = 104 \cdot 10^{-6} \text{ m}^2 \quad (9.9b)$$

For slots which host both windings or in split-phase IMs, some slots may be larger than others.

9.3 SIZING THE ROTOR MAGNETIC CIRCUIT

The rotor slots for single-phase IMs are either round or trapezoidal or in between (Figure 9.4).

With 30 rotor slots, the rotor slot pitch τ_{sr} is

$$\tau_{sr} = \frac{\pi \cdot D_r}{N_r} = \frac{\pi \cdot 82.2 \cdot 10^{-3} \cdot 4}{30} = 8.6 \cdot 10^{-3} \text{ m} \quad (9.10)$$

With a rotor tooth width $b_{tr} = 2.6 \cdot 10^{-3} \text{ m}$, the tooth flux density B_{tr} is

$$B_{tr} = \frac{\Phi \cdot 2p_l}{N_r \cdot b_{tr} \cdot L} = \frac{1.388 \cdot 10^{-3} \cdot 4}{30 \cdot 2.6 \cdot 10^{-3} \cdot 0.053} = 1.343 \text{ T} \quad (9.11)$$

The rotor slots useful area is in many cases (35%–60%) of that of the stator slot

$$A_r = 0.38 \frac{A_s \cdot N_s}{N_r} = 0.38 \cdot 104 \cdot 10^{-6} \cdot \frac{36}{30} = 47.42 \cdot 10^{-6} \text{ m}^2 \quad (9.12)$$

The trapezoidal rotor slot (Figure 9.4). with

$$h_{or} = W_{or} = 1 \cdot 10^{-3} \text{ m} \quad (9.13)$$

$$\text{has } W_{tr} \approx \tau_{sr} - b_{tr} = (8.6 - 2.6) 10^{-3} = 6 \cdot 10^{-3} \text{ m}$$

Adopting a slot height $h_2 = 10 \cdot 10^{-3} \text{ m}$, we can calculate the rotor slot bottom width W_{2r} :

$$W_{2r} = \frac{2A_r}{h_2} - W_{ls} = \frac{2 \cdot 47.40 \cdot 10^{-3}}{10 \cdot 10^{-3}} - 6 \cdot 10^{-3} = 3.48 \cdot 10^{-3} \text{ m} \quad (9.14)$$

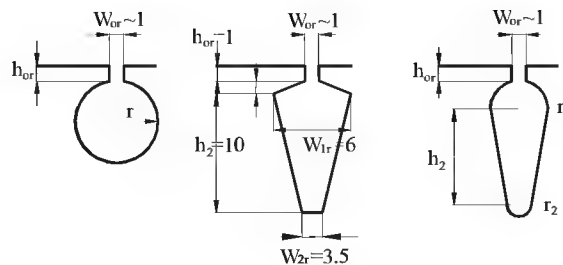


FIGURE 9.4 Typical rotor slot geometries.

A “geometrical” verification is now required.

$$\begin{aligned}
 W_{2r} &= \frac{\pi [D_r - (h_{or} + h_{or1} + h_2) \cdot 2]}{N_r} - b_{tr} \\
 &= \frac{\pi [82.2 - 2(1 + 1 + 10)]10^{-3}}{30} - 2.6 \cdot 10^{-3} = 3.49 \cdot 10^{-3} \text{ m}
 \end{aligned} \quad (9.15)$$

As (9.14) and (9.15) produce about the same value of slot bottom width, the slot height h_2 has been chosen correctly. Otherwise, h_2 should have been changed until the two values of W_{2r} converged.

To avoid such an iterative computation, (9.14) and (9.15) could be combined into a second-order equation with h_2 as the unknown after the elimination of W_{2r} .

A round slot with a diameter $d_r = W_{2r} = 6 \cdot 10^{-3} \text{ m}$ would have produced an area:

$$A_r = \frac{\pi}{4} (6 \cdot 10^{-3})^2 = 28.26 \cdot 10^{-6} \text{ m}^2.$$

This would have been a very small value unless copper bars are used instead of aluminium bars. The end-ring area A_{ring} is

$$A_{ring} \approx A_r \frac{1}{2 \sin \frac{\pi \cdot p_l}{N_r}} = 47.42 \cdot 10^{-6} \frac{1}{2 \sin \frac{\pi \cdot 2}{30}} = 114 \cdot 10^{-6} \text{ m}^2 \quad (9.16)$$

With a radial height $b_r = 15 \cdot 10^{-3} \text{ m}$, the ring axial length is $a_r = \frac{A_{ring}}{b_r} = 7.6 \cdot 10^{-3} \text{ m}$.

9.4 SIZING THE STATOR WINDINGS

By now, the number of slots in the stator is known: $N_s = 36$. As we do have a permanent capacitor motor, the auxiliary winding is always operational. It thus seems naturally to allocate both windings about the same number of slots in fact 20/16. In general, single-layer windings with concentrated coils are used. (Figure 9.5)

With 5(4) coils (slots) per pole and phase, the winding factors for the two phases (Chapter 4, Vol. 1) are

$$\begin{aligned}
 K_{wm} &= \frac{\sin \left(q_m \frac{\alpha_s}{2} \right)}{q_m \sin \frac{\alpha_s}{2}} = \frac{\sin \left(5 \cdot \frac{\pi}{36} \right)}{5 \sin \frac{\pi}{36}} = 0.9698 \\
 K_{wa} &= \frac{\sin \left(q_a \frac{\alpha_s}{2} \right)}{q_a \sin \frac{\alpha_s}{2}} = \frac{\sin \left(4 \cdot \frac{\pi}{36} \right)}{4 \sin \frac{\pi}{36}} = 0.9810
 \end{aligned} \quad (9.17)$$

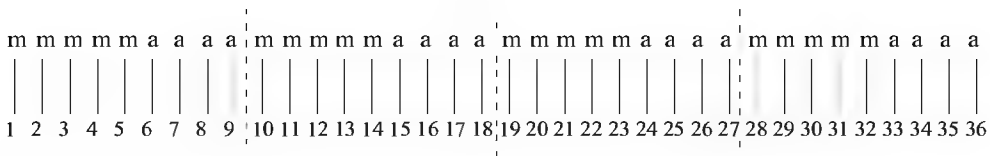


FIGURE 9.5 Stator winding: m – main phase, a – auxiliary phase, four poles.

In case the number of turns/phase in various slots is not the same, the winding factor is calculated in (9.20).

As detailed in Chapter 4, Vol. 1, two types of sinusoidal windings can be built (Figure 9.6).

When the total number of slots per pole per phase is an odd number, concentrated coils with slot axis symmetry seem adequate. In contrast, for even number of slots/pole/phase, concentrated coils with tooth axis symmetry are recommended.

Should we have used such windings for our case with $q_m = 5$ and $q_a = 4$, slot axis symmetry would have applied to the main winding and tooth axis symmetry to the auxiliary winding. A typical coil group is shown in Figure 9.7.

From Figure 9.7, the angle between the axis a-a (Figures 9.6 and 9.7) and the k th slot, β_{kv} (for the v th harmonic), is

$$\beta_{kv} = \gamma \frac{\pi}{N_s} v + (k-1) \frac{2\pi v}{N_s} \quad (9.18)$$

The effective number of conductors per half a pole Z'_v is [4]

$$Z'_v = \sum_{k=1}^n Z_k \cos \beta_{kv} \quad (9.19)$$

$$N_v = 4p_1 \cdot Z'_v$$

N_v is the effective number of conductors per phase.

The winding factor K_{wv} is simply

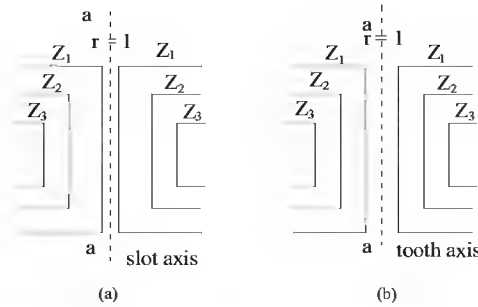


FIGURE 9.6 Sinusoidal windings: (a) with slot axis symmetry and (b) with tooth axis symmetry.

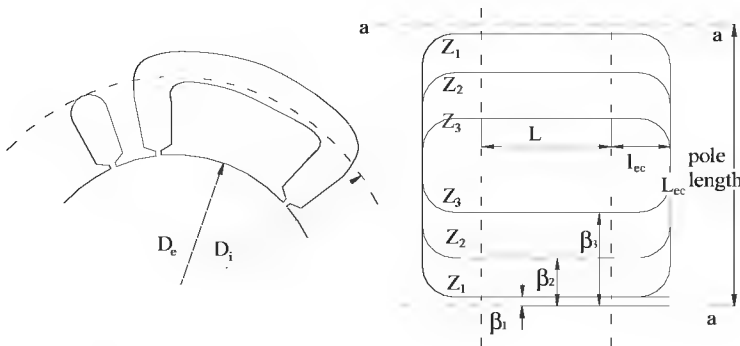


FIGURE 9.7 Typical “overlapping” winding.

$$K_{vv} = \frac{Z'_v}{\sum_{k=1}^n Z_k} = \frac{\sum_{k=1}^n Z_k \cos \beta_{kv}}{\sum_{k=1}^n Z_k} \quad (9.20)$$

The length of the conductors per half a pole group of coil, l_{cn} , is [3]

$$l_{cn} \approx \sum_{k=1}^n Z_k \left[L + (\pi - 2)L_{ec} + D_e \left(\frac{\pi}{2p_1} - \beta_{kl} \right) \right] \quad (9.21)$$

So the resistance per phase R_{phase} is

$$R_{\text{phase}} = \rho_{co} \frac{4p_1 l_{cn}}{A_{con}} \quad (9.22)$$

A_{con} is the conductor (magnet wire) cross-sectional area. A_{con} depends on the total number of conductors per phase, slot area, and design current density.

The problem is that although the supply current at rated load may be calculated as

$$I_s = \frac{P_n}{\eta_n V_s \cos \varphi} = \frac{186.5}{0.7 \cdot 115 \cdot 0.98} = 2.36 \text{ A}, \quad (9.23)$$

with an assigned value for the rated efficiency, η_n , still the rated current for the main and auxiliary currents I_m and I_a is not known at this stage.

I_m and I_a should be almost 90° phase-shifted for rated load and

$$\underline{I}_s = \underline{I}_m + \underline{I}_a \quad (9.24)$$

Now as the ratio $a = \frac{I_m}{I_a} = \frac{N_a}{N_m}$, for symmetry, we may assume that

$$I_m \approx \frac{I_s}{\sqrt{1 + \left(\frac{1}{a}\right)^2}} \quad (9.25)$$

With the turn ratio in the interval $a = 1.0\text{--}2$, in general $I_m = I_s \cdot (0.700\text{--}0.90)$.

The number of turns of the main winding N_m is to be determined by observing that the emf in the main winding $E_m = (0.96\text{--}0.98) \cdot V_s$ and thus with (9.3)

$$E_m = \pi \sqrt{2} \Phi_m N_m k_{wm} f_{ln} \quad (9.26)$$

Finally

$$N_m = \frac{0.97 \cdot 115}{\pi \sqrt{2} \cdot 1.382 \cdot 10^{-3} \cdot 0.9698 \cdot 60} = 313 \text{ turns}$$

In our case, the main winding has 10 ($p_1 q_m = 2 \cdot 5 = 10$) identical coils. So the number of turns per slot n_{sm} is

$$n_{sm} = \frac{N_m}{q_m p_l} = \frac{313}{5 \cdot 2} = 31 \text{ turns/coil} \quad (9.27)$$

Assuming the turn ratio is $a = 1.5$, the number of turns per coil in the auxiliary winding is

$$n_{sa} = \frac{N_m \cdot a \cdot k_{wm}}{q_a \cdot p_l \cdot k_{wa}} = \frac{310 \cdot 1.5 \cdot 0.9698}{4 \cdot 2 \cdot 0.9810} \approx 57 \text{ turns/coil} \quad (9.28)$$

As the slots are identical and their useful area is (from 9.9b) $A_s = 104 \cdot 10^{-6} \text{ m}^2$, the diameters of the magnetic wire used in two windings are

$$\begin{aligned} d_m &= \sqrt{\frac{4}{\pi} \cdot \frac{A_s K_{fill}}{n_{sm}}} = 10^{-3} \sqrt{\frac{4}{3.14} \cdot \frac{104 \cdot 0.4}{31}} \approx 1.3 \cdot 10^{-3} \text{ m} \\ d_a &= \sqrt{\frac{4}{\pi} \cdot \frac{A_s K_{fill}}{n_{sa}}} = 10^{-3} \sqrt{\frac{4}{3.14} \cdot \frac{104 \cdot 0.4}{57}} \approx 1.0 \cdot 10^{-3} \text{ m} \end{aligned} \quad (9.29)$$

The filling factor was considered rather large: $K_{fill} = 0.4$.

The predicted current density in the two windings would be

$$\begin{aligned} j_{com} &= \frac{I_m}{\pi d_m^2 / 4} = \frac{2.36}{\left(\sqrt{1 + \left(\frac{1}{1.5} \right)^2} \right) \cdot \pi \cdot \frac{1.3^2}{4} \cdot 10^{-6}} = 1.4847 \cdot 10^6 \text{ A/m}^2 \\ j_{coa} &= \frac{I_m}{a \cdot \pi \cdot d_a^2 / 4} = \frac{1.9696}{\frac{1.5 \cdot \pi \cdot 1^2 \cdot 10^{-6}}{4}} = 1.6726 \cdot 10^6 \text{ A/m}^2 \end{aligned} \quad (9.30)$$

The forecasted current densities, for rated power, are rather small so good efficiency (for the low power considered here) is expected.

An initial value for the permanent capacitor has to be chosen.

Let us assume complete symmetry for rated load. with $I_{an} = \frac{I_{mn}}{a} = \frac{1.96}{1.5} = 1.3 \text{ A}$.

Further on

$$V_{an} = V_{mn} \cdot a = 115 \cdot 1.5 = 172 \text{ V} \quad (9.31)$$

The capacitor voltage V_c is

$$V_c = \sqrt{V_s^2 + V_{an}^2} = 115 \sqrt{1 + 1.5^2} = 207.3 \text{ V} \quad (9.32)$$

Finally, the permanent capacitor C is

$$C = \frac{I_a}{\omega_l V_c} = \frac{1.3}{2 \cdot \pi \cdot 60 \cdot 207.3} \approx 16 \mu\text{F} \quad (9.33)$$

We may choose a higher value of C than $16 \mu\text{F}$ (say $25 \mu\text{F}$) to increase somewhat the breakdown and the starting torque.

9.5 RESISTANCES AND LEAKAGE REACTANCES

The main and auxiliary winding resistances, R_{sm} and R_{sa} , respectively, (9.22) are

$$\begin{aligned} R_{sm} &= \rho_{Co} \cdot 4 \cdot 2 \frac{l_{cnm}}{\pi d_m^2/4} \\ R_{sa} &= \rho_{Co} \cdot 4 \cdot 2 \frac{l_{cna}}{\pi d_a^2/4} \end{aligned} \quad (9.34)$$

The length of the coils/half a pole for main and auxiliary windings l_{cnm} (9.21) is

$$\begin{aligned} l_{cnm} &\approx n_{cm} \left[\left(2 + \frac{1}{2} \right) L + 3(\pi - 2)L_{ec} + D_e \left(\frac{3\pi}{2p_1} - \frac{2\pi}{36} - \frac{4\pi}{36} \right) \right] \\ &= 31 \cdot \left[\left(2 + \frac{1}{2} \right) \cdot 0.053 + 3(\pi - 2) \cdot 0.025 + 0.14 \left(\frac{3\pi}{4} - \frac{2\pi}{36} - \frac{4\pi}{36} \right) \right] = 9.805 \text{ m} \end{aligned} \quad (9.35)$$

$$\begin{aligned} l_{cna} &\approx n_{ca} \left[2L + 3(\pi - 2)L_{ec} + D_e \left(\frac{2\pi}{2p_1} - \frac{2\pi}{36} - \frac{4\pi}{36} \right) \right] \\ &= 57 \cdot \left[2 \cdot 0.053 + 3(\pi - 2) \cdot 0.025 + 0.14 \left(\frac{2\pi}{4} - \frac{\pi}{18} - \frac{\pi}{9} \right) \right] = 11.38 \text{ m} \end{aligned} \quad (9.36)$$

Finally,

$$\begin{aligned} R_{sm} &= \frac{2.1 \cdot 10^{-8} \cdot 4 \cdot 2 \cdot 9.805}{\frac{\pi}{4} \cdot 1.3^2 \cdot 10^{-6}} = 1.2417 \Omega \\ R_{sa} &= \frac{2.1 \cdot 10^{-8} \cdot 4 \cdot 2 \cdot 11.38}{\frac{\pi}{4} \cdot 1^2 \cdot 10^{-6}} = 2.435 \Omega \end{aligned}$$

The main winding leakage reactance X_{sm} is

$$X_{sm} = X_{ss} + X_{se} + X_{sd} + X_{skew} \quad (9.37)$$

where

- X_{ss} – the stator slot leakage reactance
- X_{se} – the stator end-connection leakage reactance
- X_{sd} – the stator differential leakage
- X_{skew} – skewing reactance.

$$X_{se} + X_{ss} = 2\mu_0 \omega_1 \frac{W_1^2 \cdot L}{p \cdot q} (\lambda_{ss} + \lambda_{se}) \quad (9.38)$$

with (Chapter 6, Vol. 1, and Figure 9.3)

$$\lambda_{ss} = \frac{2h_{ss}}{3(W_{1s} + W_{2s})} + \frac{2h_{ws}}{W_{1s} + W_{2s}} + \frac{h_{os}}{W_{os}} = \frac{2 \cdot 16}{3(5+8)} + \frac{2 \cdot 1}{5+8} + \frac{1}{1.8} = 1.53 \quad (9.39)$$

Also (Chapter 6, Vol. 1, Equation (6.28))

$$\lambda_{sc} = \frac{0.67 \cdot q_m}{L} (l_{ec} - 0.64 \cdot \tau) \quad (9.40)$$

It is $q_m/2$, as the q coils/pole are divided into two sections whose end connections go into opposite directions (Figure 9.5).

The end-connection average length l_{ec} is

$$l_{ec}^m = \frac{l_{cn} \cdot 2}{q_m n_{cm}} - L \quad (9.41)$$

With l_{cn} from (9.21)

$$l_{ec}^m = \frac{9.805 \cdot 2}{5 \cdot 31} - 0.053 = 0.0735 \text{ m}$$

$$\lambda_{sc} = \frac{0.67 \cdot 5}{0.053} (0.0735 - 0.64 \cdot 0.065) = 1.008$$

The differential leakage reactance X_{sd} (Chapter 6, Vol. 1, Equations (6.2) and (6.9) and (Figures 6.2–6.5)) is

$$\frac{X_{sd}}{X_{mm}} = \sigma_{ds} \cdot \Delta_d = 2.6 \cdot 10^{-2} \cdot 0.92 = 2.392 \cdot 10^{-2} \quad (9.42)$$

for $q_m = 5$, $N_s = 36$, $N_r = 30$, one slot pitch skewing of rotor.

X_{mm} is the nonsaturated magnetization reactance of the main winding (Chapter 5, Vol. 1, Equation (5.115))

$$X_{mm} = \omega_l \frac{4\mu_0}{\pi^2} (W_m \cdot K_{wm1})^2 \frac{L \cdot \tau}{p_l \cdot g \cdot K_C}$$

$$= 2\pi \cdot 60 \frac{4 \cdot 4\pi \cdot 10^{-7}}{\pi^2} (10 \cdot 31 \cdot 0.9698)^2 \frac{0.053 \cdot 0.065}{2 \cdot 0.3 \cdot 10^{-3} \cdot 1.3} = 72.085 \Omega$$

where K_C is the Carter coefficient and g the airgap.

$$\frac{X_{skew}}{X_{mm}} = (1 - K_{skew}^2) = 1 - \frac{\sin^2 \frac{\alpha_{skew}}{2}}{(\alpha_{skew})^2} = 1 - \frac{\sin^2 \frac{\pi}{18}}{\left(\frac{\pi}{18}\right)^2} = 9.108 \cdot 10^{-3} \quad (9.43)$$

Finally, the main winding leakage reactance X_{sm} is

$$X_{sm} = 2 \cdot 4\pi \cdot 10^{-7} \frac{2\pi \cdot 60 \cdot 310^2 \cdot 0.053 \cdot (1.53 + 1.008)}{2 \cdot 5}$$

$$+ 72.085 \cdot (23.92 + 9.108) \cdot 10^{-3} = 4.253 \Omega$$

The auxiliary winding is placed in identical slots, and thus, the computation process is similar. A simplified expression for X_{sa} in this case is

$$X_{sa} \approx X_{sm} \left(\frac{W_a}{W_m} \right)^2 = 4.253 \cdot \left(\frac{57 \cdot 8}{31 \cdot 10} \right)^2 = 9.204 \, \Omega \quad (9.44)$$

The rotor leakage reactance, after its reduction to the main winding (Chapter 6, Vol. 1 Equations (6.86)–(6.87)), is

$$X_{rm} = X_{rmd} + X'_{bem} = \omega_l (L_{rmd} + L'_{bem}) \quad (9.45)$$

The rotor-skewing component has been “attached” to the stator and the zig-zag component is lumped into differential one X_{rmd} .

From Equation (6.16) (Chapter 6, Vol. 1),

$$X_{rmd} = \sigma_{dr0} X_{rm} = 2.8 \times 10^{-2} X_{rm} \quad (9.46)$$

With $N_r/p_l = 30/2 = 15$ and one slot pitch skewing (from Figure 6.4 in Vol. 1)

$$\sigma_{dr0} = 2.8 \cdot 10^{-2}$$

The equivalent bar–end-ring leakage inductance L'_{ben} is (Equation 6.86 in Vol. 1):

$$L'_{bem} = L_{bem} \frac{12 \cdot K_{wm}^2 W_m^2}{N_r} \quad (9.47)$$

From (6.91)–(6.92), Vol. 1,

$$L_{ben} = \mu_0 l_b \lambda_b + 2\mu_0 \lambda_{ei} \cdot 2\pi \cdot \frac{D_{ir}}{N_r} \quad (9.48)$$

The rotor bar (slot) and end-ring permeance coefficients, λ_b and λ_{ei} , respectively, (from (6.18) and (6.46) in Vol. 1 and Figure 9.5), are

$$\lambda_b = \frac{h_{or}}{W_{or}} + \frac{2h_{or1}}{(W_{o1} + W_{o2})} + \frac{2h_2}{3(W_{1r} + W_{2r})} = \frac{1}{1} + \frac{1 \cdot 2}{1 + 6} + \frac{2 \cdot 10}{3(6 + 3.5)} = 1.987 \quad (9.49)$$

$$\lambda_{ei} = \frac{2.3 \cdot D_{ir}}{4 \cdot N_r \cdot L \cdot \sin^2 \left(\frac{\pi p_l}{N_r} \right)} \log \frac{4.7 \cdot D_{ir}}{a_r + 2 \cdot b_r} \quad (9.50)$$

with $D_{ir} \approx D_r - b = (82.8 - 15) \cdot 10^{-3} = 67.8 \cdot 10^{-3} \text{ m}$ $a_r = 7.6 \cdot 10^{-3} \text{ m}$ and $b_r = 15 \cdot 10^{-3} \text{ m}$ from (9.16)

$$\lambda_{ei} = \frac{2.3 \cdot 67.8 \cdot 10^{-3}}{4 \cdot 30 \cdot 0.053 \cdot \sin^2 \left(\frac{\pi}{15} \right)} \log \left(\frac{4.7 \cdot 67.8}{2 \cdot 15 + 7.6} \right) = 0.579$$

so from (9.48)

$$L_{ben} = 1.256 \cdot 10^{-6} \left(0.053 \cdot 1.987 + 0.579 \frac{2\pi \cdot 67.8 \cdot 10^{-3}}{30} \right) = 0.1425 \cdot 10^{-6} \text{ H}$$

Now from (9.45),

$$\begin{aligned}
 X_{rm} &= 2.8 \cdot 10^{-2} X_{mm} + \omega_l L_{ben} \frac{12 K_{wm}^2 W_m^2}{N_r} \\
 &= 2.8 \cdot 10^{-2} \cdot 72.085 + 2\pi \cdot 60 \cdot 0.1425 \cdot 10^{-6} \frac{12 \cdot 0.9698^2 \cdot 310^2}{30} \\
 &= 3.961 \Omega
 \end{aligned} \tag{9.51}$$

The rotor cage resistance is

$$R_{rm} = R_{be} \frac{12 K_{wm}^2 W_m^2}{N_r} \tag{9.52}$$

With R_{be} (Equation (6.63) in Vol. 1),

$$R_{be} = R_b + \frac{R_{ring}}{2 \cdot \sin^2 \left(\frac{\pi \cdot p_l}{N_r} \right)} \tag{9.53}$$

$$\begin{aligned}
 R_b &= \rho_{Co} \frac{l_b}{A_r} = 3 \cdot 10^{-8} \frac{0.065}{47.42 \cdot 10^{-6}} = 4.112 \cdot 10^{-5} \Omega \\
 R_{ring} &= \rho_b \frac{l_{ring}}{A_{ring}} = 3 \cdot 10^{-8} \frac{2\pi \cdot 67.8 \cdot 10^{-3}}{1.14 \cdot 10^{-4} \cdot 30} = 3.735 \cdot 10^{-6} \Omega
 \end{aligned} \tag{9.54}$$

Finally,

$$R_{rm} = \left(4.112 \cdot 10^{-5} + \frac{3.735 \cdot 10^{-6}}{2 \cdot \sin^2 \frac{\pi \cdot 2}{30}} \right) \cdot 12 \cdot \frac{0.9698^2 \cdot 310^2}{30} = 3.048 \Omega$$

Note that the rotor resistance and leakage reactance calculated above are not affected by the skin effects.

9.6 THE MAGNETIZATION REACTANCE X_{mm}

The magnetization reactance X_{mm} is affected by magnetic saturation which is dependent on the resultant magnetization current I_m .

Due to the symmetry of the magnetization circuit, it is sufficient to calculate the functional X_m (I_m) for the case with current in the main winding ($I_a = 0$, bare rotor slots).

The computation of $X_m(I_m)$ may be performed analytically (see Chapter 6 or Ref. [4]) or by finite element modelling (FEM). For the present case, to avoid lengthy calculations, let us consider X_{mm} constant for a moderate saturation level ($1 + K_s$) = 1.5

$$X_{mm} = (X_{mm})_{unsat} \frac{1}{1 + K_s} = \frac{72.085}{1.5} = 48.065 \Omega \tag{9.55}$$

9.7 THE STARTING TORQUE AND CURRENT

The theory behind the computation of starting torque and current is presented in Chapter 14, Vol. 1, Section 14.5:

$$(I_s)_{s=1} = I_{sc} \sqrt{\left[1 + \frac{t_{es}}{a} \frac{(\omega C |\underline{Z}_{sc}| \cdot a^2 - \sin \varphi_{sc})}{\cos \varphi_{sc}} \right]^2 + \frac{t_{es}^2}{a^2}} \quad (9.56)$$

$$t_{es} = \frac{T_{es}}{\left(\frac{2p_l}{\omega_l} I_{sc}^2 (R_{r+})_{s=1} \right)} = \frac{K_a \cdot \sin \varphi_{sc}}{a (1 + K_a^2 + 2K_a \cos \varphi_{sc})}$$

$$K_a = \frac{1}{\omega \cdot C \cdot a^2 \cdot |Z_{sc}|}; I_{sc} = \frac{V_s}{2 \cdot |\underline{Z}_{sc}|}$$

$$\begin{aligned} Z_{sc} &= \sqrt{(R_{sm} + R_{rm})^2 + (X_{sm} + X_{rm})^2} \\ &= \sqrt{(1.2417 + 3.048)^2 + (4.235 + 3.96)^2} = 9.26 \, \Omega \end{aligned} \quad (9.57)$$

$$\cos \varphi_{sc} = 0.463$$

$$\sin \varphi_{sc} = 0.887$$

$$K_a = \frac{1}{2\pi \cdot 60 \cdot 25 \cdot 10^{-6} \cdot 1.5^2 \cdot 9.26} = 5.095$$

$$I_{sc} = \frac{115}{2 \cdot 9.26} = 6.2095 \, \text{A}; (R_{r+})_{s=1} = R_{rm} = 3.048 \, \Omega$$

$$T_{sc} = \frac{2 \cdot 2}{2\pi \cdot 60} \cdot 6.2095^2 \cdot 3.048 = 2.495 \, \text{Nm}$$

The relative value of starting torque t_{es} is

$$t_{es} = \frac{5.095 \cdot 0.887}{1.5 (1 + 5.095^2 + 2 \cdot 5.095 \cdot 0.463)} = 0.095$$

$$T_{es} = t_{es} T_{sc} = 0.095 \cdot 2.495 = 0.237 \, \text{Nm}$$

The rated torque T_{en} , for an alleged rated slip $S_n = 0.06$, would be

$$T_{en} \approx \frac{P_n p_l}{\omega_l \cdot (1 - S_n)} = \frac{186.5 \cdot 2}{2\pi \cdot 60 \cdot (1 - 0.06)} = 1.053 \, \text{Nm} \quad (9.58)$$

So the starting torque is rather small as the permanent capacitance $C_a = 25 \cdot 10^{-6} \, \text{F}$ is too small.

The starting current (9.56) is

$$(I_s)_{s=1} = 6.2095 \sqrt{\left[1 + \frac{0.095}{1.5} \left(\frac{\frac{1}{5.095} - 0.887}{0.463} \right) \right]^2 + \left(\frac{0.095}{1.5} \right)^2} \approx 5.636 \text{ A}$$

The starting current is not large (the presumed rated source current $I_{sn} = 2.36 \text{ A}$. (Equation 9.23)), but the starting torque is small.

The result is typical for the permanent (single) capacitor IM.

9.8 STEADY-STATE PERFORMANCE AROUND RATED POWER

Although the core losses and the stray load losses have not been calculated, the computation of torque and stator currents for various slips for $S = 0.04$ – 0.20 may be performed as developed in Chapter 14, Vol. 1, Section 14.3.

To shorten the presentation, we will illustrate this point by calculating the currents and torque for $S = 0.06$.

First, the impedances $\underline{Z}_+^m, \underline{Z}_-^m$ and $\underline{Z}_a^m$ ((14.11)–(14.14) in Chapter 14, Vol. 1) are calculated

$$\begin{aligned} \underline{Z}_+ &= R_{sm} + j \cdot X_{sm} + j \frac{X_{mm} \left(jX_{rm} + \frac{R_{rm}}{S} \right)}{\frac{R_{rm}}{S} + j(X_{mm} + X_{rm})} \\ &= 1.2417 + j \cdot 4.253 + \frac{j \cdot 48.056 \left(j \cdot 3.96 + \frac{3.048}{0.06} \right)}{\frac{3.048}{0.06} + j \cdot (48.056 + 3.96)} = 23.43 + j \cdot 29.583 \end{aligned} \quad (9.59)$$

$$\begin{aligned} \underline{Z}_- &= R_{sm} + j \cdot X_{sm} + j \frac{X_{mm} \left(j \cdot X_{rm} + \frac{R_{rm}}{(2-S)} \right)}{\frac{R_{rm}}{(2-S)} + j \cdot (X_{mm} + X_{rm})} \\ &\approx 1.2417 + j \cdot 4.253 + j \cdot \frac{48.056 \cdot \left(j \cdot 3.96 + \frac{3.048}{(2-0.06)} \right)}{\frac{3.048}{(2-0.06)} + j \cdot (48.056 + 3.96)} \approx 2.7 + j \cdot 7.912 \end{aligned} \quad (9.60)$$

$$\begin{aligned} \underline{Z}_a^m &= \frac{1}{2} \left(\frac{R_{sa}}{a^2} - R_{sm} \right) + j \cdot \frac{1}{2} \left(\frac{X_{sa}}{a^2} - X_{sm} \right) - j \cdot \frac{1}{2 \cdot a^2 \cdot \omega \cdot C} \\ &= \frac{1}{2} \left(\frac{2.435}{1.5^2} - 1.2417 \right) + 0 - \frac{j}{2 \cdot 1.5^2 \cdot 2\pi \cdot 60 \cdot 25 \cdot 10^{-6}} \\ &= -0.08 - j \cdot 23.6 \approx -j \cdot 23.6 \end{aligned} \quad (9.61)$$

The current components $\underline{I}_{m+}$ and $\underline{I}_{m-}$ ((14.16) and (14.17) Chapter 14, Vol. 1) are

$$I_{m+} = \frac{V_s}{2} \cdot \frac{\left[\left(1 - \frac{j}{a} \right) \underline{Z}_{-} + 2\underline{Z}_{a}^m \right]}{\underline{Z}_{+} \cdot \underline{Z}_{-} + \underline{Z}_{a}^m (\underline{Z}_{+} + \underline{Z}_{-})} = 1.808 - j \cdot 2.414 \quad (9.62)$$

$$I_{m-} = \frac{V_s}{2} \cdot \frac{\left[\left(1 + \frac{j}{a} \right) \underline{Z}_{+} + 2\underline{Z}_{a}^m \right]}{\underline{Z}_{+} \cdot \underline{Z}_{-} + \underline{Z}_{a}^m (\underline{Z}_{+} + \underline{Z}_{-})} = 0.304 - j \cdot 8.197 \cdot 10^{-3} \quad (9.63)$$

Now, the torque components T_{+} and T_{-} are calculated from (14.18) to (14.19) (Chapter 14, Vol. 1):

$$T_{e+} = \frac{2p_1}{\omega_1} I_{m+}^2 \left[R_e(\underline{Z}_{+}) - R_{sm} \right] \quad (9.64)$$

$$T_{e-} = -\frac{2p_1}{\omega_1} I_{m-}^2 \left[R_e(\underline{Z}_{-}) - R_{sm} \right] \quad (9.65)$$

$$T_e = T_{e+} + T_{e-}$$

The source current I_s is

$$I_s = I_m + I_a = I_{m+} + I_{m-} + j \cdot \frac{(I_{m+} - I_{m-})}{a} = 3.721 - j \cdot 1.422 \quad (9.66)$$

The source power factor becomes

$$\cos \phi_1 = \frac{R_e(I_s)}{I_s} = 0.934 \quad (9.67)$$

$$T_{e+} = \frac{2 \cdot 2}{2\pi \cdot 60} \cdot 9.096 \cdot [23.43 - 1.2417] = 2.1425 \text{ Nm}$$

$$T_{e-} = -\frac{2 \cdot 2}{2\pi \cdot 60} \cdot 0.09248 \cdot [2.7 - 1.2417] = -0.967 \cdot 10^{-3} \text{ Nm}$$

Note that the current is about 60% higher than the presumed rated current (2.36 A), while the torque is twice the rated torque $T_{en} \approx 1 \text{ Nm}$.

It seems that when the slip is reduced gradually, perhaps around 4% ($S = 0.04$), the current goes down and so does the torque, coming close to the rated value, which corresponds to the rated power P_n (9.58).

On the other hand, if the slip is gradually increased, the breakdown torque region is reached.

To complete the design, Core, stray load, and mechanical losses need to be calculated. Although the computation of losses is traditionally performed as for the three-phase IMs, the elliptic travelling field of single-phase IM leads to larger losses [5]. We will not follow this aspect here in further detail.

We may now consider the preliminary electromagnetic design done. Thermal model may then be developed as for the three-phase IM (Chapter 12, Vol. 1).

Design trials may now start to meet all design specifications. The complexity of the nonlinear model of the single-phase IM makes the task of finding easy ways to meet, say, the starting torque and current, breakdown torque and providing for good efficiency, rather difficult.

This is where the design optimization techniques come into play.

However, to cut short the computation time of optimization design, a good preliminary design is useful and so are a few design guidelines based on experience (see Ref. [1]).

9.9 GUIDELINES FOR A GOOD DESIGN

- In general, a good value for the turn ratio a lies in the interval between 1.5 and 2.0, except for reversible motion when $a = 1$ (identical stator windings).
- The starting and breakdown torques may be considered proportional to the number of turns of main winding squared.
- The maximum starting torque increases with the turn ratio a .
- The flux densities in various parts of the magnetic circuit are inversely proportional to the number of turns in the main winding, for given source voltage.
- The breakdown torque is almost inversely proportional to the sum $R_{sm} + R_{rm} + X_{sm} + X_{rm}$.
- When changing gradually the number of turns in the main winding, the rated slip varies with W_m^2 .
- The starting torque may be increased, up to a point, in proportion to rotor resistance R_{rm} increase.
- For a given motor, there is a large capacitor C_{ST} which could provide maximum starting torque and another one C_{SA} to provide, again at start, maximum torque/current. A value between C_{ST} and C_{SA} is recommended for best starting performance. For running conditions, a smaller capacitor C_a is needed. In permanent capacitor IMs, a value C'_a closer to C_a ($C'_a > C_a$) is in general used.
- The torque varies with the square root of stack length. If stack length variation ratio K is accompanied by the number of turns variation by $1/\sqrt{K}$, the torque remains almost unchanged.

9.10 OPTIMIZATION DESIGN ISSUES

Optimization design implies:

- A machine model for analysis (as the one described in previous sections).
- Single or multiple objective functions and constraints.
- A vector of initial independent variables (from a preliminary design) is required in nonevolutionary optimization methods.
- A method of optimization (search of new variable vectors until the best objective function value is obtained).

Typical single objective functions F are

F_1 – maximum efficiency without excessive material cost

F_2 – minimum material (iron, copper, aluminium, capacitor) cost for an efficiency above a threshold value

F_3 – maximum starting torque

F_4 – minimum global costs (materials plus loss capitalized costs for given duty cycle over the entire life of the motor).

A combination of the above objectives functions could also be used in the optimization process.

A typical variable vector X might contain

1. Outer rotor diameter D_r
2. Stator slot depth h_{st}
3. Stator yoke height h_{cs}
4. Stator tooth width b_{ts}
5. Stack length L
6. Airgap length g
7. Airgap flux density B_g
8. Rotor slot depth h_{rt}
9. Rotor tooth width b_{tr}
10. Main winding wire size d_m
11. Auxiliary winding wire size d_a
12. Capacitance C_a
13. Effective turn ratio a .

Variables 10–12 vary in steps while the others are continuous.

Typical constraints are

- Starting torque T_{es}
- Starting current $(I_s)_{s=1}$
- Breakdown torque T_{eb}
- Rated power factor $\cos \varphi_n$
- Stator winding temperature rise ΔT_m
- Rated slip S_n
- Slot fullness k_{fill}
- Capacitor voltage V_C
- Rotor cage maximum temperature T_{rmax} .

Chapter 8 of this book presented in brief quite a few optimization methods (for more information, see Ref. [6]).

According to most of them, the objective function $F(x)$ is augmented with the SUMT [7]:

$$P(X_k, r_k) = F(X_k) + r_k \sum_{j=1}^m \frac{1}{G_j(X_k)} \quad (9.68)$$

where the penalty factor r_k is gradually decreased as the optimization search counter k increases.

There are many search engines which can change the initial variable vector towards a global optimum for $P(X_k, r_k)$ (Chapter 8).

Hooke–Jeeves modified method is a good success example for rather moderate computation time efforts.

Ref. [8] presents such an optimization design attempt for a two-pole, 150 W, 220 V, 50 Hz motor with constraints as shown in Table 9.1.

Efficiency (F_1) and material cost (F_2) evolution during the optimization process is shown in Figure 9.8a [8]. Stack length evolution during F_1 and F_2 optimization process shown in Figure 9.8b shows an increase before decreasing towards the optimum value.

Also, it is interesting to note that criteria F_1 (max. efficiency) and F_2 (minimum material costs) lead to quite different wire sizes in both the main and auxiliary windings (Figure 9.9a and b) [8].

A 3.4% efficiency increase has been obtained in this particular case. This means about 3.4% less energy input for the same mechanical work.

TABLE 9.1
Design Constraints

Constraint	Limit	Standard	F ₁	F ₂	F ₃
Power factor	>0.92	0.96	0.96	0.95	0.94
Main winding temperature (°C)	<90	86.2	82.2	86.0 (+)	84.8
Max. torque (Nm)	>1.27	1.29	1.27 (+)	1.28 (+)	1.49
Start torque (Nm)	0.84	0.878	0.876	0.84 (+)	1.06
Start current (A)	<11.5	10.5	10.9	11.5 (+)	11.5 (+)
Slot fullness $k_{\text{fill}} \frac{4}{\pi} (\text{tod}^2)$	<0.8	0.78	0.8 (+)	0.66	0.8 (+)
Capacitor voltage (V)	<280	263	264	246	261
Efficiency	>0.826	0.826	0.859	0.828 (+)	0.836

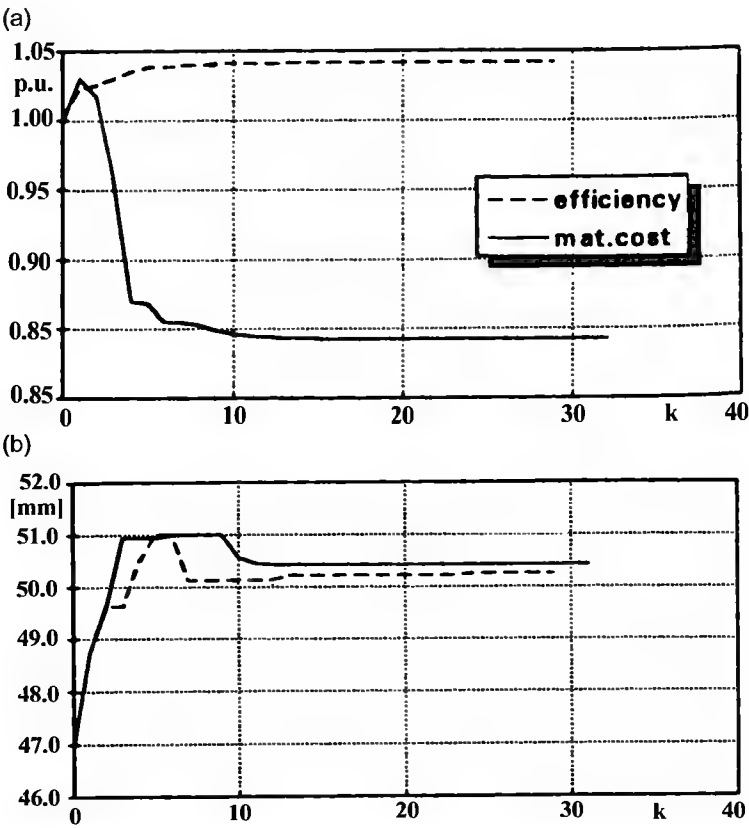


FIGURE 9.8 Evolution of the optimization process (after Ref. [8]): (a) efficiency and material costs: F₁ and F₂ and (b) stack length for F₁ and F₂.

It has to be noted that reducing the core losses by using better core material and thermal treatments (and various methods of stray load loss reduction) could lead to further increases in efficiency.

Although the power/unit of single-phase IMs is not large, their number is.

Consequently, increases in efficiency of 3% or more have a major impact on both the world's energy consumption and the environment (lower temperature motors, less power from the power plants, and thus less pollution).

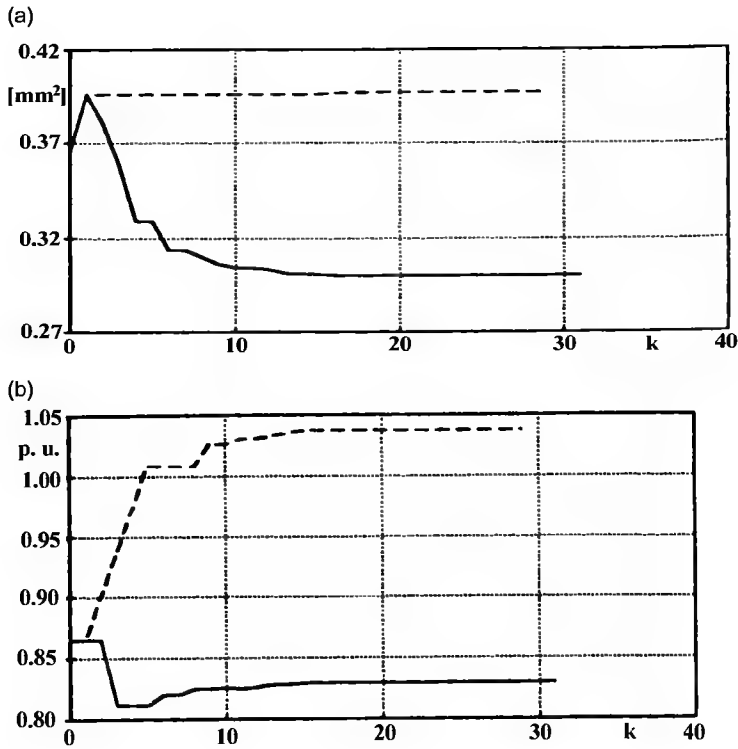


FIGURE 9.9 (a) Main winding wire sizes evolution during the optimization process. (After Ref. [8].) (b) Auxiliary winding wire sizes evolution during the optimization process.

9.11 TWO-SPEED PM SPLIT-PHASE CAPACITOR INDUCTION/SYNCHRONOUS MOTOR

To illustrate the optimal design virtues further hereby a pole-changing (four and two poles) split-phase capacitor IM is rather thoroughly optimally designed and characterized (for steady state and transients) [9].

The machine is destined to start and work most of the time in low-speed (four poles 50(60) Hz) operation mode and a short duty cycle in two-pole operation mode.

As an example, let us consider the following specifications:

For four-pole operation,

$$V_1 = 115 \text{ V}$$

$$f_{1n} = 60 \text{ Hz, efficiency } \geq 0.84$$

$$P_{n4p} = 50 \text{ W}$$

$$P_{n2p} = 100 \text{ W for two-pole operation. efficiency } \geq 0.62.$$

Based on a thorough literature list [10–31] and the mathematical models in previous chapters, the following optimal design methodology was put in place [9].

9.11.1 POLE-CHANGING AND USING PERMANENT MAGNETS

Although the use of 2/1 pole-changing method in split-phase capacitor phase induction motors is, in principle, a good old idea for most cases where speed variation is necessary, its use is not always economically viable for small fractional horsepower single-phase motors used in compressors for

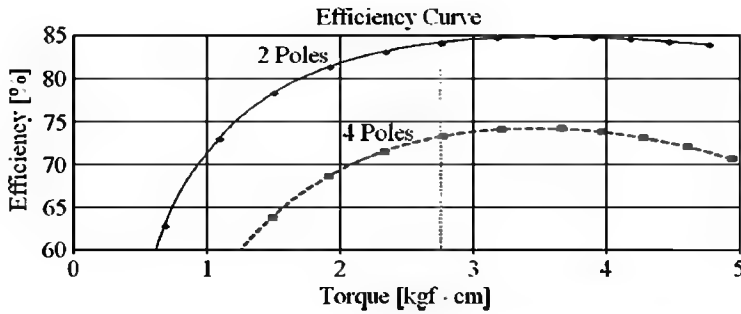


FIGURE 9.10 Comparison graph of existing 100 W IM, two- and four-pole capacitor run induction motors.

household refrigeration. The main problem is the lower efficiency at lower speed, exactly at the condition where high efficiency is required for usual refrigerator drives. One of the reasons for that is the magnetization current, which is significantly higher at a higher number of poles.

Figure 9.10 shows a theoretical comparison graph of two existing 100 W single-phase run capacitor induction motors, designed to work at two and four poles. The amount of active material was kept the same for both configurations to generate a fair comparison. The maximum efficiency reached by the two-pole configuration was considerably higher than for the four-pole machine.

For the motor shown in Figure 9.10, the advantages of using a two-speed motor are lost by the low efficiency produced at four-pole condition where the motor runs 90% of the time. To reduce or even overcome this problem, a way to increase the efficiency for four-pole operation has to be found, taking into account that the motor must have enough torque at two and four poles and also have a minimum guaranteed efficiency for two-pole operation and sufficient torque to pass through eventual overloads.

Starting from the example shown in Figure 9.10, a four-pole magnet and cage-rotor configuration in the rotor should notably increase the efficiency at low speed, but it may ruin the performance at high speed (two poles), due to PM-produced core loss and core saturation. There is no doubt that the magnets should form four poles in the rotor, since this is the condition where the efficiency should be maximized, but the difficult task is how to avoid the loss of performance (or at least minimize the loss) for two-pole operation. There are a few rules we found useful to follow when PMs are used for the four-pole operation mode:

- The magnets should not generate two-pole large saliency (L_d/L_q should be near unity).
- There must be enough space between the magnets to avoid flux concentration in two-pole operation, which may greatly increase the core losses.
- The magnets should generate enough flux for four-pole operation.
- The magnets flux should however not be too high, to avoid deterioration of the starting curve for four-pole operation, by a very high drag torque by PMs.
- The position and thickness of the magnets must be dimensioned to avoid PM demagnetization for both two- and four-pole operation.

9.11.2 THE CHOSEN GEOMETRY

The geometry shown in Figure 9.11 is proposed here to reach the minimum requirements which are necessary to generate high efficiency and good starting for four-pole operation (low speed), and acceptable efficiency and high breakdown torque for two-pole operation.

To be sure that the proposed geometry will not generate two-pole saliency, it is necessary to check the reluctance torque ($B_r = 0$ in PMs). This can be done using a FEA static analysis and a script language to shift the rotor step by step along 90° . Additionally, the FEA analysis helps to

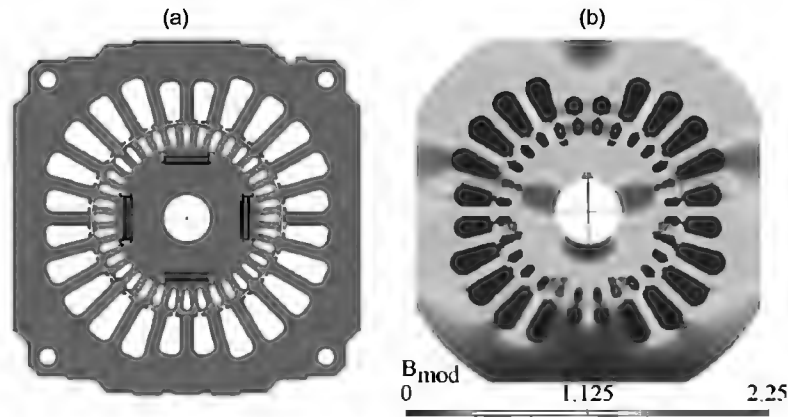


FIGURE 9.11 Proposed geometry for a two- to four-pole motor (a) and flux density map for two-pole operation, 1.3 A – peak current (b).

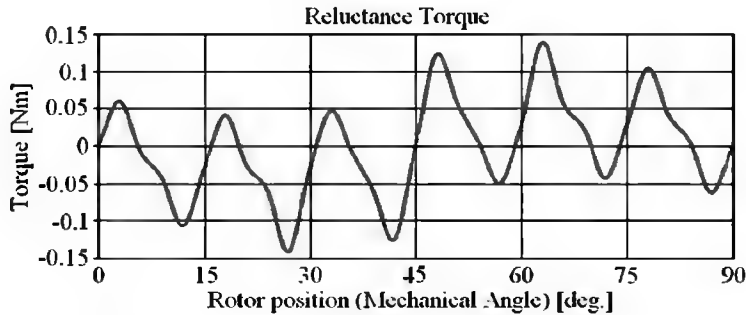


FIGURE 9.12 Reluctance torque (two poles).

identify possible flux bottlenecks of the lamination, which can cause efficiency problems for two-pole operation. Figure 9.11 shows the output of the first step of the simulation, where it is possible to see the machine cross section and the flux density map.

Figure 9.12 shows the reluctance (zero B_r in PMs) torque curve (obtained by 2D FEM), where it is possible to note that the average value is zero. This way, the rotor will not generate too a significant oscillatory torque for two-pole operation; an increase in efficiency and a reduction the motor vibration are obtained this way.

The motor windings should be arranged to generate both two and four poles, and this can be done several ways. Some configurations can be used to save copper and others to improve manufacturing ability.

The prototype has been constructed with windings configurations as shown in Figure 9.13 (to use only two switches (Figure 9.20: dual switch four pole and simple switch two pole) in order to simplify the switching required for pole count changing), for a compressor that requires 50 and 100 W of shaft power, for four-pole and two-pole operation, respectively.

Consequent pole (one pole wound and one not) four-pole windings (Figure 9.13b and c) are used to reduce copper weight.

9.11.3 EXPERIMENTAL RESULTS

Only the main winding is provided for two-pole operation, because the starting is performed in four-pole operation.

For the prototype in (Figure 9.13), the measured torques versus speed, and calculated and measured efficiency versus load are shown in Figure 9.14. To illustrate the contradictory performance in four- and two-pole operation, Figure 9.15 shows the experimental results with 13 mm and 14.3 mm long PMs per pole, respectively.

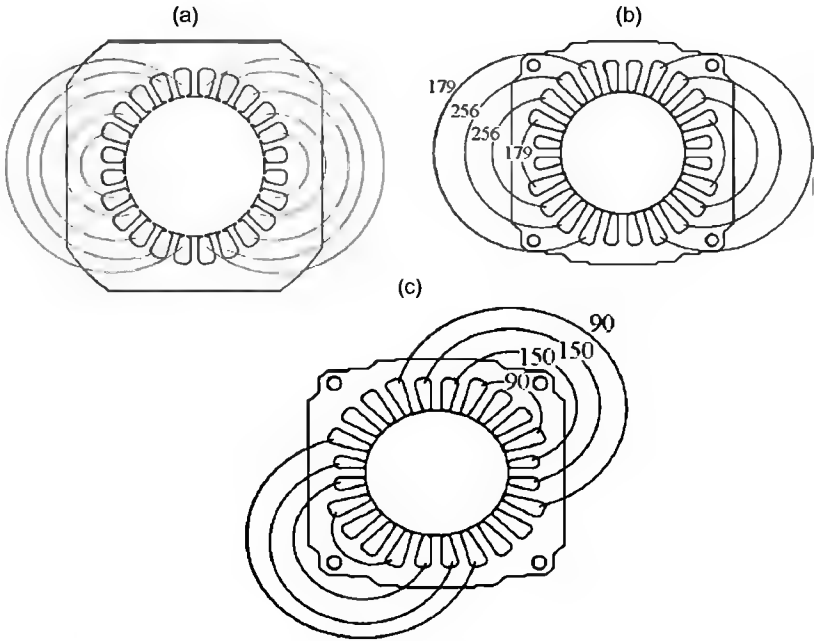


FIGURE 9.13 Two- and four-pole windings: (a) two-pole main winding, (b) four-pole main winding, and (c) four-pole auxiliary winding.

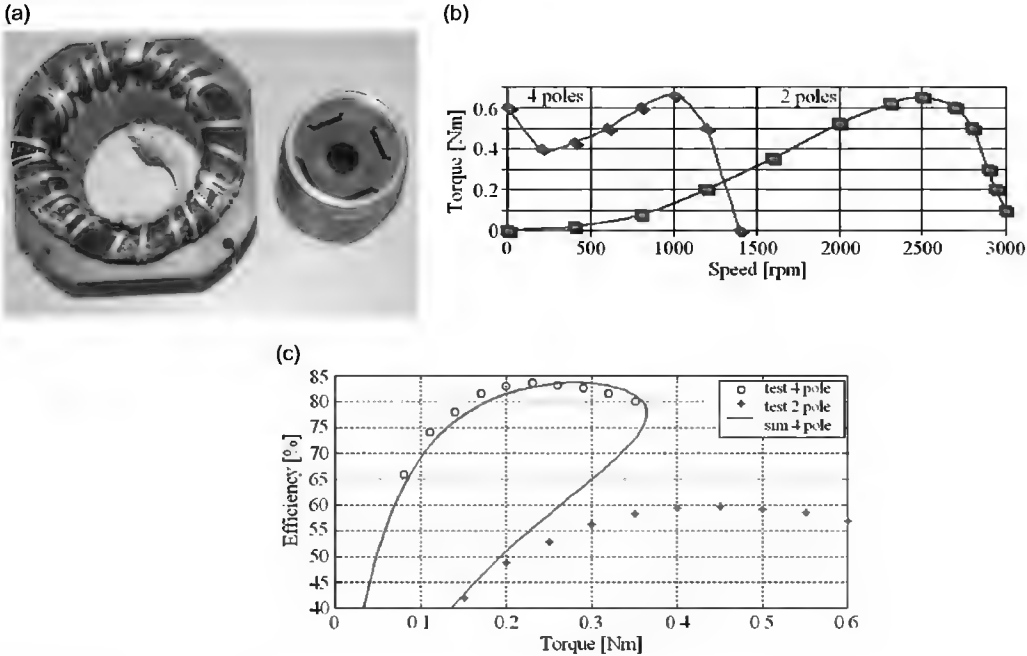


FIGURE 9.14 Prototype (a), asynchronous torque (b), and efficiencies versus load torque (c).

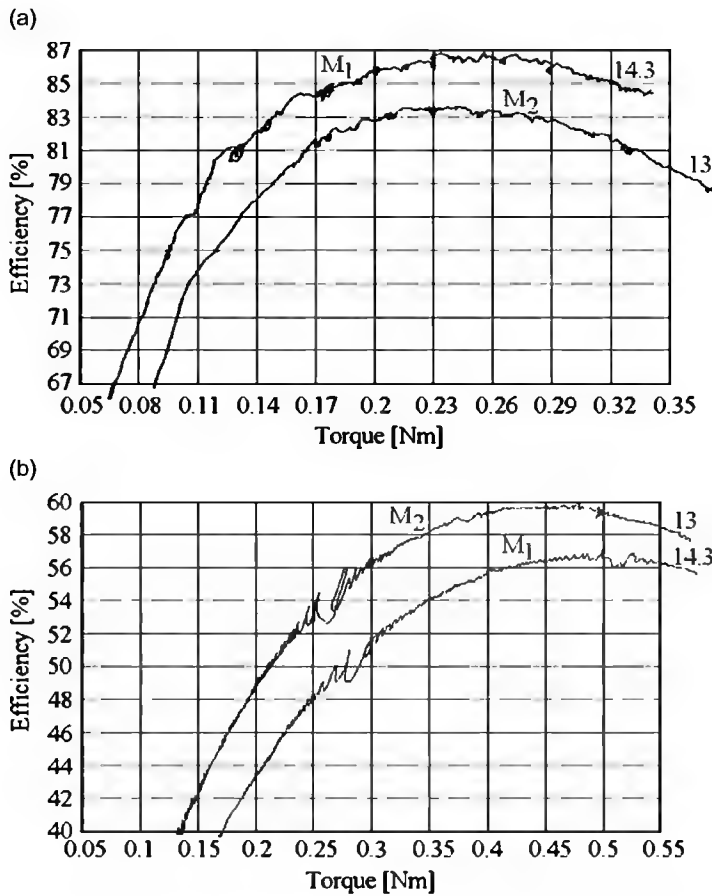


FIGURE 9.15 Measured efficiency torque curves for $2p = 4$ (a) and $2p = 2$ operation motors (b), and 13 mm M_2 and, respectively, 14.3 mm, M_1 , of PM per pole.

The prototype (Figure 9.14a) was tested in an active dynamometer environment at two poles with only main winding. At four poles, it was tested with both windings plus both capacitors (start and run).

Note that the curves in Figures 9.14b,c and 9.15 represent average values. since the real curves show torque ripple due to the magnet flux torque pulsations, slot-opening, etc.

The efficiency targets are almost met (84% tested of the desired 85% for four-pole operation), but refinements might rise the efficiency further by 1%–2% for both operation modes.

9.11.4 THEORETICAL CHARACTERIZATION: STEADY-STATE MODEL AND OPTIMAL DESIGN

So far we presented the essentials of the rationale behind the proposed two- to four-pole motor configuration and some field experimental results that validate it rather satisfactorily for industrial applications.

As expected, a thorough theoretical investigation effort was put forward also. This part will be exposed in some detail here. In essence, four main items have been followed:

- Developing a nonlinear analytical circuit model for two- and four-pole operation modes for steady state, to be used for general design
- Developing an optimal design methodology and a dedicated MATLAB code (see especially the proposed optimization function)

- Two-dimensional FEM investigation of PM flux density in the airgap and of torque pulsations for four-pole mode operation, for given stator currents
- Developing a machine circuit model with MATLAB code for the investigation of starting and load-perturbation transients embedding magnetic saturation, with transient passage from four- to two-pole operation.

9.11.5 STEADY-STATE MODEL

The magnetic equivalent circuit for four-pole operation in d axis, shown in Figure 9.16, allows to compute the permanent magnet emf, where R_{msy} and R_{mst} are stator yoke and teeth magnetic reluctances, respectively; R_{mrt} , R_{mrb} , R_{mry} , and R_{mpm} are the magnetic reluctances per pole of rotor cage teeth, rotor PM bridge, rotor yoke, and PM, respectively; and V_{PM} is the pole mmf of PM. This magnetic circuit represents a pole section and accounts iteratively for magnetic saturation at zero stator and rotor currents. Then, for steady-state operation, the magnetic saturation at no load “is frozen” when calculating constant circuit parameters for the machine model. It may be argued that stator currents in synchronous four-pole operation influence the saturation level; this aspect was checked by 2D FEM investigations and found acceptably small (Figure 9.11b) to allow constant circuit parameters of the machine, to be used in the forthcoming optimal analytical design methodology.

The equivalent circuit parameters expressions are rather standard [11], but amended by magnetic saturation coefficients derived from no-load operation (PM only contribution). The interior PM has been considered to produce a virtual airgap saliency; this way, the main inductances L_{d4} and L_{q4} , for four- and two-pole inductance L_{m2} , have been calculated with rather novel expressions not given here for lack of space. A MATLAB computer program was first developed to calculate, for given geometrical and winding data, the steady-state synchronous (for four-pole operation) and asynchronous steady state (for four- and two-pole operation). Skipping the strenuous analytical details, sample results are given in Figure 9.17.

It may be observed that the peak efficiency for both $2p = 4$ and $2p = 2$, Figure 9.17, closely match the experimental values (Figure 9.14). Also, for asynchronous four-pole operation, the machine seems capable to produce enough torque for quick-enough starting.

9.11.6 OPTIMAL DESIGN

Based on this, further on, an optimal design methodology based on Hooke–Jeeves algorithm [20] has been developed. The variables vector in the optimal design is presented in Table 9.2.

The hereby proposed intricate optimization multiple objective function is

$$C_t = C_i + C_e + C_{pdpm} + C_{pt} \quad (9.69)$$

$$C_i = C_c + C_l + C_{PM} + C_{ri} + C_{rc} + C_w \quad (9.70)$$

$$C_e = \frac{P_n}{1000} \left(p_{2p} \frac{1 - \eta_{2p}}{\eta_{2p}} + 0.5 p_{4p} \frac{1 - \eta_{4p}}{\eta_{4p}} \right) h_{py} n_y c_{pr} \quad (9.71)$$

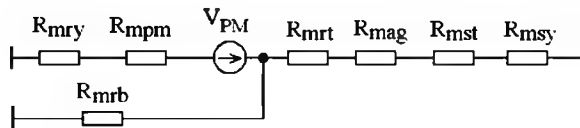


FIGURE 9.16 Magnetic circuit per pole – four-pole operation.

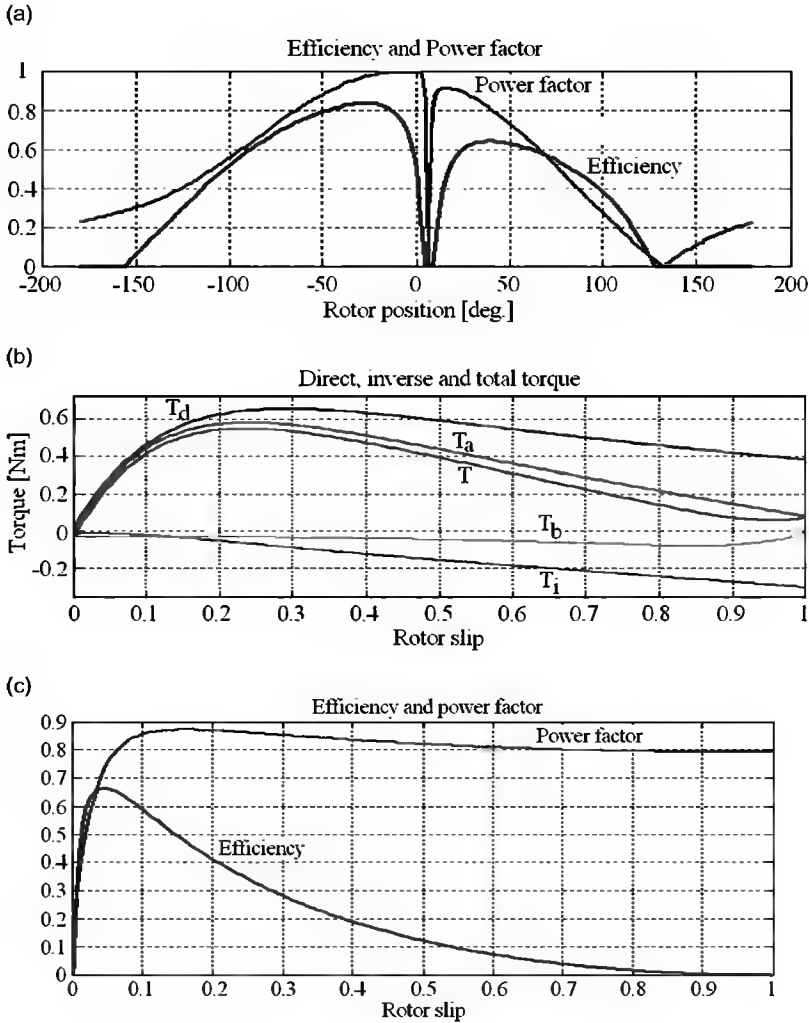


FIGURE 9.17 Steady-state efficiency (simulation results): (a) $2p = 4$ (synchronous mode – efficiency and power factor), (b) $2p = 4$ – asynchronous mode, torque components (T_d – direct torque, T_i – inverse torque, T_a – asynchronous torque, T_b – PM breaking torque, T – total torque), and (c) $2p = 2$ (asynchronous mode – efficiency and power factor). (After Ref. [9].)

$$c_{pt} = \max \left(0, \frac{k_{sdPM} H_{pm1d} - 0.9 H_c}{H_c} k_{cdpm} c_i \right) \quad (9.72)$$

$$c_{pdpm} = \max \left(0, \frac{k_{sdPM} H_{pm1d} - 0.9 H_c}{H_c} k_{cdpm} c_i \right) \quad (9.73)$$

where c_t is the total cost (objective function), c_i the initial cost, c_c the stator copper cost (USD), c_l the stator lamination cost (USD), c_{PM} the PM cost (USD), c_{ri} the rotor iron cost (USD), c_{rc} the rotor cage cost (USD), c_w the weight penalty cost (USD), c_e the energy cost (USD), P_n the rated power in W, P_{2p} and P_{4p} the probability of two- and four-pole operation, respectively, η_{2p} and η_{4p} the efficiency at two- and four-pole operation, respectively, h_{py} the number of operation hours per year, n_y the number of years, e_{pr} the energy price, c_{pt} the penalty over temperature cost (USD/C degree), T_w the winding

TABLE 9.2
Optimal Design Variables

Variables	Initial	Final	Legend
D_{si} (mm)	63.4	67.4	Stator inner diameter
L_{stack} (mm)	40	41.6	Core stack length
W_{st} pu	0.45	0.46	Stator tooth width in pu
W_{rt} pu	0.45	0.45	Rotor tooth width in pu
a_{rb} (mm ²)	14.7	17.8	Rotor bar area
w_{pm} pu	0.8	0.95	Width of PM relative to the field barrier
h_{pm} (mm)	1.5	1	PM height
K_{nm2c}	1	1	Multiplication factor for main coil turns
K_{nm4c}	1	1	Multiplication factor for main coil turns
K_{na4c}	1	1.21	Multiplication factor for auxiliary coil turns
w_{mbs}	4	4	Width of magnetic barrier in slots number
d_{m2wi}	13	13	Wire index two-pole winding ^a
d_{m4wi}	12	12	Wire index four-pole main winding ^a
d_{a4wi}	8	9	Wire index four-pole auxiliary winding ^a

^a Index in a wire diameter table.

temperature (C degree), T_{wmax} the winding maximum allowed temperature, k_{pt} the temperature penalty cost gain, c_{pdPM} the PM demagnetization penalty cost (USD), k_{cdPM} the demagnetization safety factor, H_{pml} the PM field strength at the maxim stator current (A/m), and k_{cdpm} the PM demagnetization penalty gain.

The price of main active materials considered in the optimal design are 10 USD/kg for cooper, 5 USD/kg for laminations, 5 USD/kg for aluminium, and 50 USD/kg for PMs.

It is evident by the key optimization results in Table 9.3 that the optimization design is needed, but the progress it produces is moderate (4% increase efficiency), because the initial values of variables were chosen based on solid industrial experience.

Also, the optimal design time of 14.7 s (Table 9.3) is considered reasonable (the optimal design prototype has not been fabricated yet).

The 4% increase in efficiency for four-pole operation obtained with 1 USD increase of active material cost (Table 9.3) may be translated in 2 Wh gain (per hour) which, for 0.1 USD/kWh and 5000h (33% duty cycle), would need a recovery time of about 2 years, which is satisfactory as the fabrication of the optimal configuration does not bring notable additional costs.

TABLE 9.3
Optimal Design Key Results

Table Column Subhead	Initial	Optimized
Outer diameter (mm)	63.04	66.74
Stack length (mm)	40	41.6
Efficiency (%) (4p/2p)	81.3/56.3	87.97/60.35
Power factor (4p/2p)	0.878/0.703	0.988/0.744
Active material cost (USD)	25.6	26.6
PM weight (g)/cost (USD)	24.64/1.23	21.4/1.07
Active material weight (kg)	3.98	4.13
Computation time (s)	0.59	14.7

9.11.7 2D FEM INVESTIGATIONS

In order to check the analytical design model and optimal design results, 2D FEM analysis was performed for four-pole synchronous mode and given values of currents in the main and auxiliary windings and power angle. But first the PM airgap flux density is verified as shown in Figure 9.18. Its fundamental component is rather close to the one considered in the analytical (magnetic circuit based) model (0.3568 T (analytically) versus 0.3556 T (by FEM) initial dimensions from Table 9.2).

Figure 9.19a and b shows that the stator current in the two pole only (main) winding influences notably the airgap flux density distribution, but its peak value is not unusually high at 0.85 T; magnetic saturation for two-pole operation varies with speed and, in a more refined future analytical model, this aspect may be considered in the future.

Sample, 2D FEM results shown in Figure 9.19c indicate clearly that there is a notable magnetic saliency in the four-pole operation mode (L_{m4} , L_{a4}) but the saliency is mild in L_{m2} , for two-pole operation (end-connection leakage inductances are not included).

Again, the average value of inductance L_2 (for two poles), which includes the airgap (L_{m2}), stator slot leakage, and differential leakage components, is close to the analytical model value (L_2 is around 0.41 H by 2D FEM, Figure 9.19c, and 0.43 H in the analytical model) and shows small saliency, as desired. The same acceptable agreement of analytical and FEM calculated d and q (minimum and maximum) inductances of main (L_{m4}) and auxiliary (L_{a4}) windings for four-pole operation was observed.

9.11.8 PROPOSED CIRCUIT MODEL FOR TRANSIENTS AND SIMULATION RESULTS

The circuit model for transients, developed here, integrates the d-q models for four- and two-pole windings in a single, dedicated MATLAB computer program that has to be capable of simulating the machine acceleration from zero speed to the steady-state speed of four-pole operation and then onward into two-pole steady-state operation speed. In addition, it has to handle torque perturbations at any speed. The schematic circuit model for transients is depicted in Figure 9.20, whereas the blocks “Model_4p” and “Model_2p” are described below in equation terms, for better clarity.

The four-pole model:

$$V_{\text{grid}} = R_{m4}i_{m4} + s\psi_{sm4}; \quad V'_{a4} = R'_{a4}i'_{a4} + s\psi_{am4} \quad (9.74)$$

$$i_{m4} = \frac{\psi_{sm4} - \psi_{mm4}}{L_{m\sigma 4}}; \quad i'_{a4} = \frac{\psi_{am4} - \psi_{sa4}}{L'_{a\sigma 4}} \quad (9.75)$$

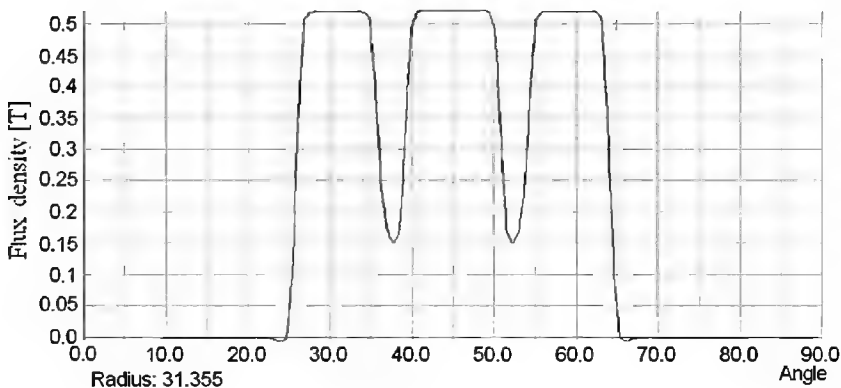


FIGURE 9.18 No-load operation: PM airgap flux density distribution for one pole (PM pole angle to vertical is 45°).

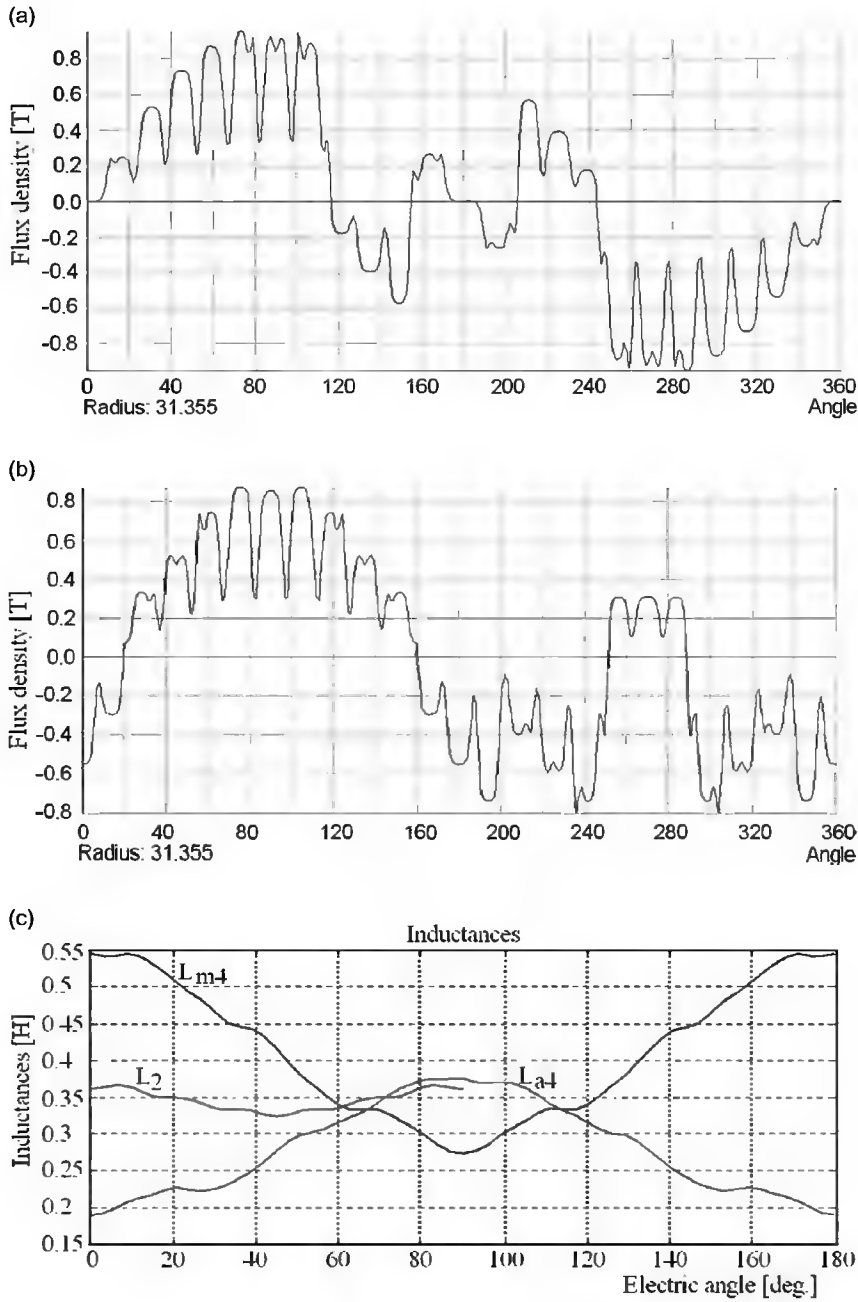


FIGURE 9.19 Airgap flux density for two-pole operation under load ($I_{m2} = 1.3$ A – peak value) for (a) (vertical) PM pole angle, (b) 45° PM pole angle, and two pole (L_{m2}) and (c) four-pole synchronous inductances (L_{m4} , L_{a4}).

$$\begin{pmatrix} i_{d4} \\ i_{q4} \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} i_{m4} \\ i_{a4} \end{pmatrix} \quad (9.76)$$

$$\psi_{d4} = L_{dm} \frac{sL_{r\sigma 4} + R_{r4}}{sL_{rd4} + R_{r4}} i_{d4} + \psi_{PM} \quad (9.77)$$

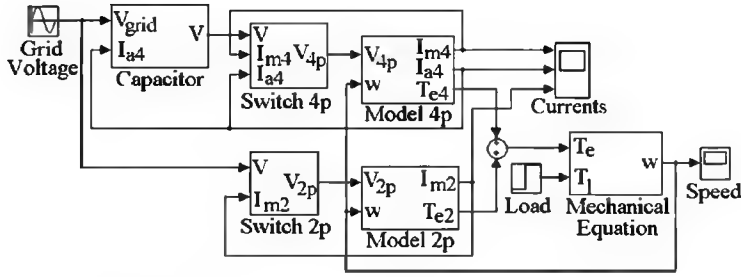


FIGURE 9.20 Dynamics model – general block diagram.

$$\Psi_{q4} = L_{qm} \frac{sL_{r\sigma 4} + R_{r4}}{sL_{rq4} + R_{r4}} i_{q4} \quad (9.78)$$

$$\begin{pmatrix} \Psi_{mm4} \\ \Psi_{am4} \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} \Psi_{d4} \\ \Psi_{q4} \end{pmatrix} \quad (9.79)$$

$$T_{em4p} = 2(\Psi_{mm} i'_a - \Psi_{am} i_m) \quad (9.80)$$

$$V'_{a4} = -\frac{V_{a4}}{k_{a4}} \quad i_{a4} = -\frac{i'_{a4}}{k_{a4}} \quad (9.81)$$

$$R'_{a4} = \frac{R_{a4}}{k_{a4}^2} \quad L'_{a\sigma 4} = \frac{L_{a\sigma 4}}{k_{a4}^2} \quad (9.82)$$

$$V_a = V_{grid} - \frac{1}{s} \cdot \frac{1}{C} \cdot i_{a4} \quad (9.83)$$

where V_{grid} , V'_a , R_m (14 Ω), R_a (26.5 Ω), $L_{m\sigma}$ (39 mH), and $L'_{a\sigma}$ (20 mH) are main and auxiliary winding voltages, resistances, and leakage inductances, reduced to the main winding, respectively; V_a is the auxiliary winding voltage; I_m , I'_a , Ψ_{sm} , and Ψ_{am} are the main and auxiliary winding currents and total flux linkages reduced to the main winding, respectively; L_{dm} (285 mH), L_{qm} (472 mH), L_{rd} , L_{rq} , and $L_{r\sigma}$ (22 mH) the d-q magnetization, total rotor inductances, and rotor leakage inductance, respectively, all reduced to the main winding; Ψ_{PM} (0.337 Wb) is the PM flux linkage in d axis; θ the rotor angle; k_a (0.819) the turn ratio factor for auxiliary to main winding; and C the capacity in series with auxiliary winding (15 μ F at start – before 0.8 s, 5 μ F running capacitor).

After notably processing the standard equations of split-phase IM [11], now with flux linkages as variables, the two-pole IM machine model is

$$s \begin{pmatrix} \Psi_{sm2} \\ \Psi_{rm2} \\ \Psi_{m2} \end{pmatrix} = \begin{pmatrix} -\frac{R_{s2}}{\sigma L_s} & R_{s2} \frac{1-\sigma}{\sigma L_{m2}} & 0 \\ R_{r2} \frac{1-\sigma}{\sigma L_{m2}} & -\frac{R_{r2}}{\sigma L_{r2}} & -\omega \\ 0 & \omega & -\frac{R_r}{L_r} \end{pmatrix} \cdot \begin{pmatrix} \Psi_{sm2} \\ \Psi_{rm2} \\ \Psi_{ra2} \end{pmatrix} + \begin{pmatrix} V_{2p} \\ 0 \\ 0 \end{pmatrix} \quad (9.84)$$

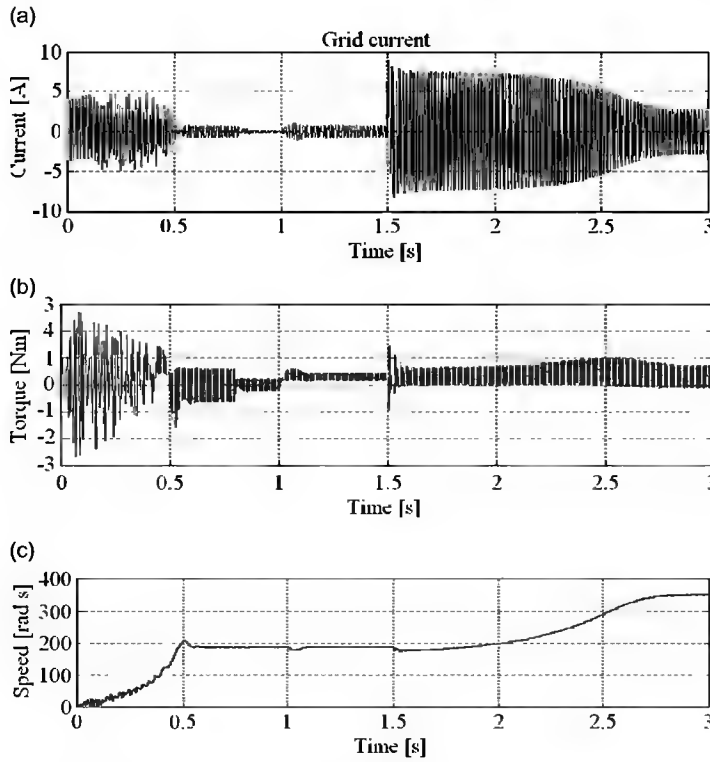


FIGURE 9.21 Acceleration process with rated torque (0.3 Nm) applied at $t = 1$ s onward through $2p = 4$, $t < 1.5$ s and $2p = 2$ ($t > 1.5$ s): (a) grid current, (b) total torque, and (c) speed versus time. (After Ref. [9].)

$$i_{m2} = \frac{1}{\sigma L_{s2}} \Psi_{sm2} + \frac{\sigma - 1}{\sigma L_{m2}} \Psi_{rm2} \quad (9.85)$$

$$T_{e2p} = -\frac{L_{m2}}{L_{r2}} \Psi_{rm2} i_{m2}; \quad \sigma = 1 - \frac{L_{m2}^2}{L_{s2} L_{r2}} \quad (9.86)$$

where R_{s2} (9.3 Ω), R_{r2} (7.47 Ω), L_{s2} (359 mH), and L_{r2} (343 mH) are stator and rotor resistances and inductances for two poles, respectively; L_{m2} (337 mH) is the magnetization inductance; Ψ_{sm2} and Ψ_{rm2} are the stator and rotor fluxes along stator winding axis; Ψ_{rma} is the rotor flux along an axis orthogonal to the stator winding axis and T_{e2p} the torque for two-pole operation. Figure 9.21 illustrates a comprehensive dynamic process including acceleration with rated load torque (0.3 Nm) applied at 1s and onward through $2p = 4$ until $t = 1.5$ s and then also for $2p = 2$ sudden connection for $t > 1.5$ s. The machine is capable (as in experiments) of accelerating quickly and switching safely from low- to high-speed mode under full load (where dual switch $4p$ is opened and single switch $2p$ is closed, Figure 9.20). More inquiries into the transients may be performed with the proposed MATLAB code, but we end here due to lack of space.

9.11.9 CONCLUSION

This investigation shows a feasible way to create a two-speed motor, without the need of an inverter, using a stator with two- and four-pole windings and a cage rotor with buried magnets (for four poles). The motor was designed to run synchronously at four poles (low speed 50 W, 84% efficiency) and asynchronously for two poles (high speed 100 W, 60% efficiency). A prototype was constructed, and

dynamometer tests were performed to confirm the theoretical simulations. Moreover, rather comprehensive models for steady state and transients accounting for various losses and for magnetic saturation have been developed to optimally design the machine according to specifications. Four percent efficiency rise by optimal design methodology for low-speed operation in comparison with the experienced standard analytical design was obtained and considered satisfactory. The theoretical effort was satisfactorily verified in part by 2D FEM inquiries and in full-scale industrial experiments.

Separate four-pole consequent split-phase windings (to save copper weight) and a two-pole main-only winding have been used to reduce the switching of the pole count to only a dual switch 4p and simple switch 2p. Although this motor can be a good option for the applications where the 2/1 ratio speed variation can generate benefits, the attractiveness of this solution can vary with the cost of the materials (mainly magnets and starting devices) and must be ultimately evaluated, case by case, by the motor designer.

9.12 SUMMARY

- The design of single-phase IM tends to be more involved as the magnet field is rather elliptical, in contrast to being circular, as for three-phase IMs.
- The machine utilization factor C_u and the tangential force density (stress) f_t tend to be higher and smaller, respectively, than for three-phase IMs. They also depend on the type of single-phase IM split-phase, dual capacitor, permanent capacitor, or split-phase capacitor.
- For new specifications, the tangential force density f_t (0.1–1.5) N/cm² is recommended to initiate the design process. For designs in given standardized frames, the machine utilization factor C_u seems more practical as an initiation constant.
- The number of stator N_s and rotor slots N_r selection goes as for three-phase IMs in terms of parasitic torque reduction. The larger the number of slots, the better the performance.
- The stator bore-to-stator outer diameter ratio D_i/D_o may be chosen as variable in small intervals, increasing with the number of poles above 0.58 for $2p_1 = 2$, 0.65 for $2p_1 = 4$, 0.69 for $2p_1 = 6$ and 0.72 for $2p_1 = 8$. These values correspond to laminations which provide maximum no-load airgap flux density for given stator mmf in three-phase IMs [2].
- By choosing the airgap, stator, rotor tooth, and core flux densities, the sizing of stator and rotor slotting becomes straightforward.
- The main winding effective number of turns is then calculated by assuming the emf $E_m/V_s \approx 0.95\text{--}0.97$.
- The main and auxiliary windings can be made with identical coils or with graded turn coils (sinusoidal windings). The effective number of turns (or the winding factor) for sinusoidal windings is computed by a special formula. In such a case, the geometry of various slots may differ. More so for the split-phase IMs where the auxiliary winding occupies only 33% of stator periphery and is active only during starting.
- The rotor cage cross-sectional total area may be chosen for start as 35%–60% of the area of stator slots.
- The turn ratio between auxiliary and main winding is $a = 1.5\text{--}2.0$, in general. It is equal to unity ($a = 1$) for reversible motors which have identical stator windings.
- The capacitance initial value is chosen for symmetry conditions at an assigned value of rated slip and efficiency (for rated power).
- Once the preliminary sizing is done, the resistance and leakage reactances may be calculated. Then, either by refined analytical methods or by FEM (with bare (cage-free) rotor and zero auxiliary winding current), the magnetization curve $\Psi_m(I_{mm})$ or reactance $X_{mm}(I_{mm})$ is obtained.
- To estimate the starting and steady-state running performance and constraints such as starting torque and current, rated slip, current, efficiency, power factor, and breakdown torque, revolving field or cross-field models are used.

- The above completes a preliminary electromagnetic design. A thermal model is used then to estimate stator and rotor temperatures.
- Based on such an analysis model, an optimization design process may be started. A good initial (preliminary) design is useful in most nonevolutionary optimization methods [5].
- The optimization design (Chapter 8) is a constrained nonlinear programming problem. The constraints may be lumped into an augmented objective function by procedures such as SUMT [7]. Better procedures for safe global optimization are currently proposed [32].
- Increases in efficiencies of a few percent may be obtained by optimization design [9]. Given the immense number of single-phase IMs, despite the low-power/unit, their total power is significant. Consequently, any improvement of efficiency of more than 1%–2% is relevant in both energy costs and environmental effects.
- Reduction of space harmonics in the design stage is yet another important issue [33].
- Winding reconfiguration with solid-state switches is presented in Ref. [34].
- The single-phase IM preliminary design case study and then the optimal design methodology and code developed in this chapter for a two-speed PM-assisted IM [9] are mere example of the complexity facing IM designers of today and tomorrow.

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10 Three-Phase Induction Generators

10.1 INTRODUCTION

In Chapter 7, Vol. 1, we alluded to the induction generator mode both in stand-alone (capacitor excited) and grid-connected situations. In essence for the cage-rotor generator mode, the slip S is negative ($S < 0$). As the induction machine (IM) with a cage rotor is not capable of producing reactive power, the energy for the machine magnetization has to be provided from an external means, either from the power grid or from constant (or electronically controlled) capacitors [1–24].

The generator mode is currently used for braking advanced PWM converter-fed drives for industrial (elevator) and traction purposes. Induction generators – grid connected or isolated-capacitor excited – are used for constant or variable speed and constant or variable voltage/frequency, in small hydropower plants, wind energy systems, emergency power supplies, etc. [1]. Both cage and wound rotor configurations are in use. For a summary of these possibilities, see Table 10.1. Cogeneration of electric power in industry at the grid (constant voltage and frequency) for small range variable speed and motor/generator operation in pump-back hydropower plants or in wind energy conversion are all typical applications for wound rotor IMs.

In what follows, we will discuss the main performance issues of IGs first in stand-alone configurations and then at power grid. Although changes in performance owing to variable speed are a key issue here, we will not deal with power electronics or control issues. We choose to do so as IG systems now constitute a mature technology whose in-depth (useful) treatment could be the subject matter of an entire book [25,26].

Typical configurations of IGs are shown in Figures 10.1–10.4.

Figure 10.1 portrays a WRIG whose rotor is connected through a bi-directional power flow converter and transformer (optional) to the power grid which has constant voltage and frequency.

For limited prime mover speed variation ($X\%$), the power converter rating is limited to $X\%$ of IG rated power. Consequently, reasonably lower costs are encountered.

Figure 10.2 shows an isolated cage-rotor IG (CRIG) with variable frequency but constant voltage for variable prime mover speed. The power electronics converter shown in Figure 10.2 has limited ratings and simplified control as it acts as a Variac to control (reduce) the capacitance reactive power flow into the IG for constant voltage control.

The A.C.–A.C. power electronics converter (PEC) in Figure 10.3 makes the transition from the variable voltage V_g and frequency f_g of the generator to the constant voltage and frequency of the

TABLE 10.1
IG Configurations

IG Type	Speed		Grid		Stator Frequency		Voltage	
	Constant	Variable	Connected	Isolated	Constant	Variable	Constant	Variable
Wound rotor	–	*	*	*	*	*	*	*
Cage rotor	*	*	*	*	*	*	*	*

– Impractical.

* Practical.

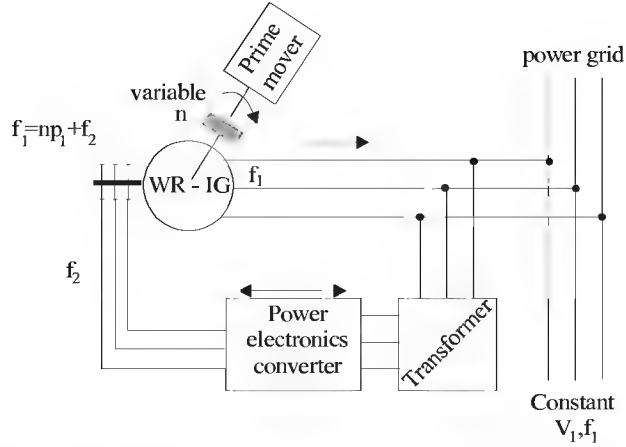


FIGURE 10.1 Advanced (bi-directional rotor power flow) wound rotor IG (WRIG) at power grid.

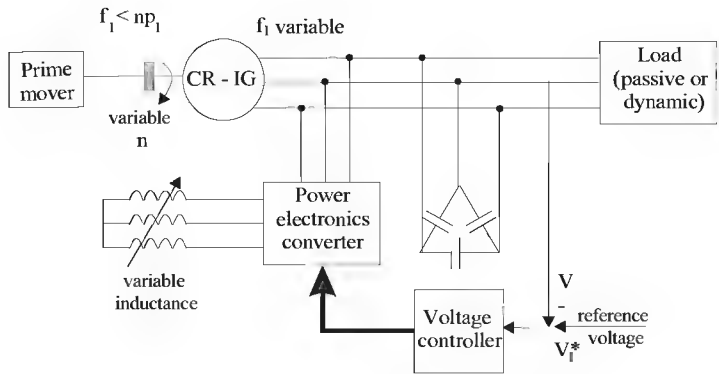


FIGURE 10.2 Isolated cage-rotor IG (CRIG) with constant voltage V_1 and variable frequency output f_1 for variable speed.

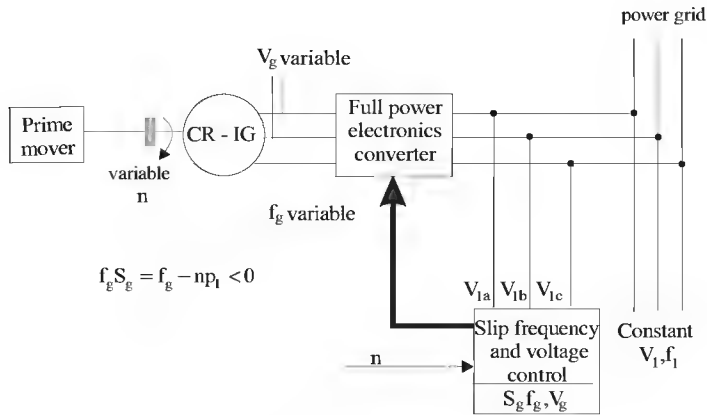


FIGURE 10.3 Power grid-connected CRIG: variable speed constant V_1 and f_1

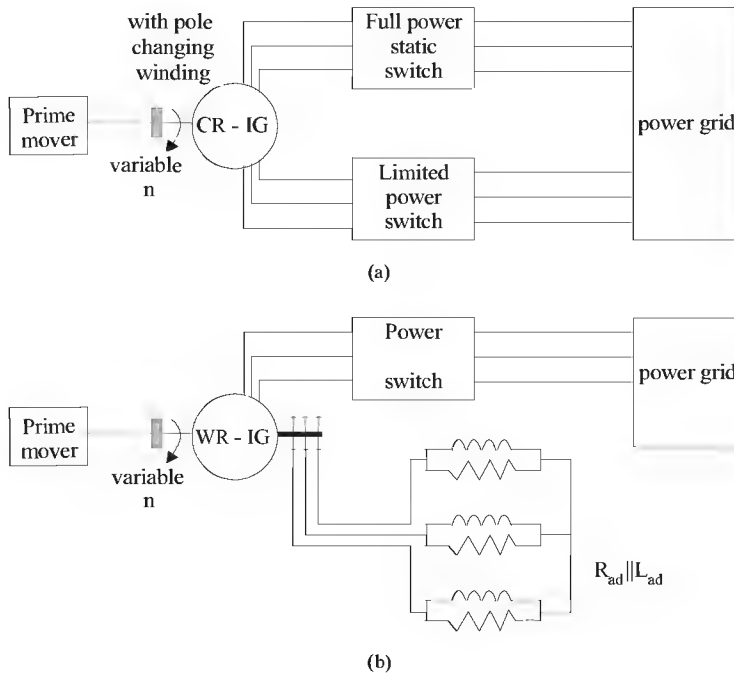


FIGURE 10.4 Simplified IG systems: (a) CRIG with pole changing winding and two static power switches for grid connection and (b) WRIG with parallel $R_{ad} \parallel L_{ad}$ in the rotor for grid connection system.

power grid. In the process, the same PEC transfers reactive power from the power grid to provide the magnetization of the cage-rotor induction generator (CRIG), or reactive power is “delivered” to the grid even when the CRIG is idle.

The PEC is rated at full power and thus adds to the total costs of the equipment, while the CR-IG is rugged and costs less.

Handling limited variable speed prime movers (wind or constant head small hydraulic turbines) to extract most of the available primary energy may be done by simpler methods such as pole changing windings for CR-IG (Figure 10.4a) or even a parallel connected R_{ad}/L_{ad} circuit in the rotor of the WRIG for grid-connected IGs (Figure 10.4b).

On the other hand, for less frequency-sensitive loads voltage regulation for variable speed variable frequency but constant voltage, long-shunt capacitor connections or saturable load interfacing transformers may be used in conjunction with CR-IGs.

Solutions like those shown in Figure 10.4 are characterized by low costs. But the power flow control and voltage regulation (for isolated systems) are only moderate. For low-/medium-power applications with limited prime mover speed variation, they may be adequate.

The system configurations presented in Figures 10.1–10.4 are meant to show the multitude of solutions that are feasible and have been proposed. Some of them are extensively used in wind power and small hydropower systems.

In the following, we will focus on IG behaviour in such schemes rather than on the systems themselves. First, we will investigate in depth the self-excitation and load performance under steady state and transients of CR-IG with capacitors for isolated systems.

Then, the WRIG with bi-directional power capability PEC in the rotor for grid-connected systems will be discussed in detail in terms of stability limits and performance.

Finally, DSW cage (or loop-cage) rotor IGs and the cascaded IG, which recently have a rich literature and strong R&D efforts, are discussed in their fundamentals.

10.2 SELF-EXCITED INDUCTION GENERATOR (SEIG) MODELLING

We describe a cage-rotor IG with capacitor excitation using SEIG (Figure 10.5a). The standard equivalent circuit of SEIG on a per phase bases is shown in Figure 10.5b.

First, with the switch S open, the machine is driven by the prime mover. As the SEIG picks up speed slowly, the no-load terminal voltage increases and settles at a certain value. This is the self-excitation process, which has been known since the 1930s [2].

In essence, first the residual magnetism in the rotor laminated core (from previous IG operation) produces by motion an emf in the stator windings. Its frequency is $f_{10} = n_0 p_1$. This emf is applied to the machine terminals and produces in the RLC circuit of each phase a magnetization current which produces an airgap field. This field adds to the remnant field of the rotor to produce a higher emf. The process goes on until an equilibrium is reached for a given speed n_0 and a given capacitance C at a voltage level V_0 . However, this process is stable as long as the machine is saturated and thus $X_m(I_m)$ is a nonlinear function (as shown already in Chapter 7, Vol. 1).

In a very simplified form, with X_{1l} , X_{2l} , R_1 , and R_2 neglected, the equivalent circuit for no load degenerates to X_m in parallel with a capacitor and a small emf determined by the remnant rotor field (Figure 10.6a).

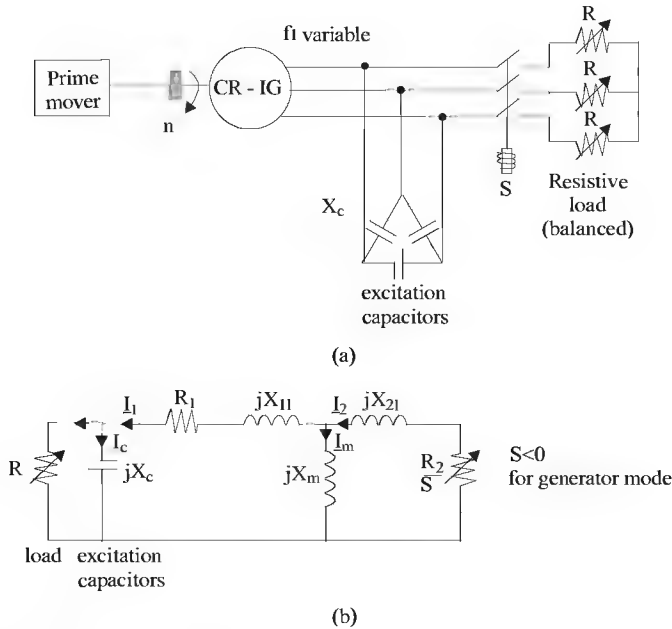


FIGURE 10.5 SEIG with resistive load: (a) general scheme and (b) standard equivalent circuit.

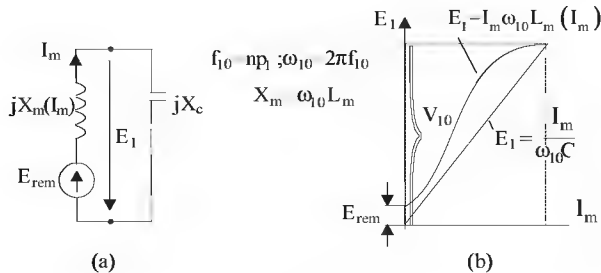


FIGURE 10.6 Oversimplified circuit for explaining self-excitation: (a) the circuit and (b) the characteristics.

The mandatory nonlinear $L_m(I_m)$ relationship is evident from Figure 10.6b where the final no-load voltage V_{10} occurs at the intersection of the no-load characteristic (to be found by the standard no-load test, or by design) with the capacitor voltage straight line. Also, the necessary presence of remnant rotor field (E_{rem}) is self-evident.

Once the SEIG is loaded, the terminal voltage changes depending on speed, SEIG parameters, the nature of the load, and its level.

The occurrence of load implies rotor currents, that is a nonzero slip, $S \neq 0$. Even if the speed of the prime mover is kept constant, the frequency f_1 varies with load

$$f_1 = \frac{np_1}{1+|S|}; \quad f_{10} = (f_1)_{S=0} = np_1 \quad (10.1)$$

The steady-state performance of SEIG can be determined by calculating the variation of voltage V_1 , frequency f_1 , stator current I_1 , power factor, efficiency, with speed n , load, and capacitor C . In this arrangement, V_1 and f_1 are fundamental unknowns.

It is also feasible to have the capacitance C and frequency f_1 as the main unknowns for given speed, load, and output voltage V_1 . Apparently, the problem is simple and implies only to use the standard circuit with given $L_m(I_m)$ function shown in Figure 10.5b. The next section deals extensively with this issue.

10.3 STEADY-STATE PERFORMANCE OF SEIG

Various analytical methods (models) have been developed in order to predict the steady-state performance of SEIG.

Among them, two are predominant:

- The impedance model
- The admittance model.

The impedance model is based on the single-phase equivalent circuit shown in Figure 10.5a, which, in general, is expressed in per unit terms.

f – frequency f_1 /rated frequency f_{1b} : $f = f_1/f_{1b}$
 v – speed/synchronous speed for f_{1b} : $v = np_1/f_{1b}$.

The final form of the circuit is shown in Figure 10.7.

The R , L character of the load, the presence of core loss resistance R_{core} (which may also vary slightly with frequency f), and the nonlinearity of $X_m(I_m)$ dependence with the unknowns X_m (the real output is V_1) and f makes the solving of this model possible only using a numerical procedure. Once X_m and f are calculated, the entire circuit model may be solved in a straightforward manner.

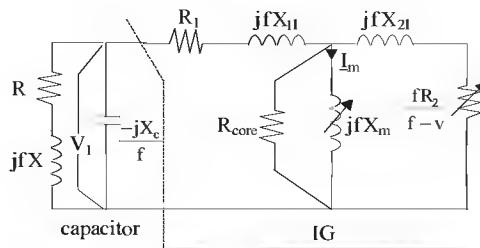


FIGURE 10.7 The impedance model of SEIG f – p.u. frequency, v = p.u. speed.

Fifth- or fourth-order polynomial equations in f or X_m are obtained from the conditions that the real and imaginary parts of the equivalent impedance are zero. A wealth of literature on this subject is available [3–5]. Rather recently a fairly general solution of the impedance model, based on the optimization approach, has been introduced [6].

However, the high order of system nonlinearity prevents an easy understanding of performance sensitivity to various parameters. In search of a simpler solution, the admittance model has been proposed [7].

While the X_m equation is simple, calculating f involves a complex procedure. In [8,9], admittance models, that lead to quadratic equations for the unknowns, are obtained for balanced resistive load, without additional simplifying assumptions.

10.4 THE SECOND-ORDER SLIP EQUATION MODEL FOR STEADY STATE

The second-order equation model may be obtained from the standard circuit shown in Figure 10.8.

The slip is negative for generator mode:

$$S = \frac{(f - v)}{f} \quad (10.2)$$

The airgap voltage E_a for frequency f (in p.u.):

$$\underline{E}_a = f \underline{E}_t = I_2 \left(\frac{R_2}{S} + jX_{21} \right) \quad (10.3)$$

where E_t is the airgap voltage at rated frequency.

For simplicity, the core loss resistance is neglected, while the load is purely resistive. The parallel capacitor-load resistance circuit may be transformed into a series one:

$$R_L - jX_L = \frac{R \left(-j \frac{X_C}{f} \right)}{R - j \frac{X_C}{f}} = \frac{R}{1 + \frac{R^2 X_C^2}{f^2}} - \frac{j \frac{X_C}{f}}{1 + \frac{X_C^2}{f^2 R^2}} \quad (10.4)$$

Now, we may lump R_1 and R_L into $R_{1L} = R_1 + R_L$ and fX_{11} and X_L into $X_{1L} = fX_{11} - X_L$ to obtain the simplified equivalent circuit shown in Figure 10.9.

Notice that frequency f will be given and the new unknowns are S and X_m while $E_t(I_m)$ or $X_m(E_t)$ comes from the no-load curve of IG. Also the load resistance R , the capacitance C , and the values of R_2 , X_{21} , R_1 , and X_{11} are given. Consequently, with f known and S calculated, the speed v can be computed as

$$v = f(1 - S) \quad (10.5)$$

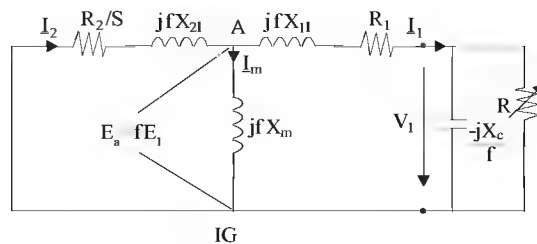


FIGURE 10.8 Equivalent circuit of SEIG with slip S and frequency f (p.u.) shown.

If the speed is known, a simpler iterative procedure is required to change f until the desired v is obtained.

We should mention that, with given f , the presence of any type of load does not complicate the problem rather than the expressions of R_L and X_L in (10.4). An induction motor load is such a typical dynamic load.

For self-excitation, the summation of currents in node A (Figure 10.9) must be zero (implicitly $E_1 \neq 0$):

$$0 = -I_2 + I_1 + I_m; \quad (10.6)$$

$$fE_1 \left(\frac{1}{jfX_m} + \frac{1}{R_{IL} + jX_{IL}} + \frac{S}{R_2 + jSfX_{21}} \right) = 0 \quad (10.7)$$

The same result is obtained in [9] after introducing a voltage source in the rotor.

The real and imaginary parts of (10.7) have to be zero:

$$\frac{R_{IL}}{R_{IL}^2 + X_{IL}^2} + \frac{SR_2}{R_2^2 + S^2f^2X_{21}^2} = 0 \quad (10.8)$$

$$\frac{1}{fX_m} - \frac{X_{IL}}{R_{IL}^2 + X_{IL}^2} + \frac{SfX_{21}}{R_2^2 + S^2f^2X_{21}^2} = 0 \quad (10.9)$$

For a given f , load, and IG parameters, (10.8) has the slip S as the only unknown:

$$aS^2 + bS + c = 0 \quad (10.10)$$

$$\text{with } a = f^2X_{21}^2R_{IL}; \quad b = +R_2(R_{IL}^2 + X_{IL}^2); \quad c = R_{IL}R_2^2 \quad (10.11)$$

Equation (10.10) has two solutions, but only the smaller one (S_1) refers to a real generator mode. The larger one refers to a braking regime (all the power is consumed in the machine losses).

$$S_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} < 0 \quad (10.12)$$

Complex solutions $S_{1,2}$ imply that self-excitation cannot take place.

Once S_1 is known, the corresponding speed v , for a given frequency f , capacitance, and load, is calculated from (10.5). When the speed is given, f is changed until the desired speed v is obtained.

Now, with S , f , etc. known, the only unknown in (10.9) is X_m , which is given by

$$X_m = \frac{R_2(R_{IL}^2 + X_{IL}^2)}{-(SfX_{21}R_{IL} + R_2X_{IL})f}; \quad S < 0 \quad (10.13)$$

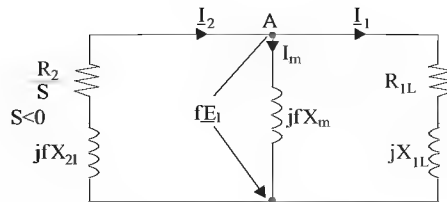


FIGURE 10.9 A simple nodal form of SEIG equivalent circuit.

With X_m determined, E_1 may be directly obtained from the no-load curve at the rated frequency (Figure 10.10).

As E_1 and f , S , X_m are known, from the parallel equivalent circuit of Figure 10.8, we may simply calculate I_2 and I_1 , as I_m comes directly from the magnetization curve (Figure 10.10):

$$I_2 = \frac{-fE_1}{\frac{R_2}{S} + jfX_{2l}} \quad (10.14)$$

$$I_1 = \frac{fE_1}{R_{1l} + jX_{1l}}; \quad X_{1l} < 0 \quad (10.15)$$

It is now simple to construct the terminal voltage phasor $\underline{V}_1$ as

$$\underline{V}_1 = I_1 [(R_{1l} - R_1) + j(X_{1l} - X_{2l})] \quad (10.16)$$

$$\text{or } \underline{V}_1 = fE_1 - (R_1 + jfX_{1l})I_1 \quad (10.17)$$

Capacitor current I_c and load current I_L are, respectively,

$$I_c = +\underline{V}_1 jf/X_c; \quad I_L = I_1 - I_c \quad (10.18)$$

Equations (10.14)–(10.18) are illustrated in the phasor diagram of Figure 10.11.

As the direction of the stator current, I_1 , and voltage, $\underline{V}_1$, has been chosen for the generator, the power factor angle ϕ_1 shows the current ahead of voltage. This is a clear sign that the machine is magnetized from outside.

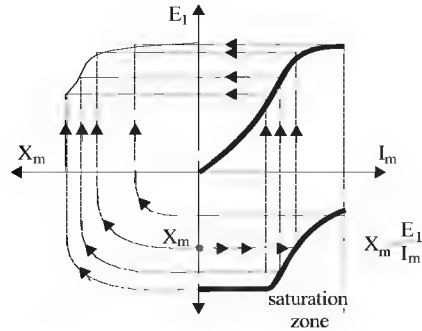


FIGURE 10.10 No-load curve of IG at rated frequency.

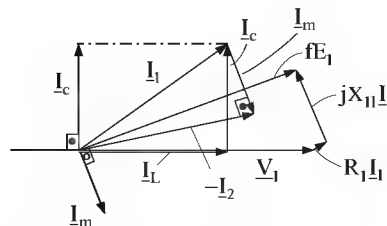


FIGURE 10.11 SEIG phasor diagram under resistive load.

As expected, for a resistive load, the terminal voltage V_L and the load current I_L end up in phase (Figure 10.11). We may approximately calculate the core losses, p_{core} , as

$$p_{\text{core}} = \frac{3(fE_t)^2}{R_{\text{core}}} \quad (10.19)$$

with R_{core} determined from no-load tests at the rated frequency. In reality, R_{core} varies slightly with frequency, and a no-load test for different frequencies would give the $R_{\text{core}}(f)$ functional.

The SEIG efficiency η_g is

$$\eta_g = \frac{3V_L I_L \cos \phi_L}{3V_L I_L \cos \phi_L + 3R_1 I_1^2 + 3R_2 I_2^2 + p_{\text{core}} + p_{\text{mec}}} \quad (10.20)$$

Mechanical losses are determined separately from the standard no-load tests at variable voltage and, eventually, frequency.

As we can see, the direct steady-state problem with f_l and capacitance load, machine parameters and no-load characteristic given, and S and X_m as unknowns may be solved in a straightforward manner. In the case when the speed v , instead of f , is fixed, the problem may be solved the same way through a few iterations by changing f in (10.10) until the value of $S < 0$ will produce the required speed $v = f(1 - S)$.

The solution to (10.10) leads to a few solution existence conditions:

$$fX_{ll} < X_L; \quad X_L = \frac{X_C/f}{1 + \frac{X_C^2}{f^2 R^2}} \quad (10.21)$$

where X_L refers to the capacitor in parallel with the load impedance.

Without capacitive predominance, the voltage collapses. With given X_C (capacitor), (10.21) reduces to a load resistance R condition given by

$$R \geq X_C \sqrt{\frac{X_{ll}}{X_C - f^2 X_{ll}}} \quad (10.22)$$

The load resistance R has to be a real number in (10.22):

$$X_C \geq f^2 X_{ll} \quad (10.23)$$

Thus, (10.22) sets the value of the minimum load resistance R_{min} for a given capacitance, frequency f , and stator leakage reactance X_{ll} . The smaller the X_{ll} , the lower the R_{min} , that is the larger the maximum load.

Also the slip Equation (10.10) solutions have to be real such that

$$b^2 - 4ac = R_2^2 (R_{lL}^2 + X_{lL}^2)^2 - 4f^2 X_{2l}^2 R_{lL}^2 R_2^2 > 0 \quad (10.24)$$

$$\text{or } 2fX_{2l} \leq \frac{(R_{lL}^2 + X_{lL}^2)}{R_{lL}} \quad (10.25)$$

A low rotor leakage reactance X_{2l} seems beneficial from this point of view.

The maximum possible value of the slip S_{max} corresponds to $b^2 - 4ac = 0$, and from (10.12) with (10.25), it becomes

$$S_{\text{max}} = -\frac{b}{2a} = -\frac{R_2}{fX_{2l}} \quad (10.26)$$

For this limiting value of the slip, (10.21) is not satisfied any more. So the composite capacitor-load reactance is not capacitive any more. Consequently, the voltage collapses at $S_{\max}$. However, the elegant expression (10.26), which corresponds essentially to an ideal maximum load power, represents a design constraint condition. We may remember that $S_{\max}$ of (10.26) corresponds to the peak torque slip of constant airgap flux vector-controlled IM [10].

Example 10.1 The Slip S and X_m Problem Solution

Let us consider an IM with the following data: $P_n = 7.36 \text{ kW}$, $f_{1b} = 60 \text{ Hz}$, $2p_1 = 2$ poles, star connection, $V_{1n} = 300 \text{ V}$, stator resistance $R_1 = 0.148 \Omega$, rotor resistance $R_2 = 0.144 \Omega$, stator leakage reactance (at 60 Hz) $X_{1l} = 0.42 \Omega$, and the rotor leakage reactance $X_{2l} = 0.25 \Omega$. The no-load curve $E(X_m)$ at 60 Hz (Figure 10.10) may be broken into a few straight-line segments [9].

$$E_1 = \begin{cases} 0; & X_m \geq 24.0 \Omega \\ 304.1 - 7.67X_m; & 19 \leq X_m \leq 24 \\ 220 - 3.16X_m; & 16 \leq X_m \leq 19 \\ 206 - 2.17X_m; & X_m < 16 \end{cases} \quad (10.27)$$

Let us determine the voltage and speed for $f_g = 43.33 \text{ Hz}$ and for $f_g = 58.33 \text{ Hz}$ and zero load ($R = \infty$), for $C = 180 \mu\text{F}$.

Solution

We start with a given frequency for no load $R = \infty$ in (10.4), with $f \approx 43.33/60 \approx 0.722$, $R_l = 0$, $X_l = X_c/f$, $R_{1l} = R_1 + R_l = R_1 = 0.148 \Omega$.

$$X_{1l} = fX_{1l} - \frac{X_c}{f} = 0.722 \cdot 0.42 - \frac{1}{2\pi 60 \cdot 0.722 \cdot 180 \cdot 10^{-6}} \approx -20.11 \Omega$$

Now from (10.11), a , b , and c are

$$a = f^2 X_{2l}^2 R_{1l} = 0.722^2 \cdot 0.25^2 \cdot 0.148 = 4.821 \cdot 10^{-2}$$

$$b = R_2 (R_{1l}^2 + X_{1l}^2) = 0.144 \cdot (0.148^2 + 33.42^2) = 160.833$$

$$c = R_{1l} R_2^2 = 0.148 \cdot 0.144^2 = 3.069 \cdot 10^{-3}$$

From (10.12)

$$S_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

For $4ac \ll b$, we can approximate S_1 by

$$S_1 = -\frac{c}{b} = \frac{3.069 \cdot 10^{-3}}{2 \cdot 160.33} \approx 1 \cdot 10^{-5} \approx 0$$

This small value was expected.

From (10.13), the magnetization reactance is

$$X_m = \frac{R_2 (R_{1l}^2 + X_{1l}^2)}{-R_2 X_{1l} f} \approx \frac{-X_{1l}}{f} = \frac{20.11}{0.722} = 27.864 > 24.0 \Omega$$

with zero rotor current, X_m is expected to be fully compensated by the capacitance reactance minus the stator leakage reactance. The emf value E_1 , for $X_m = 27.864 > 24$ in (10.27), is $E_1 = 0$; that

is, at a low frequency (speed), with a capacitance $C = 180 \mu\text{F}$ (per phase), the machine does not self-excite even on no load!

For $f_g = 58.33\text{Hz}$ ($f = 0.9722$), however, and $R = \infty$, X_{IL} becomes

$$X_{IL} = fX_{II} - \frac{X_C}{f} = 0.9722 \cdot 0.42 - \frac{1}{2\pi 60 \cdot 0.9722 \cdot 180 \cdot 10^{-6}} = -14.75 \Omega$$

In this case,

$$X_m = -\frac{X_{IL}}{f} = -\frac{14.75}{0.9722} = 15.179 \Omega$$

Now, from (10.27),

$$E_1 = 206 - 2.27 \cdot 15.179 = 171.54 \text{ V}$$

The phase current I_1 (equal, in this case, to capacitor current) is (from (10.15))

$$I_1 = \frac{fE_1}{R_{IL} + jX_{IL}} = \frac{0.9722 \cdot 171.54}{0.148 - j14.75} = 1.13 \cdot 10^{-2} + j1.306$$

The terminal voltage V_1 from (10.17) is

$$\begin{aligned} V_1 &= fE_1 - (R_1 + jfX_{II})I_1 \\ &= 0.9722 \cdot 171.54 - (0.148 + j0.9722 \cdot 0.42)(1.13 \cdot 10^{-2} + j1.306) \\ &= 171.422 - j1.67 \end{aligned}$$

Due to capacitor current, $V_1 > fE_1$ as expected. Note also that with $S = 0$, $I_2' = 0$.

When the machine is loaded ($R = 40 \Omega$), as expected, at $f = 43.33\text{ Hz}$ and same capacitor, the machine again will not self-excite (it did not under no load).

However, for $f = 58.33\text{ Hz}$, it may self-excite.

With $R = 40 \Omega$ and $X_C = \frac{1}{2\pi 60 \cdot 180 \cdot 10^{-6}} = 14.744 \Omega$ from (10.4),

$$R_L = \frac{R}{1 + \frac{R^2 f^2}{X_C^2}} = \frac{40}{1 + \frac{40^2 \cdot 0.9722^2}{14.744^2}} = 5.027 \Omega$$

$$X_L = \frac{\frac{X_C}{f}}{1 + \frac{X_C^2}{R^2 f^2}} = \frac{\frac{14.744}{0.9722}}{1 + \frac{14.744^2}{0.9722^2 \cdot 40^2}} = 13.26 \Omega$$

So

$$R_{IL} = R_1 + R_L = 0.148 + 5.027 = 5.175 \Omega$$

$$X_{IL} = fX_{II} - X_L = 0.9722 \cdot 0.42 - 13.26 = -12.851 \Omega$$

The coefficients a–c are obtained again from (10.11):

$$a = f^2 X_{21}^2 R_{IL} = 0.9722^2 \cdot 0.25^2 \cdot 5.175 = 0.3057$$

$$b = R_2 (R_{IL}^2 + jX_{IL}^2) = 0.144 (5.475^2 + 12.551^2) = 27.6377$$

$$c = R_{IL} R_2^2 = 5.175 \cdot 0.25^2 = 0.323$$

Still $b^2 \gg 4ac$, so S_1 is

$$S_1 = \frac{-c}{b} = \frac{-0.323}{27.6377} = -1.17 \cdot 10^{-2}$$

Now, the speed n is

$$n = f_g (1 - S_1) = 58.33 \cdot (1 + 1.17 \cdot 10^{-2}) = 59.126 \text{ rps} = 3540.75 \text{ rpm}$$

The magnetizing reactance X_m is (10.13)

$$\begin{aligned} X_m &= \frac{R_2 (R_{IL}^2 + X_{IL}^2)}{-(SfX_{2L}R_{IL} + R_2X_{IL})f} \\ &= \frac{0.144(5.175^2 + 12.851^2)}{-(-1.17 \cdot 10^{-2} \cdot 0.9722 \cdot 0.25 \cdot 5.175 + 0.144 \cdot (-12.851)) \cdot 0.9722} = 15.241 \Omega \end{aligned}$$

The emf E_1 is thus (10.27)

$$E_1 = 206 - 2.27 \cdot 15.241 = 171.40 \text{ V}$$

The stator current I_1 (10.15) can be written as

$$\underline{I}_1 = \frac{fE_1}{R_{IL} + jX_{IL}} = \frac{0.9722 \cdot 171.54}{5.175 - j12.851} = 4.493 + j11.157$$

The terminal phase voltage $\underline{V}_1$ (10.16) is

$$\begin{aligned} \underline{V}_1 &= fE_1 - (R_1 + jfX_{1L})\underline{I}_1 \\ &= 0.9722 \cdot 171.40 - (0.148 + j0.9722 \cdot 0.148)(4.493 + j11.157) \\ &= 166.729 - j2.29; \quad V_1 = 166.74 \text{ V} \end{aligned}$$

Notice that the terminal voltage decreased from no load to $R = 40 \Omega$ load, from 171.54 to 166.74 V, while the speed required for the same frequency $f_g = 58.33 \text{ Hz}$ had to be increased from 3500 to 3540.75 rpm.

The load is still rather small as the output power P_2 is

$$P_2 = 3R_L I_1^2 = 3 \cdot 5.027 \cdot (4.493^2 + 11.157^2) = 2.181 \text{ kW} \ll 7.36 \text{ kW}$$

If the speed v is given, the computation of slip from (10.12) is done starting with a few frequencies lower than the speed until the required speed is obtained.

Once computerized, the rather straightforward computation procedure presented here allows for all performance computation, for given frequency f (or speed), capacitance, load, and machine parameters. Any load can be handled directly putting it in form of a series impedance.

The slip and capacitor problem occurs when the voltage V_1 , speed, and load are given. To retain simplicity in the computation process, the same algorithm as above may be used repeatedly for a few values of frequency and then for capacitance (above the no-load value at same voltage) until the required voltage and speed are obtained.

The iterative procedure converges rapidly as voltage in general increases with capacitance C .

10.5 STEADY-STATE CHARACTERISTICS OF SEIG FOR GIVEN SPEED AND CAPACITOR

The main steady-state characteristics of SEIGs are to be obtained at constant (given) speed though a prime mover, such as a constant speed small hydroturbine, does not have constant speed if unregulated.

They are

- Voltage versus current characteristic for given speed and load power factor
- Voltage versus power for given speed and load power factor
- Frequency (slip) versus load power for given speed and power factor.

All these curves may be obtained by using the second-order slip equation model.

Such qualitative characteristics are shown in Figure 10.12a–c.

The slight increase and then decrease in output voltage with power (and current) for leading power factor is expected. For lagging power factor, the voltage drops rapidly, and thus, the critical slip is achieved at lower power levels. The frequency is influenced only a little by the power factor for given load. On the other hand, for low value of slips, S_1 , from (10.12), becomes

$$S_1 = \frac{-c}{b} = \frac{-R_{IL}R_2}{R_{IL}^2 + X_{IL}^2} \quad (10.28)$$

The larger the X_{IL} , the smaller the S_1 . Note that X_{IL} includes the self-excitation capacitance in parallel with the load.

The characteristics notably change if the prime mover speed is not regulated. The same methodology as above may be used, with the speed versus power curve as given, to obtain results as shown in [1].

For constant head hydroturbines, the speed decreases with power, and thus, the voltage regulation is even more pronounced. Variable capacitance is needed to limit the voltage regulation.

10.6 PARAMETER SENSITIVITY IN SEIG ANALYSIS

The influence of IG resistances R_1 and R_2 and reactances X_{1l} , X_{2l} , and X_m and of parallel or series-parallel excitation capacitor on performance is described using parameter sensitivity studies.

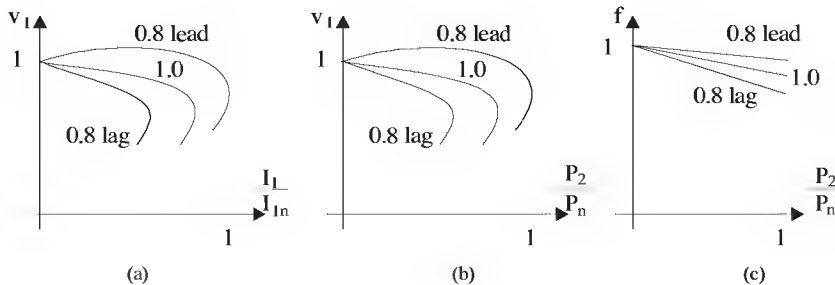


FIGURE 10.12 Steady-state curves for given speed, capacitor, and load power factor: (a) voltage/current, (b) voltage/power, and (c) frequency/power.

Parameter sensitivity studies of SEIG may be performed for unregulated (variable speed) [1] or regulated (constant speed) prime movers [6]. The main conclusions of such studies are as follows:

For constant speed:

- The no-load voltage increases with capacitance.
- The terminal voltage and maximum output power increase notably with capacitance.
- For constant voltage, the needed excitation capacitance increases with power.
- At no load, the magnetization reactance decreases with the supply voltage increase.
- In the stable load area, voltage increases with IG leakage reactance and decreases with rotor resistance.

For variable speed (constant head hydroturbine),

- Along a rather large (40%) load resistance variation range, the output power varies only a little.
- Within the range of rather constant power, the voltage drops sharply with load, more so with inductive-resistive loads.
- Up to peak load power, the frequency and speed decrease with power. They tend to increase after that, which is to be translated into stable operation from this point of view.
- The maximum power depends approximately on $C^{2.2}$ [3] and the corresponding voltage on $C^{0.5}$ [11].
- For no load, the minimum capacitance requirement is inversely proportional to speed squared.
- As expected, under load the minimum required capacitance depends on load impedance, load power factor, and speed.
- When the capacitance is too small, it produces negligible capacitive current for magnetization and thus the SEIG cuts off. At the other end, with too large a capacitance, the rotor impedance of the generator causes de-excitation and the voltage collapses again. In between, there should be an optimal value of capacitance C for which both output power and efficiency are high.
- When feeding an IM load, the SEIG's power rating should be two to three times higher than that of the IM and about twice the capacitance needed for a 0.8 lag power factor passive load. These requirements are mainly imposed by the IM transients during starting [12].
- The long-shunt connection decreases the voltage regulation and increases the maximum load accordingly. Avoiding low-frequency oscillations makes subsynchronous resonance important [13].
- Saturable reactances (or transformers) have been proposed also to reduce voltage regulation at low costs [14].

10.7 POLE CHANGING SEIGS

Pole changing winding stator SEIGs have been proposed for isolated generator systems with variable speed operation when an up to two-to-one speed ratio is required. To keep the flux density in the airgap about the same for both pole numbers, the numbers of poles have to be carefully chosen.

Adequate numbers are 4/6, 6/8, 8/10, etc. The connection of phases for the two pole numbers is also important for the same airgap flux density. Parallel star and series star for 4/6 pole combination was found adequate for the scope [8].

The pole switching should occur near (but before) the drop-out speed for the low pole number.

The voltage drop (for single capacitor) with load, for both numbers of poles, is about the same. Operating at lower number of poles (higher speed), the power is doubled with respect to the case of lower number of poles. Also the efficiency is higher, as expected, at higher speed.

For constant voltage, when the speed is doubled, the capacitance and voltage amps reactive (VAR), demands in both six- and four-pole configurations drop to 1/7th and 1/3rd. When the load increases, the capacitance and VAR demands increase almost twofold [5].

High winding factor pole changing windings with simple (2 three-phase ends) pole switching power switch configurations are required (see Chapter 4, Vol. 1). SEIGs with dual windings (of different power ratings) are commercially available.

10.8 UNBALANCED STEADY-STATE OPERATION OF SEIG

Failure of one capacitor or unbalanced load impedances in a three-phase SEIG lead to unbalanced operation.

A general approach to unbalanced operation is provided here by using the symmetrical component method [15]. Both the SEIG and the load may be either *delta*- or *star*-connected.

The equivalent circuit of IG for the positive and negative sequence, with frequency f and speed v in p.u. (as in Section 10.3 (Figure 10.7)) is shown in Figure 10.13.

For the negative sequence, the slip changes from $S^+ = f/(f - v)$ to $S^- = f/(f + v)$.

Also, as the slip frequency is different for the two sequences, $f_s^+ = S^+ \cdot f \cdot f_{lb}$; $f_s^- = S^- \cdot f \cdot f_{lb}$, the skin effect in the rotor will in reality change R_2 in the negative sequence circuit accordingly. To the first approximation, this aspect may be neglected. The superposition of effects in the (+, -) method implies that X_m is the same also, though still influenced by magnetic saturation.

Core loss is neglected for simplicity.

10.8.1 THE DELTA-CONNECTED SEIG

Let us consider a delta-connected IG tied to a delta-connected admittance network Y_{ab} , Y_{ac} , and Y_{bc} (Figure 10.14).

The expression of load and excitation capacitive admittance is

$$Y_{ab} = \frac{1}{Z_{ab}} + j \frac{f}{X_c} \quad (10.29)$$

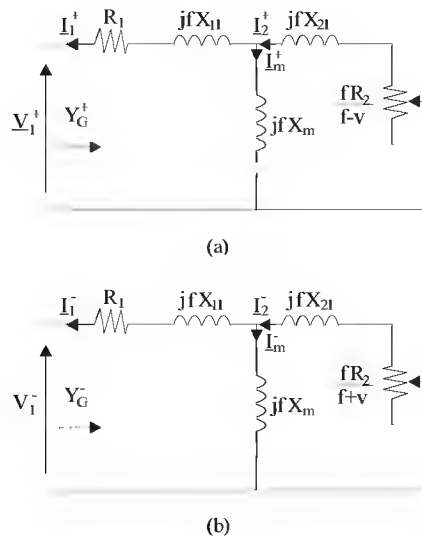


FIGURE 10.13 +/− sequence equivalent circuits of SEIG.

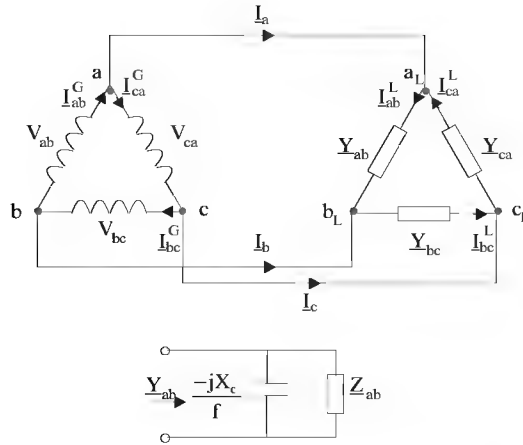


FIGURE 10.14 SEIG with delta-connected unbalanced load and excitation capacitors.

The total load-capacitor supposedly unbalanced admittances are first decomposed into the sequence components as

$$\begin{bmatrix} \underline{Y}^0 \\ \underline{Y}^+ \\ \underline{Y}^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} \underline{Y}_{ab} \\ \underline{Y}_{bc} \\ \underline{Y}_{ca} \end{bmatrix} \quad (10.30)$$

The total load-capacitor phase currents $\underline{I}_{ab}^L$, $\underline{I}_{bc}^L$, and $\underline{I}_{ca}^L$ can also be transformed into their sequence components as

$$\begin{bmatrix} \underline{I}_{ab}^{L0} \\ \underline{I}_{ab}^{L+} \\ \underline{I}_{ab}^{L-} \end{bmatrix} = \begin{bmatrix} \underline{Y}^0 & \underline{Y}^- & \underline{Y}^+ \\ \underline{Y}^+ & \underline{Y}^0 & \underline{Y}^- \\ \underline{Y}^- & \underline{Y}^+ & \underline{Y}^0 \end{bmatrix} \cdot \begin{bmatrix} \underline{V}_{ab}^0 \\ \underline{V}_{ab}^+ \\ \underline{V}_{ab}^- \end{bmatrix} \quad (10.31)$$

In the delta connection, there is no zero sequence voltage $\underline{V}_{ab}^0 = 0$. Consequently, the line currents $\underline{I}_a$ components $\underline{I}_a^+$ and $\underline{I}_a^-$ are

$$\begin{aligned} \underline{I}_a^+ &= \underline{I}_{ab}^{L+} - \underline{I}_{ca}^{L+} = (1-a)\underline{I}_{ab}^{L+} = (1-a)(\underline{Y}^0 \underline{V}_{ab}^+ + \underline{Y}^- \underline{V}_{ab}^-) \\ \underline{I}_a^- &= \underline{I}_{ab}^{L-} - \underline{I}_{ca}^{L-} = (1-a^2)\underline{I}_{ab}^{L-} = (1-a^2)(\underline{Y}^+ \underline{V}_{ab}^+ + \underline{Y}^0 \underline{V}_{ab}^-) \end{aligned} \quad (10.32)$$

From Figure 10.13, the generator phase current components $\underline{I}_1^+$ and $\underline{I}_1^-$ are

$$\underline{I}_1^+ = \underline{Y}_G^+ \underline{V}_1^+; \quad \underline{I}_1^- = \underline{Y}_G^- \underline{V}_1^- \quad (10.33)$$

Similar to (10.32), we may write the line current $\underline{I}_a$ components based on generator phase current components as

$$\begin{aligned} \underline{I}_a^+ &= (1-a)\underline{I}_{ab}^{G+} = -(1-a)\underline{V}_{ab}^+ \underline{Y}_G^+ \\ \underline{I}_a^- &= (1-a)\underline{I}_{ab}^{G-} = -(1-a^2)\underline{V}_{ab}^- \underline{Y}_G^- \end{aligned} \quad (10.34)$$

Note that the generator phases are balanced. Equating (10.32) to (10.34) and then dividing them lead to the elimination of V_{ab}^+ and V_{ab}^- to yield

$$(\underline{Y}_G^+ + \underline{Y}_0)(\underline{Y}_G^- + \underline{Y}_0) - \underline{Y}^+ \underline{Y}^- = 0 \quad (10.35)$$

Equation (10.35) represents the self-excitation condition and leads to two real coefficient nonlinear equations. In general, the solution to (10.35) for a given capacitance, frequency (or speed), and load is similar to the solution for balanced condition, although a bit more involved. For balanced operation, $\underline{Y}^+ = \underline{Y}^- = \underline{Y}$ and $\underline{Y}^0 = 0$. So (10.35) degenerates to $\underline{Y}_G^+ - \underline{Y} = 0$, as expected.

If the load is $\underline{Y}$ connected, it should be first transformed into an equivalent delta connection and then (10.35) applied. Notice that in general, the capacitors are delta connected to exploit the higher voltage available to them.

10.8.2 STAR-CONNECTED SEIG

This time the capacitors are star-connected to the load. If in reality they are delta-connected, X_C for delta connection is replaced by $3X_C$ for star connection ($X_{CY} = 3X_{C\Delta}$) (Figure 10.15).

In this case, $\underline{Y}^0$, $\underline{Y}^+$, and $\underline{Y}^-$, the total capacitive load admittance components, refer to $\underline{Y}_a$, $\underline{Y}_b$, and $\underline{Y}_c$, respectively.

The symmetrical components of the line current $\underline{I}_a$ calculated from the load are

$$\begin{bmatrix} \underline{I}_a^0 \\ \underline{I}_a^+ \\ \underline{I}_a^- \end{bmatrix} = \begin{bmatrix} \underline{Y}^0 & \underline{Y}^- & \underline{Y}^+ \\ \underline{Y}^+ & \underline{Y}^0 & \underline{Y}^- \\ \underline{Y}^- & \underline{Y}^+ & \underline{Y}^0 \end{bmatrix} \cdot \begin{bmatrix} \underline{V}_a^0 \\ \underline{V}_a^+ \\ \underline{V}_a^- \end{bmatrix} \quad (10.36)$$

with $\underline{I}_a^0 = 0$ (no neutral connection)

$$\underline{V}_a^0 = - \frac{(\underline{Y}^- \underline{V}_a^- + \underline{Y}^+ \underline{V}_a^+)}{\underline{Y}^0} \quad (10.37)$$

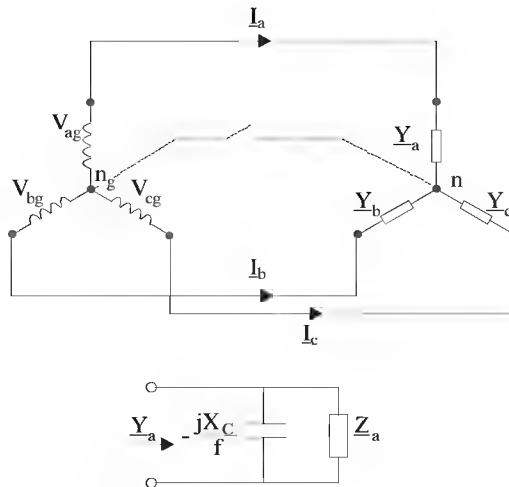


FIGURE 10.15 Star-connected SEIG.

Making use of $\underline{V}_a^0$ in the star two equations of (10.36), we obtain

$$\begin{aligned}\underline{I}_a^+ &= \left(\underline{Y}^0 - \frac{\underline{Y}^+ \underline{Y}^-}{\underline{Y}^0} \right) \underline{V}_a^+ + \left(\underline{Y}^- - \frac{\underline{Y}^+ \underline{Y}^-}{\underline{Y}^0} \right) \underline{V}_a^- \\ \underline{I}_a^- &= \left(\underline{Y}^+ - \frac{\underline{Y}^- \underline{Y}^-}{\underline{Y}^0} \right) \underline{V}_a^+ + \left(\underline{Y}^0 - \frac{\underline{Y}^+ \underline{Y}^-}{\underline{Y}^0} \right) \underline{V}_a^-\end{aligned}\quad (10.38)$$

The same line current components, calculated from the generator side, are simply

$$\underline{I}_a^+ = -\underline{Y}_G^+ \underline{V}_{ag}^+; \quad \underline{I}_a^- = \underline{Y}_G^- \underline{V}_{ag}^- \quad (10.39)$$

As the generator and the load line voltages have their sum equal to zero, even with unbalanced load, it may be shown that

$$\begin{aligned}\underline{V}_{ab}^+ &= (1 - a^2) \underline{V}_a^+ = (1 - a^2) \underline{V}_{ag}^+ \\ \underline{V}_{ab}^- &= (1 - a) \underline{V}_a^- = (1 - a) \underline{V}_{ag}^-\end{aligned}\quad (10.40)$$

$$\text{Consequently,} \quad \underline{V}_a^+ = \underline{V}_{ag}^+ \quad \text{and} \quad \underline{V}_a^- = \underline{V}_{ag}^- \quad (10.41)$$

From (10.38) to (10.39), with (10.41), we obtain the self-excitation equation:

$$\left(\underline{Y}_G^+ + \underline{Y}^0 - \frac{\underline{Y}^+ \underline{Y}^-}{\underline{Y}^0} \right) \left(\underline{Y}_G^- + \underline{Y}^0 - \frac{\underline{Y}^+ \underline{Y}^-}{\underline{Y}^0} \right) - \left(\underline{Y}^+ - \frac{\underline{Y}^- \underline{Y}^-}{\underline{Y}^0} \right) \left(\underline{Y}^- - \frac{\underline{Y}^+ \underline{Y}^-}{\underline{Y}^0} \right) = 0 \quad (10.42)$$

Note that if the SEIG is star-connected and the capacitor-load combination is delta-connected with original $+0$ sequence admittances as $\underline{Y}^0$, $\underline{Y}^+$, $\underline{Y}^-$, then a self-excitation equation similar to (10.35) is obtained

$$\left(\frac{1}{3} \underline{Y}_G^+ + \underline{Y}^0 \right) \left(\frac{1}{3} \underline{Y}_G^- + \underline{Y}^0 \right) - \underline{Y}^+ \underline{Y}^- = 0 \quad (10.43)$$

Also when n_g and n are connected, the zero sequence current $\underline{I}_a^0$ is not zero, generally. However there is no zero sequence generator voltage $\underline{V}_a^0 = \underline{V}_{ag}^0 = 0$.

Consequently, the situation is in this case similar to that of delta/delta connection, but the total load admittances $\underline{Y}^0$, $\underline{Y}^+$, and $\underline{Y}^-$ correspond to the star-connected load admittances $\underline{Y}_a$, $\underline{Y}_b$, and $\underline{Y}_c$, respectively.

Whereas the above treatment is fairly standard and general, a few examples are in order.

10.8.3 TWO PHASES OPEN

Let us consider the case of delta/delta connection (Figure 10.16) with a single capacitor between a_L and b_L and a single load

$$\underline{Y}_{ab} = \underline{Y} = \frac{1}{2}; \quad \underline{Y}_{bc} = \underline{Y}_{ca} = 0 \quad (10.44)$$

For this case

$$\underline{Y}^0 = \underline{Y}^+ = \underline{Y}^- = \frac{\underline{Y}}{3} \quad (10.45)$$

With (10.45), the self-excitation equation (10.35) becomes

$$3\mathbf{Y}_G^+ \mathbf{Y}_G^- + (\mathbf{Y}_G^+ + \mathbf{Y}_G^-) \mathbf{Y} = 0 \quad (10.46)$$

$$\text{or } \frac{3}{\mathbf{Y}} + \frac{1}{\mathbf{Y}_G^+} + \frac{1}{\mathbf{Y}_G^-} = 0 \quad (10.47)$$

Equation (10.47) illustrates the series connection of + and – generator sequence circuits to $\frac{3}{\mathbf{Y}} = 3\mathbf{Z}$ (Figure 10.16).

Other unbalanced conditions may occur. For example, if phase C opens for some reason, while the load $\mathbf{Z}$ is balanced, the SEIG performs as with a single capacitor and load impedance but with new load admittance $\mathbf{Y}_{ab} = 1.5 \mathbf{Y}$ and $\mathbf{Y}_{bc} = \mathbf{Y}_{ca} = 0$ [15].

For a short circuit across phase, a Δ/Δ connection $\mathbf{Y}_{ab} = \infty$ and thus $\mathbf{Y}^0 = \mathbf{Y}^+ = \mathbf{Y}^- = \infty$. Consequently (10.35) becomes (Figure 10.16a).

$$\mathbf{Y}_G^+ + \mathbf{Y}_G^- = 0 \quad (10.48)$$

As the capacitor is short-circuited, there will be no self-excitation. The same is true for a three-phase short circuit.

In essence, the computation of steady-state performance under unbalanced conditions includes solving first the self-excitation nonlinear Equations (10.35) or (10.42) or (10.43), (10.47) for given speed (frequency), load, and capacitance, to find the frequency (speed) and the magnetization reactance X_m . Then, as for the balanced conditions, equivalent circuits are solved, provided the magnetization curve $E_1(X_m)$ is known. It was proved in Ref. [15] that the positive sequence magnetization curves obtained for balanced no load and one-line-open load differ from each other in the saturation zone (Figure 10.17).

It turns out that, whenever a zero line current occurs, the one-line-open characteristic in Figure 10.17 should be applied [15].

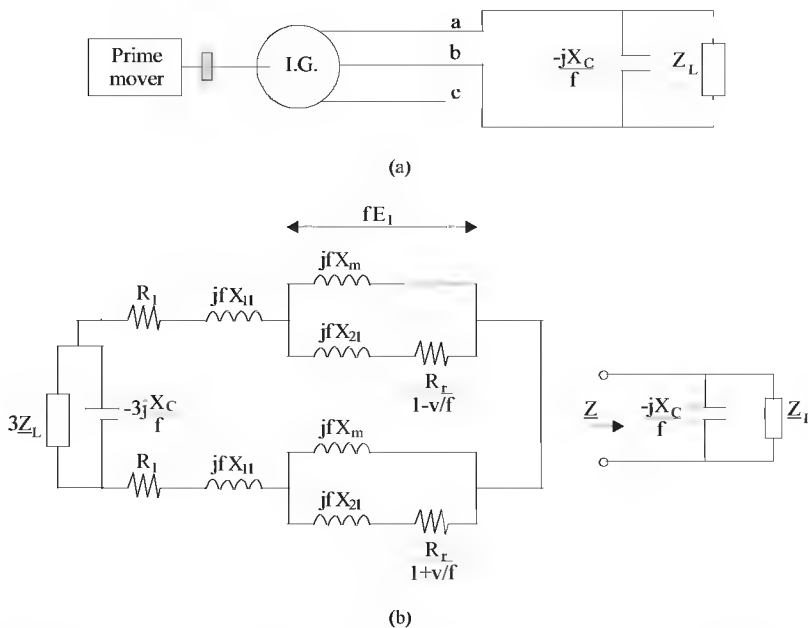


FIGURE 10.16 Unbalanced SEIG with Δ/Δ connection: single capacitor and single load.

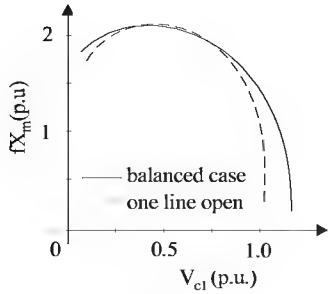


FIGURE 10.17 Positive sequence magnetizing characteristics. (After Ref. [15].)

Sometimes, the range of self-excitation capacitance is needed. Specifying the maximum magnetization reactance $X_{\max}$, the speed (frequency), machine parameters, and load, the solution of self-excitation equations developed in this section produces the required frequency (speed) and capacitances.

There are two solutions for the capacitances, $C_{\max}$ and $C_{\min}$, and they depend strongly on load and speed. For maximum load, the two values become equal to each other. Beyond that point, the voltage collapses.

Sample results obtained for $C_{\max}$ and $C_{\min}$ on no load under balanced and unbalanced (one capacitor only) conditions for $r_1 = 0.09175$ p.u., $r_2 = 0.06354$ p.u., $p_l = 2$, $f_{lb} = 60$ Hz, $x_{1l} = x_{2l} = 0.2112$ p.u., $x_{\max} = 2.0$ p.u. are shown in Figure 10.18 [15].

Meticulous investigation of various load and capacitor unbalances on load has proved [15] that the theory of symmetrical components works well.

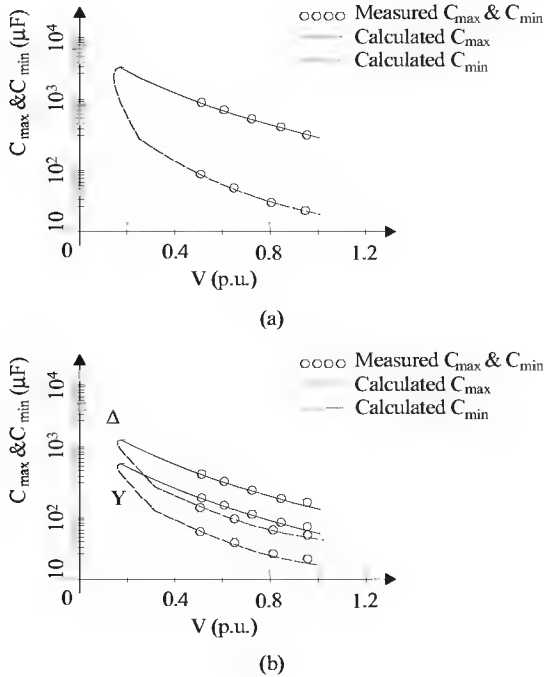


FIGURE 10.18 Min-max capacitors on no load (continued): (a) balanced capacitor (after Ref. [15]) and (b) one capacitor only.

The range of $C_{\min}$ to $C_{\max}$ for delta connection is three times smaller than for the star connection. In terms of parameter sensitivity analysis, the trends for unbalanced load are similar to those for balanced load. However, the efficiency is notably reduced and stays low with load variation.

10.9 TRANSIENT OPERATION OF SEIG

SEIGs work alone or in parallel on variable loads. Load and speed perturbations, voltage build-up on no load, and excitation capacitance changes during load operation are all causes for transients. To investigate the transients, the d-q model is in generally preferred. Also stator coordinates are used to deal easily with eventual parallel operation or active loads (A.C. motors).

The d-q model (in stator coordinates) for the SEIG alone, including cross-coupling saturation [16,17], is given as

$$[V_G] = -[R_G][i_G] - [L_G]p[i_G] - \omega_{rG}[G_G][i_G] \quad (10.49)$$

$$\text{with } [V_G] = [V_d, V_q, 0, 0] \quad (10.50)$$

$$[i_G] = [i_d, i_q, i_{dr}, i_{qr}] \quad (10.51)$$

The torque

$$T_{eG} = \frac{3}{2} P_{IG} L_m (i_q i_{dr} - i_d i_{qr}) \quad (10.52)$$

$$\frac{J}{P_{IG}} \frac{d\omega_{rG}}{dt} = T_{PM} - T_{eG} \quad (10.53)$$

$$pV_d = \frac{1}{C} i_{dc}; \quad pV_q = \frac{1}{C} i_{qc} \quad (10.54)$$

$$i_{dc} = i_d + i_{dL}; \quad i_{qc} = i_q + i_{qL} \quad (10.55)$$

i_{dc} , i_{qc} , i_{dL} , and i_{qL} are d, q components of capacitor and load currents, respectively.

The association of signs in (10.49) corresponds to generator mode.

Now, the parameter matrices R_G , L_G , and G_G are

$$R_G = \text{Diag}[R_1, R_1, R_2, R_2]$$

$$L_G = \begin{bmatrix} L_{1d} & L_{dq} & L_{md} & L_{dq} \\ L_{dq} & L_{1q} & L_{dq} & L_{mq} \\ L_{md} & L_{dq} & L_{2d} & L_{dq} \\ L_{dq} & L_{mq} & L_{dq} & L_{2q} \end{bmatrix} \quad (10.56)$$

$$G_G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & L_m & 0 & L_2 \\ -L_m & 0 & -L_2 & 0 \end{bmatrix} \quad (10.57)$$

where L_m is the magnetization inductance and i_m the magnetization current.

$$i_{md} = i_d + i_{dr}; \quad i_{mq} = i_q + i_{qr}; \quad i_m = \sqrt{i_{md}^2 + i_{mq}^2} \quad (10.58)$$

The cross-coupling transient inductance L_{dq} [16,17] is

$$L_{dq} = \frac{i_{md}i_{mq}}{i_m} \frac{dL_m}{di_m} \quad (10.59)$$

Also the transient magnetization inductances along the two axes L_{md} and L_{mq} [16,17] are

$$L_{md} = L_m + \frac{i_{md}^2}{i_m} \frac{dL_m}{di_m} \quad (10.60)$$

$$L_{mq} = L_m + \frac{i_{mq}^2}{i_m} \frac{dL_m}{di_m} \quad (10.61)$$

Also

$$\begin{aligned} L_{1d} &= L_{1l} + L_{md}; & L_{1q} &= L_{1l} + L_{mq} \\ L_{2d} &= L_{2l} + L_{md}; & L_{2q} &= L_{2l} + L_{mq} \end{aligned} \quad (10.62)$$

The SEIG equations for constant speed ω_r on no load may be written also in the form (cross-coupling saturation is neglected here):

$$0 = [Z][I] + [V_0] \quad (10.63)$$

with

$$[Z] = \begin{bmatrix} R_s + pL_s + 1/sC & 0 & pL_m & 0 \\ 0 & R_s + pL_s + 1/sC & 0 & pL_m \\ pL_m & -\omega_r L_m & R_r + pL_r & -\omega_r L_r \\ \omega_r L_m & pL_m & \omega_r L_r & R_r + pL_r \end{bmatrix} \quad (10.64)$$

$$[I] = \begin{bmatrix} i_{qs} & i_{ds} & i_{qr} & i_{dr} \end{bmatrix}; \quad [V_0] = \begin{bmatrix} V_{cq0} & V_{cd0} & V_{remq} & V_{remd} \end{bmatrix}$$

where V_{remq} and V_{remd} are the remnant flux density emfs (small values), V_{cq0} and V_{cd0} are the no-load terminal d-q voltages, and p is the Laplace operator; L_m is a function of $i_m = \sqrt{(i_{sd} + i_{rd})^2 + (i_{sq} + i_{rq})^2}$. For self-excitation at no load, $\text{Det}[Z] = 0$ and all the roots in “ p ” should have negative real parts. An eight-order polynomial equation in “ s ” is to be solved [18]. All machine parameters, capacitor C , and speed influence the self-excitation process, but especially the magnetic saturation expressed by $L_m(i_m)$ function and the remnant (initial) no-load voltages V_{remq} and V_{remd} .

Finally also, in practice, the machine acceleration ($a_m = d\omega/dt$) when too large may impede on self-excitation as active current components (q components) occur in the stator and they may hamper self-excitation.

10.10 SEIG TRANSIENTS WITH INDUCTION MOTOR LOAD

There are practical situations, in remote areas, when the SEIG is supplying an induction motor which in turns drives a pump or a similar load.

A pump load is characterized by increasing torque with speed such that

$$T_{load} \approx T_{L0} + K_L \omega_{rM}^2 \quad (10.65)$$

The most severe design problem of such a system is the starting (transient) process. The rather low impedance of IM at low speeds translates into high transient currents from the SEIG. On the other hand, a large excitation capacitor is needed for rated voltage with the motor on load. When the motor is turned off, this large capacitor produces large voltage transients. Part of it must be disconnected, or an automatic capacitance reduction to control the voltage is required. An obvious way out of this difficulty would be to have a smaller (basic) capacitance (C_G) connected to the generator and one (C_M) to the motor (Figure 10.19).

First, the SEIG is accelerated by the prime mover until it establishes a certain speed with a certain output voltage. Then, the IM is connected. Under rather notable voltage and current transients, the motor accelerates and settles at a speed corresponding to the case when the motor torque equals the pump torque.

To provide safe starting, the generator rated power should be notably in excess of the motor rated power. In general, even an 1/0.6 ratio marginally suffices.

Typical such transients for a 3.7kW SEIG and a 2.2kW motor with $C_G = 18 \mu\text{F}$, $C_M = 16 \mu\text{F}$, are shown in Figure 10.20 [19].

Voltage and current transients are evident, but the motor accelerates and settles on no load smoothly. Starting on pump load is similar but slightly slower.

On the other hand, the voltage build-up in the SEIG takes about 1–2s. Sudden load changes are handled safely up to a certain level which depends on the ratio of power ratings of the motor and SEIG.

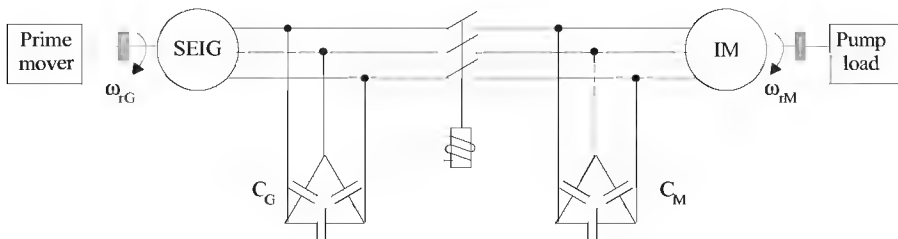


FIGURE 10.19 SEIG with IM load and split excitation capacitor.

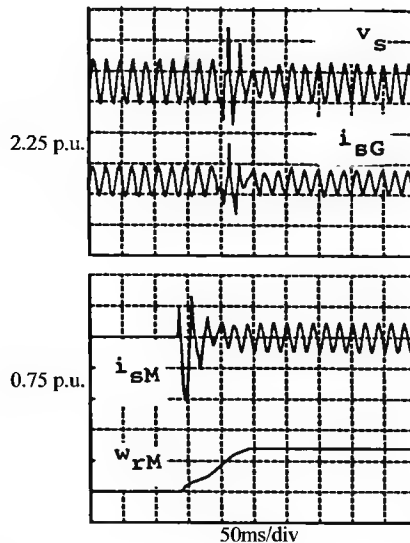


FIGURE 10.20 Motor load starting transients for a SEIG, $C_G = 18 \mu\text{F}$, $C_m = 16 \mu\text{F}$. (After Ref. [19].)

For the present case, a 0%–40% motor step load is handled safely, but an additional 60% step load is not sustainable [19]. Very low overloading is acceptable unless the SEIG to motor power rating is larger than $3 \div 4$.

In case of overloading, an overcurrent relay set at 1.2–1.3 p.u. for 300–400 ms can be used to trip the circuit.

During a short circuit at generator terminals, surges of 2 p.u. in voltage and 5 p.u. in current occur for a short time (up to some 20 ms) before the voltage collapses.

Although the split capacitor method (Figure 10.19) is very practical, for motors rated <20% of SEIG rating a single capacitor will do.

10.11 PARALLEL OPERATION OF SEIGS

An isolated power system made of a few SEIGs is a practical solution for upgrading the generating power rating upon demand. Such a stand-alone power system may have a single excitation capacitor, but most probably, it has to be variable, to match the load variations which occur inevitably. Capacitor change may be done in steps through a few power switches or continuously through power electronics control of a paralleled inductor current (Figure 10.21).

The operation of such a power grid on steady state is to be approached as done for the single SEIG. However, the magnetization curves and parameters of all SEIGs are required. Finally, the self-excitation condition will be under the form of an admittance summation equal to zero. Neglecting the “de-excitation” inductance L_{ex} , the terminal voltage equation is

$$\underline{V}_s \left(\frac{1}{R_L} - \frac{j}{fX_L} + \frac{jf}{X_C} \right) = \sum_{i=1}^N \underline{I}_{si} \quad (10.66)$$

The SEIG's equations (Section 10.3 and Figure 10.7) may be expressed as

$$\underline{I}_{mi} + \underline{I}_{si} = \underline{I}_{2i} = \frac{-f \underline{E}_{ti}}{\frac{R_{2i}}{1 - \frac{v_i}{f}} + jfX_{2li}} \quad (10.67)$$

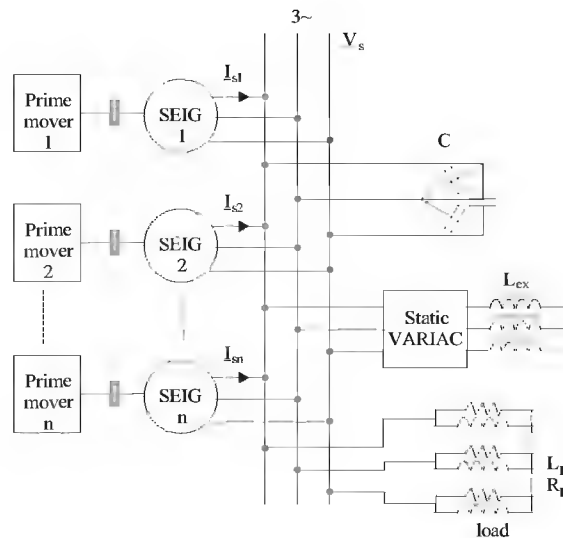


FIGURE 10.21 Isolated power grid with paralleled SEIGs.

v_i is the p.u. speed of the i th generator. $X_{mi}(E_{fi})$ represents the magnetization curve of the i th machine. All these curves have to be known. Again, for a given excitation capacitance C , motor parameters and speeds of various SEIGs and load, the voltage, frequency, SEIG currents, capacitor currents, and load currents and load powers (active and reactive) could be iteratively determined from (10.66) to (10.67). A starting value of f is in general adopted to initiate the iterative computation. For sample results, see Ref. [20].

The SEIGs speeds in p.u. have to be close to each other to avoid that some of them switch to motoring mode. Transient operation may be approached by using the d-q models of all SEIGs with the summation of the stator phase currents equal to the capacitor plus load current. So the solution of transients is similar to that for a single SEIG, as presented earlier in this chapter, but the number of equations is larger. Only numerical methods such as Runge–Kutta–Gill, etc. may succeed in solving such stiff nonlinear systems.

10.12 THE DOUBLY FED IG (DFIG) CONNECTED TO THE GRID

As shown in Figure 10.1, a wound-rotor induction generator (dubbed as doubly fed) may deliver power not only through the stator but also through the rotor provided a bi-directional power flow converter is used in the rotor circuit. The main merit of a doubly fed IG is its capability to deliver constant voltage and frequency output for $\pm(20\text{--}40)\%$ speed variation around conventional synchronous speed. If the speed variation range is $(20\text{--}40)\%$ so is the rating of static power converter and transformer in the rotor circuit (Figure 10.1). For bi-directional power flow in the rotor circuit, single stage converters (cycloconverters or matrix converters) or dual-stage back-to-back voltage source PWM inverters have to be used.

Independent reactive and active power flow control – with harmonics elimination – may be achieved this way for both super and subsynchronous speed operation. Less costly solutions use a unidirectional power flow converter in the rotor when only super or subsynchronous operation as generator is possible.

Here, we are interested mainly in the IG behaviour when doubly fed. So we will consider sinusoidal variables with rotor voltage amplitude, frequency, and phase open to free change through an ideal bi-directional power electronics voltage-source converter.

10.12.1 BASIC EQUATIONS

To investigate the DFIG, the d-q (space-phaser) model as presented in Chapter 1, Vol. 2, on transients is used.

In synchronous coordinates,

$$\begin{aligned}\bar{V}_1 &= (R_1 + (p + j\omega_l)L_1)\bar{i}_1 + (p + j\omega_l)L_m\bar{i}_2 \\ \bar{V}_2 &= (p + jS\omega_l)L_m\bar{i}_1 + (R_2 + (p + jS\omega_l)L_2)\bar{i}_2\end{aligned}\quad (10.68)$$

with

$$\begin{aligned}\bar{V}_1 &= V_d + jV_q; \quad \bar{V}_2 = V_{dr} + jV_{qr} \\ \bar{i}_1 &= i_d + ji_q; \quad \bar{i}_2 = i_{dr} + ji_{qr} \\ L_1 &= L_{1l} + L_m; \quad L_2 = L_{2l} + L_m \\ T_e &= \frac{3}{2}p_l L_m \text{Imag}[\bar{i}_1 \bar{i}_2^*]; \quad \frac{J_p \omega_r}{p_l} = T_{PM} - T_e\end{aligned}$$

We should add the Park transformation:

$$\begin{vmatrix} V_d \\ V_q \end{vmatrix} = \frac{2}{3} \begin{vmatrix} \cos \theta_{1,2} & \cos\left(-\theta_{1,2} + \frac{2\pi}{3}\right) & \cos\left(-\theta_{1,2} - \frac{2\pi}{3}\right) \\ \sin(-\theta_{1,2}) & \sin\left(-\theta_{1,2} + \frac{2\pi}{3}\right) & \sin\left(-\theta_{1,2} - \frac{2\pi}{3}\right) \end{vmatrix} \begin{vmatrix} V_a \\ V_b \\ V_c \end{vmatrix} \quad (10.69)$$

The transformation in synchronous coordinates, valid for stator and rotor, is

$$\begin{aligned} \theta_1 &= \int \omega_1 dt = \omega_1 t \quad \text{for constant frequency stator} \\ \theta_2 &= \int (\omega_1 - \omega_r) dt = \theta_1 - \int \omega_r dt \quad \text{for the rotor with variable speed} \\ \text{So, } \frac{d\theta_2}{dt} &= \omega_1 - \omega_r = S\omega_1 \end{aligned} \quad (10.70)$$

For constant speed ($\omega_r = \text{constant}$),

$$\theta_2 = S\omega_1 t + \delta \quad (10.71)$$

Also, for an infinite power bus

$$\begin{aligned} V_{1abc} &= V_1 \sqrt{2} \cos\left(\omega_1 t - (i-1) \frac{2\pi}{3}\right) \\ V_{2abc} &= V_2 \sqrt{2} \cos\left(\omega_1 t - \delta - (i-1) \frac{2\pi}{3}\right) \end{aligned} \quad (10.72)$$

In (10.72), the rotor phase voltages V_{2abc} are expressed in stator coordinates. This explains the same frequency in V_{1abc} and V_{2abc} ($\theta_1 = 0$, $\theta_2 = -\omega_1 t$). Using (10.72) and (10.71) in (10.69), we obtain

$$V_{1d} = \sqrt{2} V_1; \quad V_{1q} = 0 \quad (10.73)$$

For constant speed.

$$V_{2d} = \sqrt{2} V_2 \cos \delta; \quad V_{2q} = -\sqrt{2} V_2 \sin \delta$$

As shown in (10.73), in synchronous coordinates, the DFIG voltages in the stator are D.C. quantities. Under steady-state constant δ , the rotor voltages are also D.C. quantities, as expected. The angle δ may be considered as the power angle and thus DFIG operates as a synchronous machine. The rotor voltage $\sqrt{2} V_2$ and its phase δ with respect to stator voltage in same coordinates (synchronous in our case) and slip S are thus the key factors which determine the machine operation mode (motor or generator) and performance.

The proper relationship between V_2 and δ for various speeds (slips) represents the fundamental question for DFIG operation.

The various phase displacements in (10.71)–(10.72) are shown in Figure 10.22.

The d-q model equations serve to solve most transients in the time domain by numerical methods with $\bar{V}_1$ and $\bar{V}_2$ and prime mover torque T_{PM} as inputs and the various currents and speed as variables.

However to get an insight into the DFIG behaviour, we here investigate steady-state operation and its stability limits.

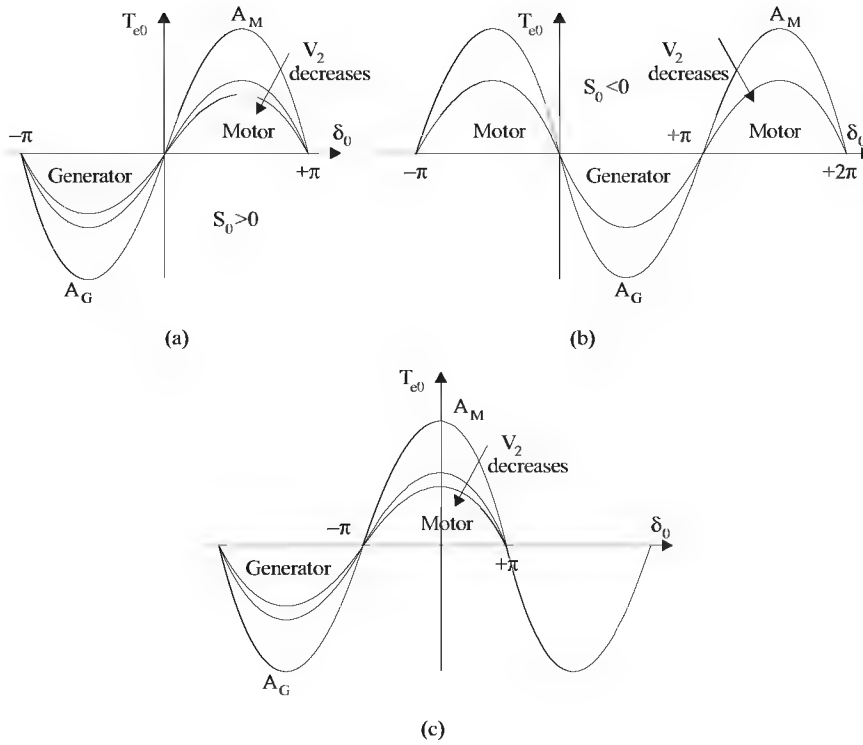


FIGURE 10.23 Ideal torque-angle curves (a) and (b) at nonzero (rather large slips $|S| > 0.1$), $R_1 = R_2 = 0$ and (c) at zero slip ($R_1 = 0$, $R_2 \neq 0$).

The statically stable operation zones occur between points A_M and A_G .

For $S_0 \neq 0$ motor and generator operation is in principle feasible for both super and subsynchronous speeds ($S_0 < 0$ and $S_0 > 0$). However, the complete expression of torque for $S_0 \neq 0$, which includes the resistances, will alter the simple expression of torque shown in (10.76). A better approximation would be obtained for $R_2 \neq 0$ with $R_1 = 0$. For low values of slip S_0 , the rotor resistance is important even for high-power IMs.

After some manipulations, (10.74) and (10.75) yield

$$(T_{e0})_{R_1=0} = 3 \left(\frac{p_1 V_1 V_2}{\omega_1} \right) \left(\frac{L_m}{L_1} \right) \frac{R_2 \cos \delta + S_0 \omega_1 L_{sc} \sin \delta - \frac{V_1}{V_2} R_2 S_0 \frac{L_m}{L_1}}{\left[R_2^2 + (S_0 \omega_1 L_{sc})^2 \right]} \quad (10.78)$$

As expected, when $R_2 = 0$, expression (10.76) is reobtained, and when $S_0 = 0$, expression (10.77) follows from (10.78).

Expression (10.78) does not collapse either for $V_2 = 0$ or for $S_0 = 0$. It only neglects the stator resistances, for simplicity, (Figure 10.24).

For $\delta_0 = 0$, the torque is

$$(T_{e0})_{\delta_0=0, R_1=0} = \frac{3p_1 V_1 V_2}{\omega_1} \frac{\left(\frac{L_m}{L_1} - S_0 \frac{V_1}{V_2} \right)}{R_2^2 + (S_0 \omega_1 L_{sc})^2} \quad (10.79)$$

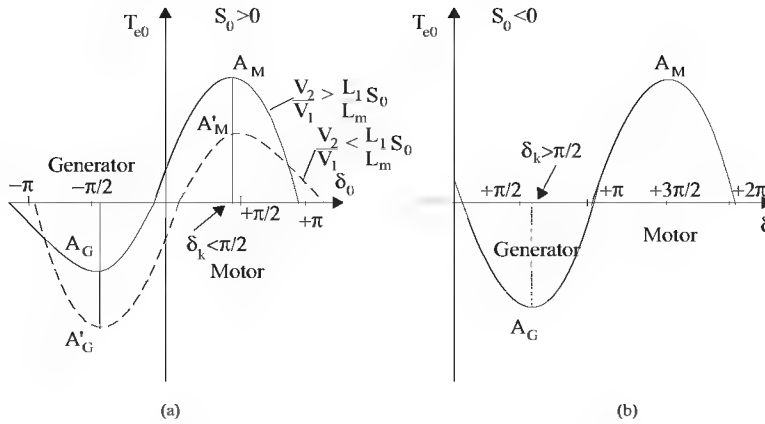


FIGURE 10.24 Realistic torque-angle curves ($R_2 \neq 0$, $R_1 = 0$) (a) $S_0 > 0$ and (b) $S_0 < 0$.

So $(T_{e0})_{\delta_0=0}$ may be either positive or negative

$$\frac{V_2}{V_1} \gtrless \frac{L_1 S_0}{L_m}; \quad (T_{e0})_{\delta_0=0} \gtrless 0 \quad (10.80)$$

For S_0 negative, the situation in (10.80) is reversed. As shown in Figure 10.24, the stability zones of motoring and generating modes are no longer equally large due to rotor resistance R_2 and also due to the ratio V_2/V_1 and the sign of slip $S_0 < 0$.

The extension of the stability zone $A_G - A_M$ in terms of δ_{KG} to δ_{0KM} may be simply found by solving the equation:

$$\frac{\partial T_{e0}}{\partial \delta_0} = 0; \quad -R_2 \sin \delta_K + S_0 \omega_1 L_{sc} \cos \delta_K = 0 \quad (10.81)$$

$$\tan \delta_{K,G,M} = \frac{S_0 \omega_1 L_{sc}}{R_2} \quad (10.82)$$

The lower the value of the rotor resistance, the larger the value of $\delta_{K,G,M}$ and thus the larger the total stability angle zone travel ($A_G - A_M$).

Slightly larger stability zones for generating occur for negative slips ($S_0 < 0$). As the slip decreases, the band of the stability zones also decreases (see (10.82)).

The static stability problem may be approached through the eigenvalues method [20] of the linearized system only to obtain similar results even with nonzero stator resistance.

The complete performance of DFIG may be calculated by directly solving the system (10.74) for currents. Saturation may be accounted for by noting that $L_1 = L_{1l} + L_m$, $L_2 = L_{2l} + L_m$ and $L_m(i_m)$ – magnetization curve – is given, with

$$i_m = \sqrt{i_{dm}^2 + i_{qm}^2}; \quad i_{dm} = i_d + i_{dr}; \quad i_{qm} = i_q + i_{qr}.$$

The core losses may be added also, in both the stator and rotor as slip frequency is not necessarily small.

The stator and rotor power equations are

$$S_1 = \frac{3}{2} (V_d i_{d0} + V_q i_{q0}) + j \frac{3}{2} (-V_d i_{q0} - V_q i_{d0}) \quad (10.83)$$

$$\underline{S}_2 = \frac{3}{2}(\underline{V}_{dr}\underline{i}_{dr0} + \underline{V}_{qr}\underline{i}_{qr0}) + j\frac{3}{2}(-\underline{V}_{dr}\underline{i}_{qr0} - \underline{V}_{qr}\underline{i}_{dr0}) \quad (10.84)$$

In principle, the active and reactive power in the stator or rotor may be positive or negative depending on the capabilities of the PEC connected in the rotor circuit. Absorbed active and reactive power are positive in (10.83) and (10.84) as the d-q model is written for motoring association of signs.

Advanced vector control methods with decoupled active and reactive power control of DFIG have been proposed recently ([22], Chapter 4, Vol. 2).

Slip recovery systems, which use unidirectional power flow power electronics converters in the rotor, lead to limited performance in terms of power factor, voltage and current harmonics, and control flexibility. Also, they tend to have equivalent characteristics different from those discussed here and are well documented in the literature [23]. Special doubly fed IMs – with DSWs (p_1 and p_2 pole pairs) and nested-cage rotor with $p_1 + p_2$ poles – have been rather recently proposed [24]. Also DSWs with p_1 and p_2 pole pairs and a regular cage-rotor may be practical. The lower power winding is inverter fed. With the wind energy systems on the rise and low-power hydropower plants gaining popularity, the IG systems with variable speed are expected to have a dynamic future in the 1–5 MVA range with cage rotors and up to tens and hundreds of MVA with wound rotors [25,26].

10.13 DFIG SPACE-PHASOR MODELLING FOR TRANSIENTS AND CONTROL

The space-phasor equations (10.68) can be written in terms of stator and rotor flux linkages $\bar{\mathbf{i}}_s$ and $\bar{\mathbf{i}}_r$:

$$\begin{aligned} \bar{\Psi}_s &= \bar{\Psi}_m + L_{sl}\bar{\mathbf{i}}_s; \quad \bar{\Psi}_m = L_m(\bar{\mathbf{i}}_s + \bar{\mathbf{i}}_r); \quad L_{mf} = \frac{\partial \bar{\Psi}_m}{\partial L_m} \\ \bar{\Psi}_r &= \bar{\Psi}_m + L_{rl}\bar{\mathbf{i}}_r; \quad \bar{\mathbf{i}}_m = \bar{\mathbf{i}}_s + \bar{\mathbf{i}}_r; \quad \bar{\mathbf{i}}_s = \bar{\mathbf{i}}_1, \bar{\mathbf{i}}_r = \bar{\mathbf{i}}_2 \\ \bar{\mathbf{V}}_s &= \bar{\mathbf{V}}_1, \bar{\mathbf{V}}_r = \bar{\mathbf{V}}_2; \quad \mathbf{R}_s = \mathbf{R}_1, \mathbf{R}_r = \mathbf{R}_2; \quad L_1 = L_{sl} + L_m; \quad L_2 = L_{rl} + L_m \end{aligned} \quad (10.85)$$

where $s = p$ = Laplace operator.

Consequently, equations (10.68) become

$$\begin{aligned} (\mathbf{R}_s + (p + j\omega_b)L_{sl})\bar{\mathbf{i}}_s + \bar{\mathbf{V}}_s &= -L_{mf}p(\bar{\mathbf{i}}_s + \bar{\mathbf{i}}_r) - j\omega_b L_m(\bar{\mathbf{i}}_s + \bar{\mathbf{i}}_r) \\ (\mathbf{R}_r + p + j(\omega_b - \omega_r)L_{rl})\bar{\mathbf{i}}_r + \bar{\mathbf{V}}_r &= -L_{mf}p(\bar{\mathbf{i}}_s + \bar{\mathbf{i}}_r) - j(\omega_b - \omega_r)L_m(\bar{\mathbf{i}}_s + \bar{\mathbf{i}}_r) \end{aligned} \quad (10.86)$$

By using (10.85) in (10.86) to eliminate the currents, we end up with two space-phasor equations with $\bar{\Psi}_s$ and $\bar{\Psi}_r$ as state variables:

$$\begin{aligned} \tau'_s \frac{d\bar{\Psi}_s}{dt} + (1 + j\omega_b\tau'_s)\bar{\Psi}_s &= -\tau'_s\bar{\mathbf{V}}_s + k_s\bar{\Psi}_r \\ \tau'_r \frac{d\bar{\Psi}_r}{dt} + (1 + j(\omega_b - \omega_r)\tau'_r)\bar{\Psi}_r &= -\tau'_r\bar{\mathbf{V}}_r + k_s\bar{\Psi}_s \\ \tau'_s &= \frac{L_s}{\mathbf{R}_s}\sigma; \quad \tau'_r = \frac{L_r}{\mathbf{R}_r}\sigma; \quad \sigma = \left(1 - \frac{L_m^2}{L_s L_r}\right); \quad k_s = \frac{L_m}{L_s}; \quad k_r = \frac{L_m}{L_r} \end{aligned} \quad (10.87)$$

k_s and k_r vary in general in the 0.91–0.96 interval.

The structural diagram shown in Figure 10.25 illustrate Equations (10.87).

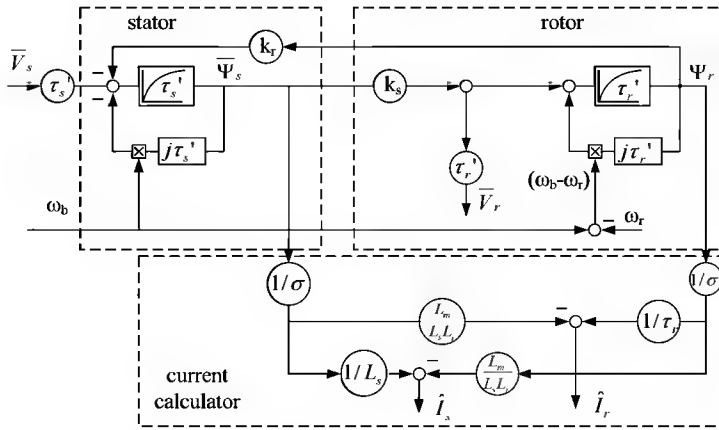


FIGURE 10.25 DFIG structural diagram with $\bar{\Psi}_s$ and $\bar{\Psi}_r$ as space-phasor state variables.

The structural diagram reveals two small time constants τ_s' and τ_r' : one related to stator and the other to rotor.

As expected, by keeping one of the flux linkage amplitudes constant, the number of time constants is reduced by one; this is the key of field-oriented control (FOC).

Decomposing the space-phasor equations (10.86) in d-q components and assuming $\bar{\Psi}_s = \Psi_s = \Psi_d$; $\Psi_q = 0$ and $\frac{d\Psi_q}{dt} = 0$ (stator flux orientation), the stator equation becomes

$$\begin{aligned} i_d R_s + V_d &= -\frac{d\Psi_d}{dt}; \\ i_q R_s + V_q &= -\omega_r \Psi_d \end{aligned} \quad (10.88)$$

Now, with the stator flux considered constant (for grid normal operation), $\frac{d\Psi_d}{dt} = 0$ and with $R_s = 0$,

$$V_d \approx 0, V_q \approx -\omega_r \Psi_d \quad (10.89)$$

Consequently, the active and reactive stator powers P_s and Q_s , in d-q variables are

$$\begin{aligned} P_s &= 3(V_d i_d + V_q i_q) = \frac{3}{2} V_q i_q = \frac{3}{2} \omega_l \Psi_d \frac{L_m i_{qr}}{L_s}; \\ Q_s &= 3(V_d i_q - V_q i_d) = \frac{3}{2} \omega_l \Psi_d i_d = \frac{3}{2} \omega_l \frac{\Psi_d}{L_s} (\Psi_d - L_m i_{dr}) \end{aligned} \quad (10.90)$$

Using the rotor equation in (10.86) yields, for steady state,

$$\begin{aligned} V_{dr0} &= R_r i_{dr} + L_{sc} S \omega_l i_{qr}; \quad S = 1 - \frac{\omega_r}{\omega_l} \\ V_{qr0} &= -R_r i_{qr} - S \omega_l \left(\frac{L_m}{L_s} \Psi_d + L_{sc} i_{dr} \right) \end{aligned} \quad (10.91)$$

Equations 10.90 express the essence of FOC in the sense that the stator active power P_s is controlled by the q axis rotor current I_{qr} , whereas stator reactive power Q_s is controlled by the d axis rotor current i_{dr} .

10.14 REACTIVE-ACTIVE POWER CAPABILITY OF DFIG

As well known, D.C.-excited synchronous generators are capable to “produce” and “consume” notable reactive power in order to provide for voltage amplitude regulation in A.C. power grids. Once DFIGs are connected to the power grid (Figure 10.26), their reactive power capabilities are important too [27].

The currently installed WT-DFIGs range from 1.5 to 6 MW, 1000–2000rpm (four-pole DFIGs), and 650–1350rpm (six pole DFIGs) with a slip range of -0.35 to 0.35 p.u., nominal slip: -0.15 to -0.2 p.u., voltage range of 0.9 – 1.1 p.u., and power factor 0.9 leading/lagging.

The rotor side converter (RSC) and line side converter (LSC) (Figure 10.25) may share the reactive current (power) contribution, both “extracting” the reactive power from the properly sized D.C. link capacitor.

While the LSC “produces/consumes” reactive power at grid frequency ω_1 (constant so far), the RSC “produces” reactive power to magnetize the DFIG, and more, at slip frequency ω_2 , which is then “magnified” in the DFIG stator at stator frequency $|\omega_1| \gg |\omega_2|$. But then, the LSC acts directly on power grid and RSC through DFIG with its losses.

Due to limited RSC ratings, the rotor current of DFIG is limited. But so is the LSC current. The two may have different kVA ratings.

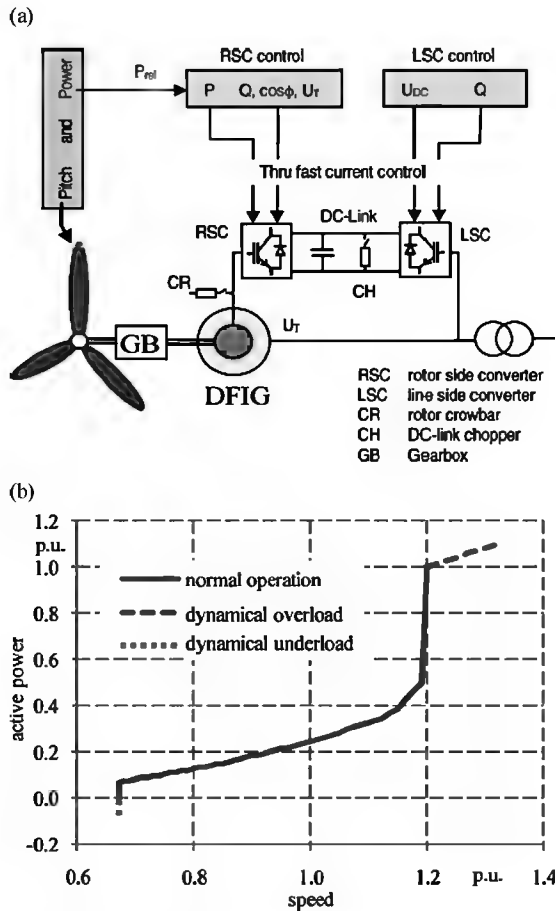


FIGURE 10.26 Wind turbine (WT) driven DFIG connected to the power grid (a) and WT active power versus speed in p.u. values (b).

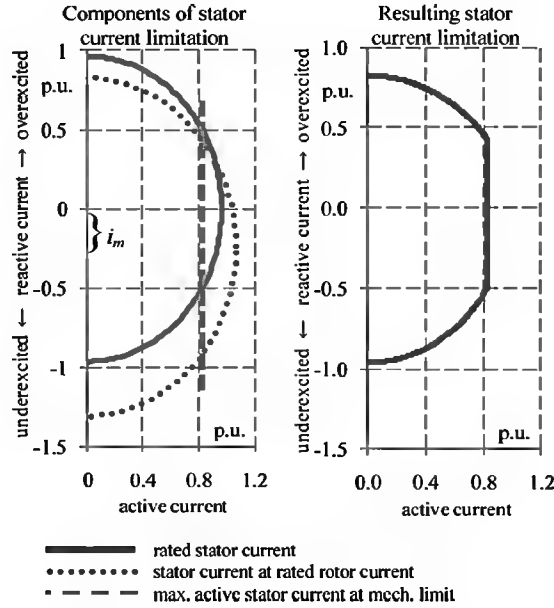


FIGURE 10.27 Typical DFIG reactive-active current capability. (After Ref. [27].)

Figure 10.27 [27] shows the DFIG reactive-active current capability for rated stator current, without considering the machine losses, but a rated active stator current of 0.8 p.u.

The LSC has to supply the slip active power first ($i_{Lsc \text{ active}}$) to the RSC and then eventually to “produce/consume” some reactive power ($i_{Lsc \text{ reactive}}$) for a rated value $i_{Lsc \text{ rated}}$.

In general, the maximum L_{sc} current is around 0.2 p.u. while L_{sc} active current increases and the maximum L_{sc} reactive current decreases with stator active current.

Finally, the combined reactive-active currents and powers are summarized in Figure 10.28a–c [27].

In the dynamic speed range, lower reactive power may allow increased active power (Figure 10.28b and c); however, the DFIG with its two converters RSC and LSC may regulate independently the active and reactive power. When the WT has to run in lower noise mode (around DFIG synchronism), the thermal limitation on static power switches in RSC implies a current (power) reduction.

Also, low grid frequency implies large slip and thus large rotor voltage which may not always be realized by the D.C. link voltage limitation.

Moreover, the operation of DFIG at variable stator frequency (ω_1) – proposed to reduce the max slip (S_{max}) and thus reduce the RSC and LSC ratings to as low values as 0.05 p.u. – leads to a voltage “deficit” that has to be corrected by additional power electronics means or DFIG measures. Stand-alone DFIG with diode rectifiers for D.C. output faces this problem.

Finally, the use of direct ac–ac PWM static converters to DFIG rotor circuit (or to the stator of CRIG) such as matrix converters demand a special treatment in terms of their reactive power capability.

10.15 STAND-ALONE DFIGS

DFIGs have been rather early proposed for Diesel engine-driven autonomous generators with constant frequency and voltage for a limited speed range due to its RSC + LSC converters reduced kVA, despite the problems of mechanical brushes used to transmit the slip power to the rotor [29–31] (Figure 10.29a–c).

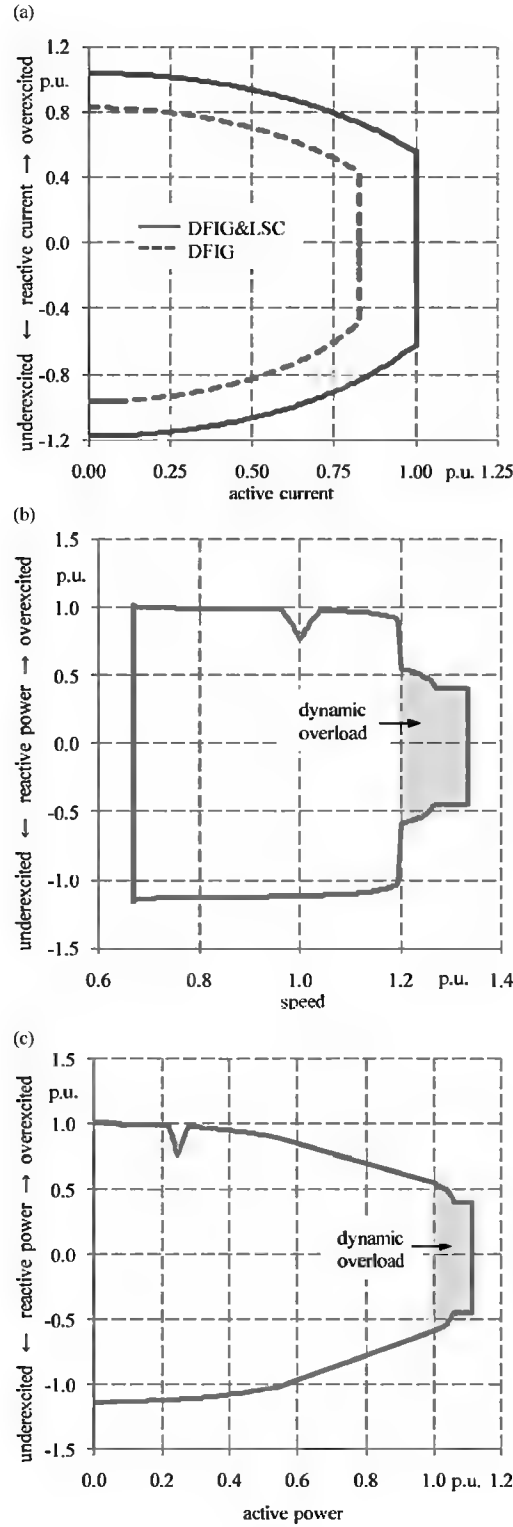


FIGURE 10.28 Combined (DFIG + L_{sc}) reactive/active currents and powers of DFIG: (a) currents and powers with optimal PWM but without (b) and (c) thermal limitations in RSC around synchronism. (After Ref. [27].)

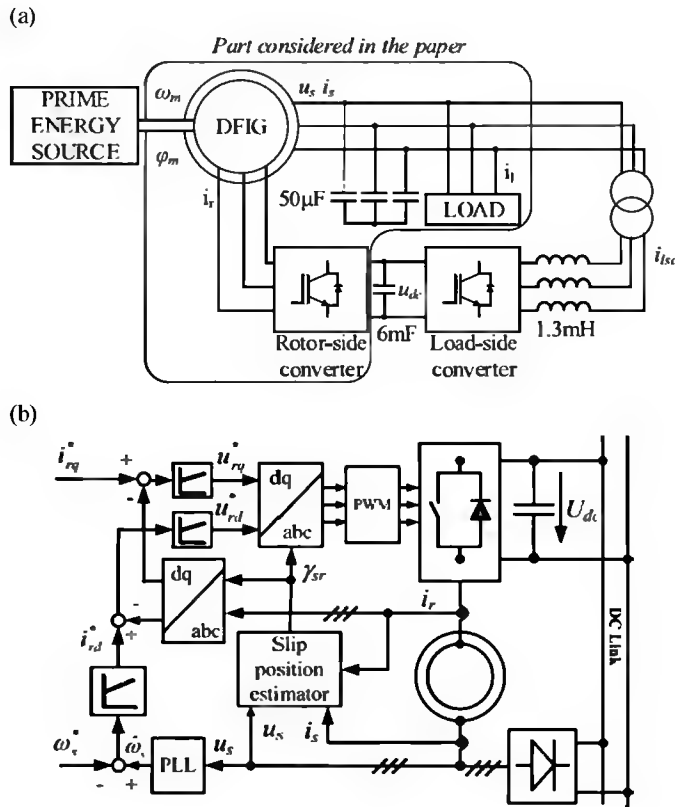


FIGURE 10.29 Stand-alone DFIG: (a) with A.C. output (after Ref. [30]) and (b) with D.C. output (after Ref. [32]).

The two basic schemes for stand-alone DFIG may be characterized by

- Both schemes based on DFIG self-excitation remnant emf and machine saturation ($L_m(i_m)$) are required, together with limited rotor acceleration at start.
- Both schemes are designed in general to operate with constant stator frequency ω_1 and variable (with speed ω_r) slip (rotor) frequency ω_2 ; however, in both by special control ω_1^* may be modified but observing the requirement for constant output A.C. or D.C. voltage.
- Both schemes are suitable for small- and medium-power wind parks and for avionics and ship generator systems at variable speed but constant output voltage.
- For the D.C. output scheme (Figure 10.29b), a full-power diode rectifier is used, which on one side reduces the costs but on other side introduces notable stator current harmonics and requests a unity power factor in the stator. A 6 phase stator winding with a dual 3 phase diode rectifier might be considered for the situation.
- Allowing for variable frequency ω_1 in the stator will limit the kVA ratings of the PWM converters in the schemes allowing for larger speed range (2.5 to 1 as needed in avionics) and constant voltage control.

Results shown in Figure 10.30, both with sensorless control, from [30,32] illustrate the stand-alone DFIG remarkable dynamic capabilities.

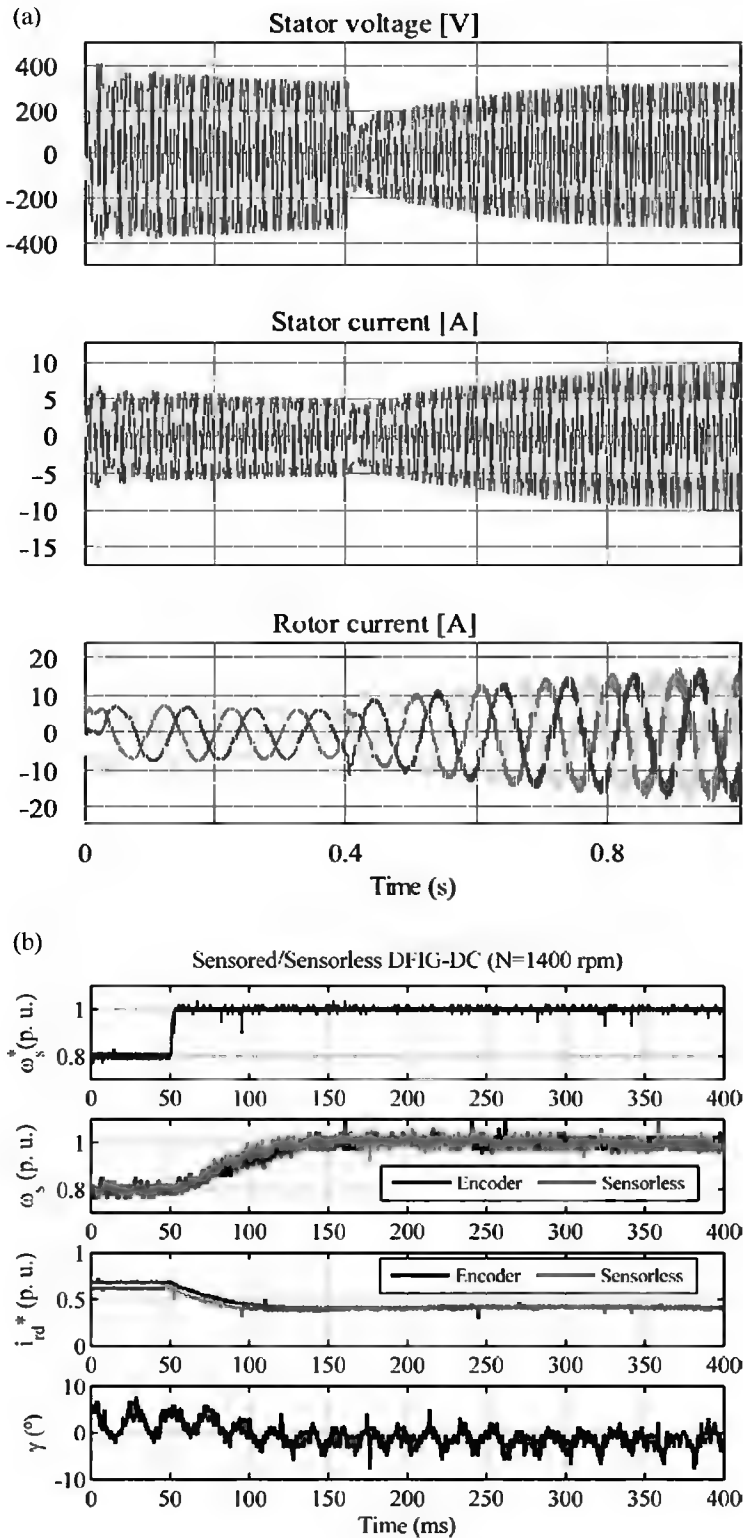


FIGURE 10.30 Stand-alone DFIG transients: (a) step load with nonlinear load (after Ref. [30]) and (b) step stator frequency transients (after Ref. [32]).

10.16 DSW CAGE AND NESTED-CAGE ROTOR INDUCTION GENERATORS

There are quite a few configurations for DSW, all intended to eliminate the rotor mechanical brushes (Figure 10.31).

The two main configurations in Figure 10.31 are

- DSW ($p_1 = \text{or } \neq p_2$) cage-rotor IGs [33–37]: DSW-CRIG
- DSW ($p_1 \neq p_2$) loop-cage $p_1 + p_2$ pole rotor IG: so-called brushless BDFIG (M) [38–40].

In general, the DSW-CRIG [35–37, 45] relies – when $p_1 = p_2$ – on the main field interaction, and thus, the control winding may be used fully but the speed of the rotor is close to ω_1/p_1 and the slip frequency is small.

In contrast, the brushless doubly fed induction machine (BDFIM) with $p_1 \neq p_2$ pole pairs of stator windings and $p_1 + p_2$ loop-cage poles rotor couples the two stator winding through a harmonic

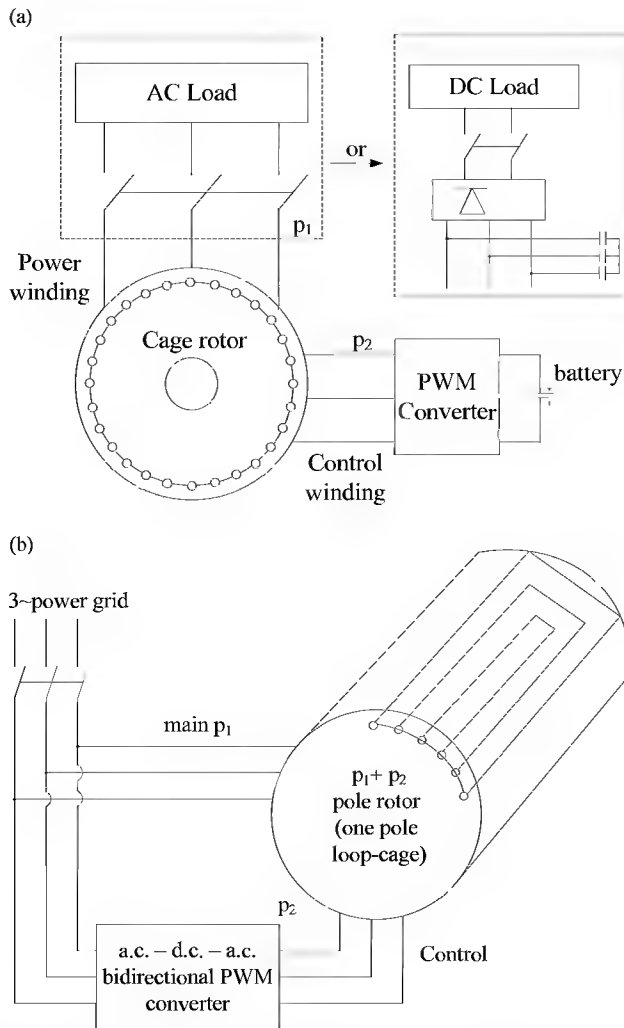


FIGURE 10.31 DSW cage (loop-cage) IMs: (a) with standard cage rotor and A.C. output and D.C. output (battery storage) and A.C. or D.C. main power output ($p_1 = \text{or } \neq p_2$), (b) with $p_1 \neq p_2$ DSW and $p_1 + p_2$ pole rotor with loop cage.

(whose maximum winding factor in the winding is around 0.5) which results in smaller torque density. However, the synchronous (operation) speed $\omega_r = (\omega_1 + \omega_2)/(p_1 + p_2)$ is reduced notably and thus the machine is usable in low speed drives, primarily.

To illustrate the performance of such machines in generator mode, Figure 10.32 shows the output power (P_1 , P_2 , P_{total}), stator currents and generator efficiency versus machine speed for a DSW-CRIG

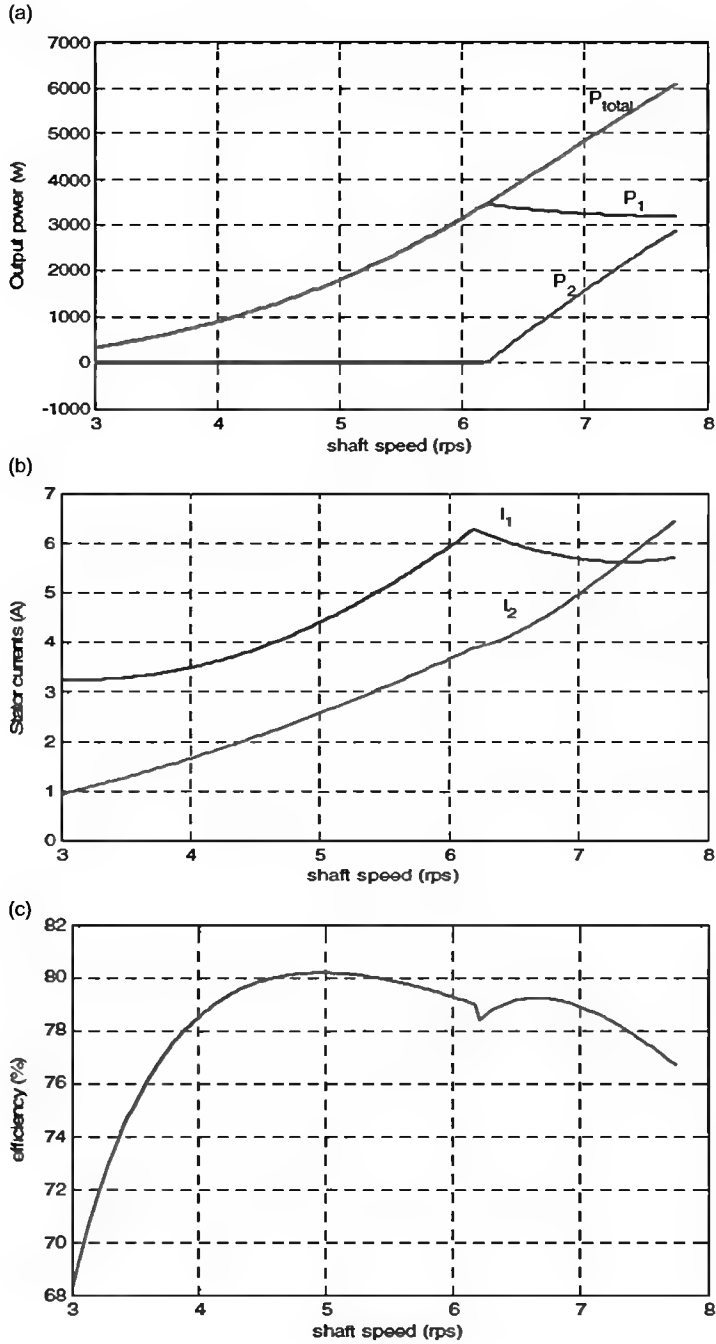


FIGURE 10.32 DSW-CRIG with “diode rectifier” main winding and 50% sizing of each winding: (a) powers, (b) currents, (c) efficiency versus rotor speed in rps. (After Ref. [37].)

with diode rectifier after the main winding and 50% rating of the control winding PWM converter capable to maintain rather constant D.C. output voltage (eight poles: turns ratio: $R_{21} = 24/31$). For $f_1 = 23\text{--}27.5\text{ Hz}$ and $f_2 < 3\text{ Hz}$, the D.C. voltage and power winding current vary little [37].

As the speed may not vary more than 15% to secure reasonable efficiency, the DSW-CRIG may be practical mostly with diesel engine power generation where such a speed range is enough to reduce fuel consumption (or pollution).

On the other hand, the BDFIG may handle large speed variations while reducing the speed (by its topology: $p_1, p_2, p_1 + p_2$) (Figure 10.33) [40].

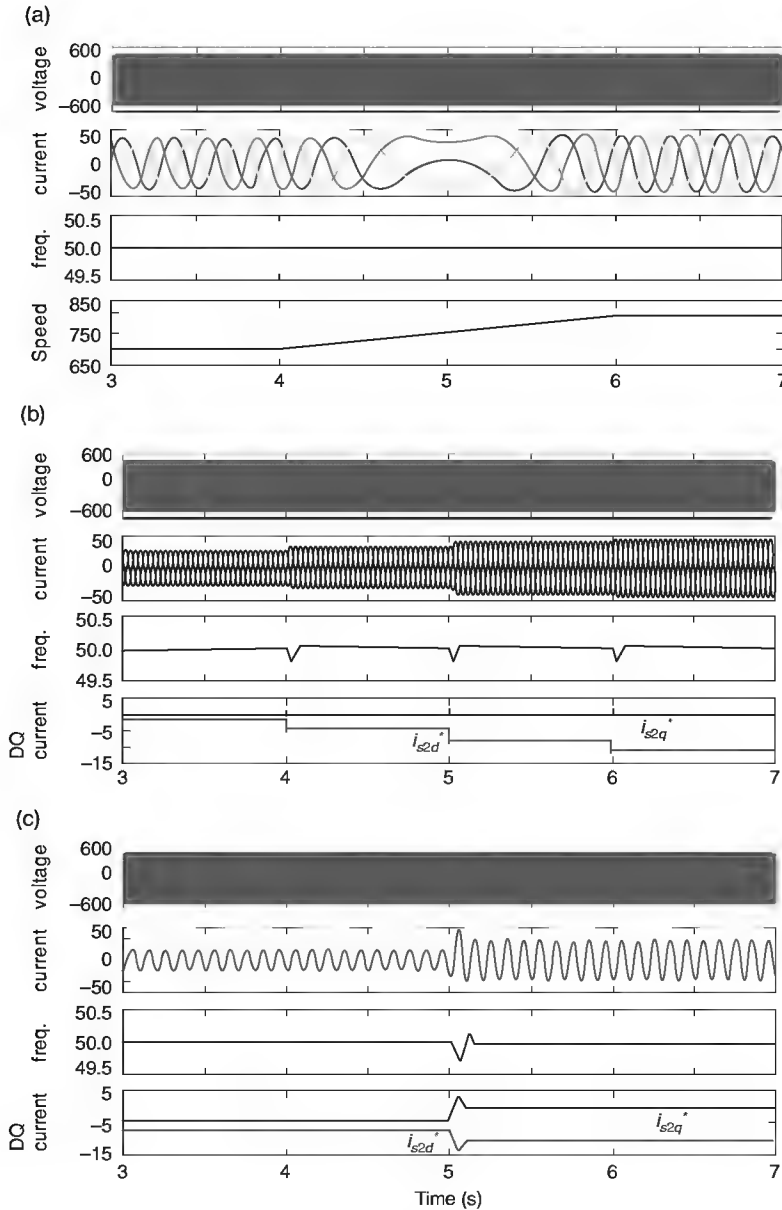


FIGURE 10.33 BDFIG transients: (a) PW voltage for rotor speed changes, (b) load multiple step increase (unity power factor), and (c) transients for 5 kW induction motor load. (After Ref. [40].)

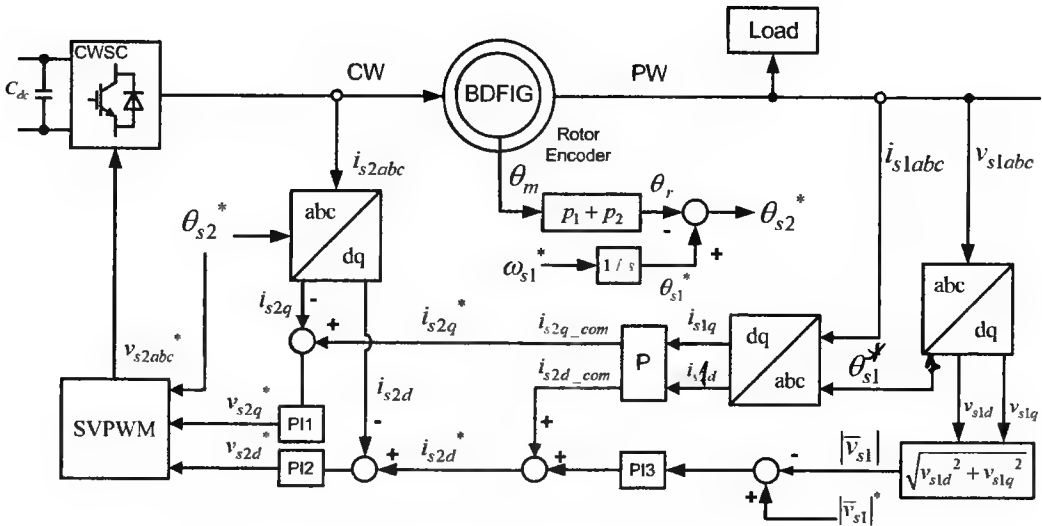


FIGURE 10.34 BDFIG FOC for stand-alone operation. (After Ref. [40].)

A few remarks are in order

- The speed ramping in Figure 10.33a goes through synchronism ($\omega_2 = 0$) when the sequence of control winding current (from under-synchronous to over-synchronous operation).
- The voltage and frequency f_1 of PW are maintained rather constant during the speed ramping (the prototype has $p_1 = 1$, $p_2 = 3$).
- The control winding (CW) frequency $\omega_2(f_2)$ is to be considered negative for under-synchronous operation $\Omega_r = \frac{\omega_1 + \omega_2}{p_1 + p_2}$ and positive for over-synchronous operation; this is in contrast with DFIG because the CW is now placed in the stator.
- Consequently, the CW power is added to PW power under synchronism ($\omega_2 = 0$) and subtracted from the latter for over synchronism; this tends to improve efficiency somewhat at lower speeds but reduce it somewhat at higher speeds.
- The machine efficiency is smaller than that of a conventional IM of same size and so are its ratings (the IM in same frame size D250 is rated at 25 kW, not 10 kW, for example).
- The basic FOC scheme is illustrated in Figure 10.34.
- As shown in Figure 10.34, the reference CW d-q angle θ_{s2}^* is

$$\theta_{s2}^* = \int \omega_{s1}^* dt + \theta_m (p_1 + p_2); \theta_{s1}^* = \int \omega_{s1}^* dt \quad (10.92)$$

where ω_{s1}^* – PW reference (output) frequency and θ_m – rotor mechanical position (measured or estimated).

- The d-q reference currents of CW i_{s2q}^* and i_{s2d}^* are obtained, respectively, from measured d-q PW current i_{s1q} and from a combination of the output of the V_{s1} regulator plus a feedforward contribution of i_{s1d} multiplied by a gain P.

10.17 DFIG WITH DIODE-RECTIFIED OUTPUT

DFIGs may be used in stand-alone applications with a diode-rectified constant (regulated) D.C. output voltage for variable speed in a minimal PEC cost scheme [42] (Figure 10.35).

In essence, in this case the stator frequency varies with speed to maintain small enough slip frequency (in the rotor) and constant D.C. output voltage by adequate control, without increasing the pulse width modulation voltage source converter (PWM VSC) ratings above 20%–30% for a 2.5 to 1 speed range for a 2 to 1 stator frequency variation range.

Typical results for such a system control case study at 2 kW output power (Figure 10.36) show such capability with <20% kVA ratings of the rotor connected voltage source converter (VSC) [43].

Also the control may improve the efficiency by selecting an optimal stator frequency for given speed and load power (Figure 10.37) [43], again for limited kVA rating of the converter.

Variable stator frequency DFIG control was also proposed in large-power wind generators A.C. connected in parallel and then through a voltage boost transformer to a full-power VSC (rectifier) as the sending end of a M(H) VDC transmission system [44]. The DFIG-PWM A.C.–D.C.–D.C. converter rating was reduced to 5%, but the deficit of D.C. stator output voltage at low speeds remained to be compensated by the full-power VSC rectifier planted as interface to M(H) VDC transmission system.

A further simplified scheme for M(H) VDC transmission with a <30% rating A.C.–D.C.–A.C. PWM converter of DFIG, but with a voltage boosting transformer and a full-power diode rectifier, may provide direct M(H) VDC interface with controlled constant D.C. output voltage. This way, the cost of power electronics seems to be reduced drastically (Figure 10.38).

All controls are exercised through the partial rating (<30%) A.C.–D.C.–D.C. PWM converter, while A.C. paralleling of DFIGs in a wind park is eliminated.

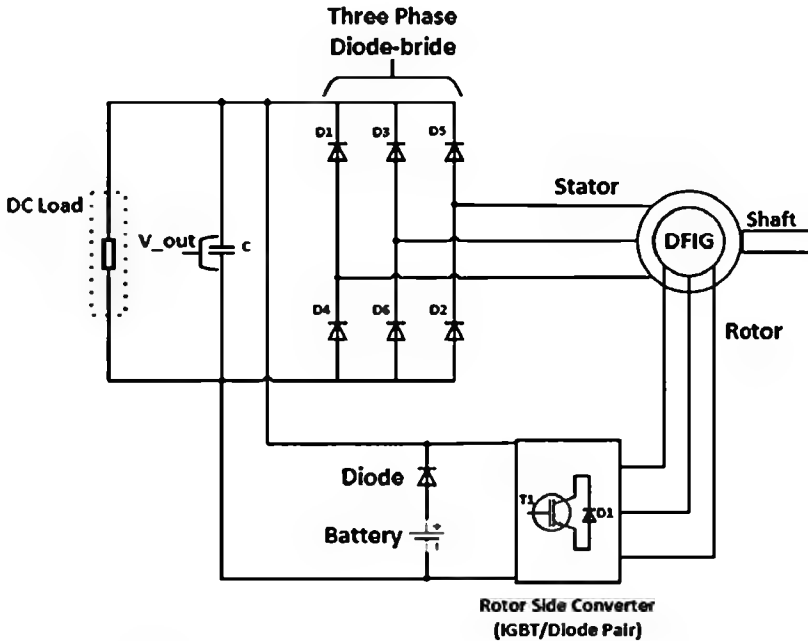


FIGURE 10.35 Stand-alone DFIG with diode-rectified output and single PWM VSC connected to the rotor.

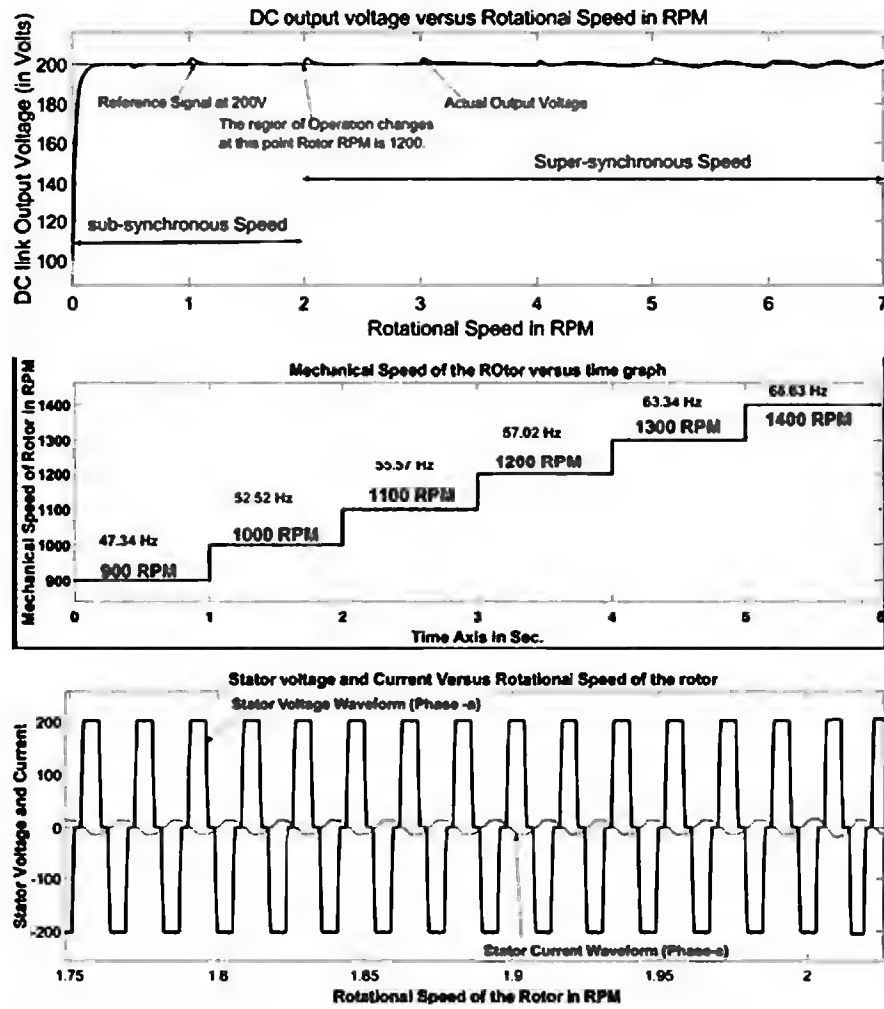


FIGURE 10.36 DC link voltage control at 2 kW, from 900–1400rpm; D.C. output voltage, rotor speed, stator voltage, and current. (After Ref. [43].)

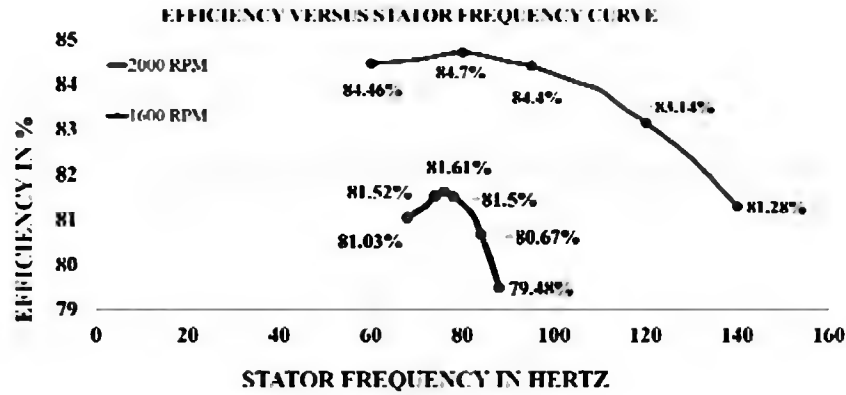


FIGURE 10.37 DFIG efficiency versus stator frequency at 1600–2000rpm for 2kW D.C. output power. (After Ref. [43].)

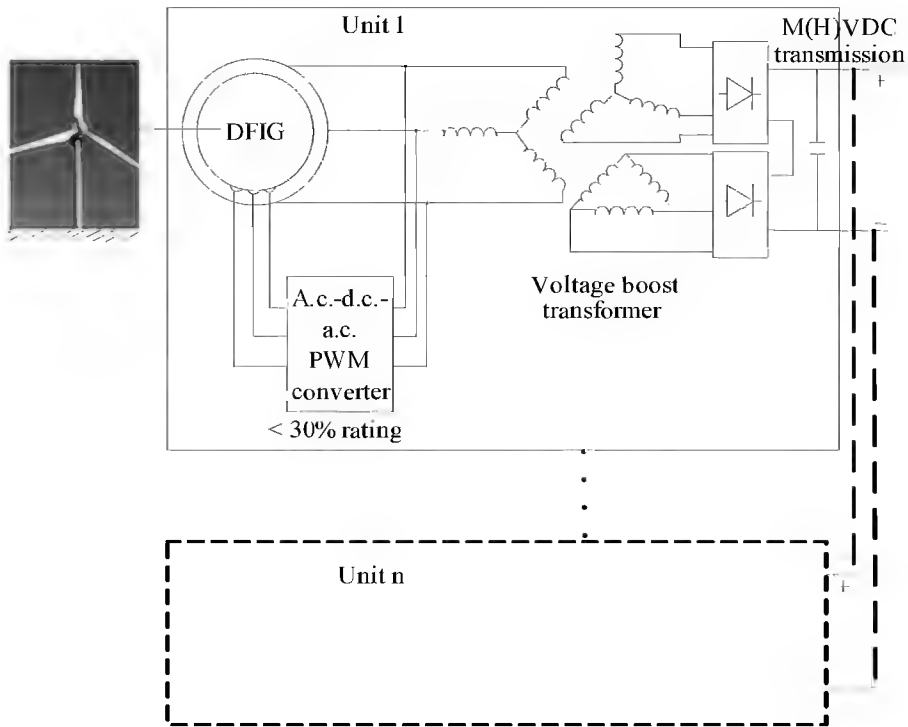


FIGURE 10.38 DFIG with variable stator frequency control and voltage boosting transformer plus full-power diode rectifier as sending end of M(H) VDC power transmission systems.

Behind the obvious simplicity and cost reduction may hide demerits in the control that need to be carefully examined (“the devil is in the details”).

10.18 SUMMARY

- Three-phase IMs are used as generators both in wound rotor (doubly fed) and in cage-rotor configurations.
- IGs may work in isolation or grid-connected.
- IGs may be used in constant or variable speed, constant or variable frequency, and constant or variable output voltage systems [28].
- As a rule, less frequency-sensitive isolated systems use cage-rotor IGs with self-excitation (SEIG).
- Grid-connected systems use both cage-rotor IGs (CRIGs) or doubly fed (wound rotor) IGs (DFIGs).
- SEIGs use parallel capacitors at the terminals. For a given speed IG and a given no-load voltage, there is a specific value of the capacitor that can produce it. Magnetic saturation is a must in SEIGs.
- When SEIGs with constant speed prime mover are loaded, the frequency and voltage decrease with load. The slip (negative) increases with load.
- The basic problem for steady-state operation of SEIG starts with a given frequency, load, IG parameters, and self-excitation capacitance to find the speed (slip), voltage, currents, power, efficiency, etc.
- Impedance and admittance methods are used to study SEIG steady state. While the impedance methods lead to 4(5)th-order equations in S (or X_m), an admittance method

that leads to a second-order equation in slip is presented. This greatly simplifies the computational process as X_m (the magnetization reactance) is obtained from a first-order equation.

- There is a minimum load resistance which still allows SEIG self-excitation which is inversely proportional to the self-excitation capacitance C (10.22).
- The ideal maximum power slip is $S_{\max} = -R_2/(fX_{2l})$, with rotor resistance R_2 and leakage reactance X_{2l} at rated frequency and stator frequency f in p.u. This expression is typical for constant airgap flux conditions in vector-controlled IM drives.
- The SEIG steady-state curves are voltage/current, voltage/power, and frequency/power for given speed and self-excitation capacitance.
- SEIG parameter sensitivity studies show that voltage and maximum output power increase with capacitance.
- For no load, the minimum capacitor that can provide self-excitation is inversely proportional to speed squared.
- When feeding an IM from a SEIG, in general the rating of SEIG should be two to three times the motor rating though, for pump-load starting, even 1.6 times overrating will do. To avoid high overvoltages when the IM load is disconnected, the excitation capacitor may be split between the IM and the SEIG.
- Pole changing stator SEIGs are used for wide speed range harnessing of SEIG's prime mover energy more completely (wind turbines, for example).
- Pole changing winding design has to maintain large enough airgap flux densities that saturate the cores to secure self-excitation over the entire design speed range.
- Unbalanced operation of SEIGs may be analysed through the method of symmetrical components and is basically similar to that of balanced load, but efficiency is notably lower and the phase voltages may differ from each other notably. When one line current is zero, the single capacitor no-load curve of IG should be considered.
- Transients of SEIG occur for voltage build-up or load perturbation. They may be investigated by the d-q model. At short circuit, for example, with IM load, the SEIG receives for a few periods 2 p.u. voltage and up to 5 p.u. current transients before the voltage collapses.
- Parallel operation of SEIGs in isolated systems makes use of a single but variable capacitor to control the voltage for variable load.
- The slip of all SEIGs in parallel has to be negative to avoid motoring. So the speeds (in p.u.) have to be very close to each other.
- Full usage of DFIG occurs when it allows bi-directional power flow in the rotor circuit.
- Generating is thus feasible to both sub- and overconventional synchronous speed. Separate active and reactive power control may be commanded.
- The study of DFIG requires the d-q model which allows for an elegant treatment of steady state with rotor voltage V_2 and voltage power angle δ (angle between $\bar{V}_1$ and $\bar{V}_2$ in synchronous coordinates) as variables and no-load slip S_0 as parameter. The DFIG works as a synchronous machine with constant stator voltage V_1 and frequency. The rotor voltage A.C. system has the slip frequency. Thus, variable speed is handled implicitly.
- The stability angle δ_K extension increases with slip and leakage reactance and decreases with rotor resistance. Also V_2/V_1 (voltage ratio) plays a crucial role in stability. Constant $(V_2/S\omega_r)$ ratio – constant rotor flux – tends to hold the peak torque less variable with slip and thus enhances stability margins.
- Eigenvalues of the linearized d-q model may serve well to investigate the DFIG stability boundaries. Vector control with separate active and reactive rotor power control change the behaviour of the DFIG for the better in terms of both steady state and transients ([25], Chapter 4, Vol. 2).

- New topologies such as dual pole stator winding (p_1, p_2 pole pairs) with $p_1 + p_2$ rotor cage nests [23] and conventional cage rotors [24] have been proposed. The IG subject seems far from exhausted.
- New control schemes of 1 phase SEIGs in stand-alone applications are proposed to reduce costs but preserve performance [41] – no speed governor but controlled ballast load for constant frequency and some voltage regulation with load.

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11 Single-Phase Induction Generators

11.1 INTRODUCTION

Small, portable or fixed, single-phase generators are built for up to 10–20 kW.

Traditionally, they use a synchronous single-phase generator with rotating diodes.

Self-excited, self-regulated single-phase induction generators (IGs) provide, in principle, good voltage regulation, more power output/weight, and more sinusoidal output voltage.

In some applications, where tight voltage control is required, power electronics may be introduced to vary the capacitors “used” by the induction machine (IM). Among the many possible configurations [1,2], we investigate here only one, which holds a high degree of generality in its analysis and seems very practical in the same time (Figure 11.1).

The auxiliary winding is connected over a self-excitation capacitor C_{ea} and constitutes the excitation winding.

The main winding has a series connected capacitor C_{sm} for voltage self-regulation and delivers output power to a given load.

With the power (main) winding open, the IG is rotated to the desired speed. Through self-excitation (in the presence of magnetic saturation), it produces a certain no-load voltage at main terminals. To adjust the no-load voltage, the self-excitation capacitor C_{ea} may be changed accordingly, for a given IG.

After that, the load is connected, and the main winding delivers power to the load.

The load voltage/current curve depends on the load impedance and its power factor, speed, IG parameters, and the two capacitors C_{ea} and C_{sm} . Varying C_{sm} , the voltage regulation may be reduced to desired values.

In general, increasing C_{sm} tends to increase the voltage at rated load, with a maximum voltage in between. This peak voltage for intermediate load may be limited by a parallel saturable reactor.

To investigate the steady-state performance of single-phase IGs, the revolving theory seems to be appropriated. Saturation has to be considered as no self-excitation occurs without it. On the other hand, to study the transients, the d-q model, with saturation included, as shown in Chapter 2, Vol. 2, may be used.

Let us deal with the steady-state performance first.

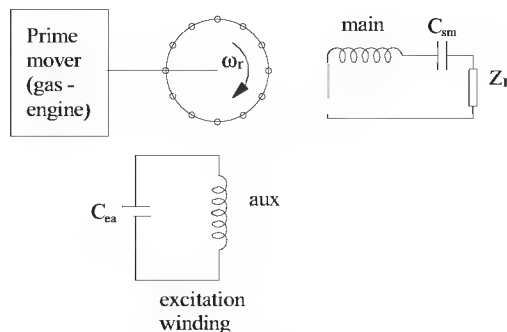


FIGURE 11.1 Self-excited self-regulated single-phase induction generator.

11.2 STEADY-STATE MODEL AND PERFORMANCE

Examining carefully the configuration shown in Figure 2.1, we notice that

- The self-excitation capacitor may be lumped in series with the auxiliary winding whose voltage is then $V_a = 0$.
- The series (regulation) capacitor C_{sm} may be lumped into the load:

$$\underline{Z}'_L = \underline{Z}_L - \frac{jX_c}{F} \quad (11.1)$$

where F is the P.U. frequency with respect to rated frequency. In general,

$$\underline{Z}_L = R_L + jX_L \cdot F \quad (11.2)$$

Now with $V_a = 0$, the forward and backward voltage components, reduced to the main winding, are ($V_a = 0$, $V_m = V_s$)

$$V_{m+} = V_{m-} \quad (11.3)$$

$$V_{m+} = V_s/2 = -\underline{Z}_L (I_{m+} + I_{m-})/2 \quad (11.4)$$

Equation (11.4) can be written as

$$\begin{aligned} \underline{V}_{AB} = V_{m+} &= -\frac{\underline{Z}'_L}{2} (I_{m+} + I_{m-}) = -\underline{Z}'_L I_{m+} + \frac{\underline{Z}'_L}{2} (I_{m+} - I_{m-}) \\ \underline{V}_{BC} = V_{m-} &= -\frac{\underline{Z}'_L}{2} (I_{m+} + I_{m-}) = -\underline{Z}'_L I_{m-} - \frac{\underline{Z}'_L}{2} (I_{m+} - I_{m-}) \end{aligned} \quad (11.5)$$

Consequently, it is possible to use the equivalent circuit shown in Figure 11.5 with $\underline{Z}'_L$ in place of both $\underline{V}_{m+}$ ($\underline{V}_{AB}$) and $\underline{V}_{m-}$ ($\underline{V}_{BC}$) as shown in Figure 11.2. Notice that $-\underline{Z}'_L/2$ also enters the circuit, flowed by $(I_{m+} - I_{m-})$, as suggested in Equations (11.5).

All parameters shown in Figure 11.2 have been divided by the P.U. frequency F . U is p.u. speed.

Denoting

$$\begin{aligned} \underline{Z}_{lmL} &= \frac{R_{sm}}{F} + jX_{sm} + \frac{R_L}{F} + jX_L - j\frac{X_{csm}}{F^2} \\ \underline{Z}'_{aL} &= \frac{R_{sa}}{2Fa^2} + j\frac{X_{sa}}{2a^2} - j\frac{X_{cea}}{2F^2a^2} - \frac{\underline{Z}_{lmL}}{2} \\ \underline{Z}_+ &= \frac{jX_{mm} \left(\frac{R_{rm}}{F-U} + jX_{rm} \right)}{\frac{R_{rm}}{F-U} + j(X_{mm} + X_{rm})} \\ \underline{Z}_- &= \frac{jX_{mm} \left(\frac{R_{rm}}{F+U} + jX_{rm} \right)}{\frac{R_{rm}}{F+U} + j(X_{mm} + X_{rm})} \end{aligned} \quad (11.6)$$

the equivalent circuit shown in Figure 11.2 can be simplified as shown in Figure 11.3.

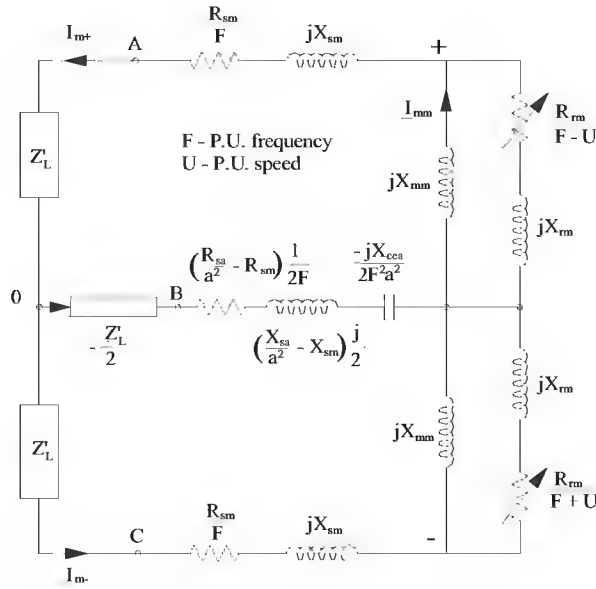


FIGURE 11.2 Self-excited self-regulated single-phase IG (Figure 11.1): equivalent circuit for steady state.

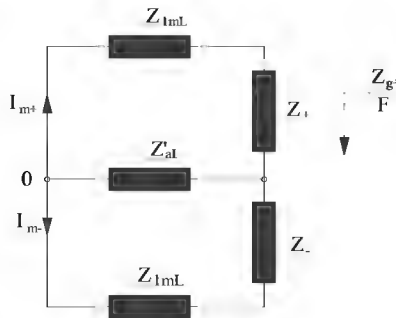


FIGURE 11.3 Simplified equivalent circuit of single-phase generator.

The self-excitation condition implies that the sum of the currents in node 0 (Figure 11.3) is zero:

$$\frac{1}{\underline{Z}_{1mL} + \underline{Z}_+} = \frac{1}{\underline{Z}_{1mL} + \underline{Z}_-} + \frac{1}{\underline{Z}'_{al}} \quad (11.7)$$

Two conditions are provided in (11.7) to solve for two unknowns. We may choose F and X_{mm} as unknowns provided the magnetization curve: $V_{g+}(X_{mm})$ is known from measurements or from finite element modelling (FEM) calculations.

In reality, X_{mm} is a known function of the magnetization current: I_{mm} : $X_{mm}(I_{mm})$ (Figure 11.4).

To simplify the computation process, we may consider that $\underline{Z}_-$ is

$$\underline{Z}_- \approx \frac{R_{rm}}{F+U} + jX_{rm} \quad (11.8)$$

As, except for X_{mm} , all other parameters are considered constant, we can express $\underline{Z}_+$ from (11.7) as

$$\underline{Z}_+ = \frac{(\underline{Z}_{1mL} + \underline{Z}_-) \cdot \underline{Z}'_{al}}{\underline{Z}_{1mL} + \underline{Z}_- + \underline{Z}'_{al}} - \underline{Z}_{1mL} \quad (11.9)$$

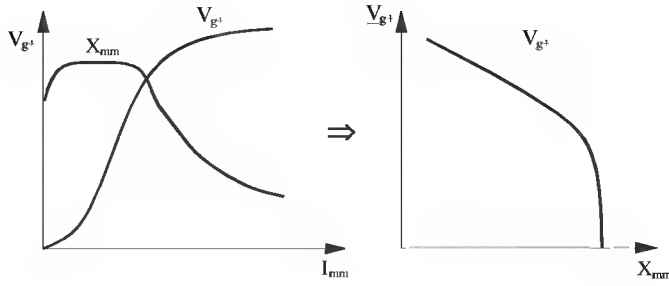


FIGURE 11.4 The magnetization curves of the main winding.

All impedances on the right side of Equation (11.9) are solely dependent on frequency F , if all motor parameters, speed n , and C_{ae} , C_{sm} , and X_L are given, for an adopted rated frequency f_{ln} .

For a row of values for F , we can simply calculate from (11.9) $\underline{Z}_+ = f(F)$ for given speed n , capacitors, load:

$$\underline{Z}_+(F, X_{mm}) = \frac{jX_{mm} \left(\frac{R_{rm}}{F-U} + jX_{mm} \right)}{j(X_{mm} + X_m) + \frac{R_{rm}}{F-U}} \quad (11.10)$$

We may use only the imaginary part of (11.10) and determine rather simply the $X_{mm}(F)$ function.

Now, from Figure (11.4), we may determine for each F , that is for every X_{mm} value, the airgap voltage value V_{g+} and thus the magnetization

$$I_{mm} = \frac{V_{g+}}{X_{mm}} \quad (11.11)$$

As we now know F , X_{mm} and V_{g+} , the equivalent circuit shown in Figure 11.3 may be solved rather simply to determine the two currents I_{m+} and I_{m-} .

From now on, all steady-state characteristics can be easily calculated.

$$\underline{I}_{m+}^{(F)} = \frac{V_{g+}}{F} \cdot \frac{1}{\left(\underline{Z}_{lmL} + \frac{\underline{Z}'_{al} \cdot \underline{Z}_{lmL}}{\underline{Z}'_{al} + \underline{Z}_{lmL}} \right)} \quad (11.12)$$

$$\underline{I}_{m-}^{(F)} = \underline{I}_{m+}^{(F)} \cdot \frac{\underline{Z}'_{al}}{\underline{Z}_- + \underline{Z}_{lmL}} \quad (11.13)$$

The load current $\underline{I}_m$ is

$$\underline{I}_m = \underline{I}_{m+} + \underline{I}_{m-} \quad (11.14)$$

The auxiliary winding current is

$$\underline{I}_a = j(\underline{I}_{m+} - \underline{I}_{m-}) \quad (11.15)$$

The output active power P_{out} is

$$P_{out} = I_m^2 \cdot R_L \quad (11.16)$$

The rotor+current component $\underline{I}_{r+}$ becomes

$$\underline{I}_{r+} = -\underline{I}_{m+} \cdot \frac{jX_{mm}}{\frac{R_m}{F-U} + j(X_m + X_{mm})} \quad (11.17)$$

The total input active power from the shaft P_{input} is

$$P_{input} \approx 2I_{m+}^2 \cdot \frac{R_m \cdot U}{F-U} - 2I_{m-}^2 \cdot \frac{R_m U}{F+U} \quad (11.18)$$

For a realistic efficiency formula, the additional core and mechanical losses $p_{iron} + p_{stray} + p_{mec}$ have to be added to the ideal input of (11.18).

$$\eta = \frac{P_{out}}{P_{input} + p_{iron} + p_{stray} + p_{mec}} \quad (11.19)$$

As the speed is given, we change the slip by varying F . We might change the load resistance with frequency (slip) to yield realistic results from the beginning.

As $X_{mm}(V_{g+})$ may be given as a table, the values of $X_{mm}(F)$ function may be looked up simply into another table. If no X_{mm} is found from the given data, it means that either the load impedance or the capacitors, for that particular frequency and speed, are not within the existence domain.

So either the load is modified or the capacitor is changed to re-enter the existence domain.

The above algorithm may be synthesized as shown in Figure 11.5.

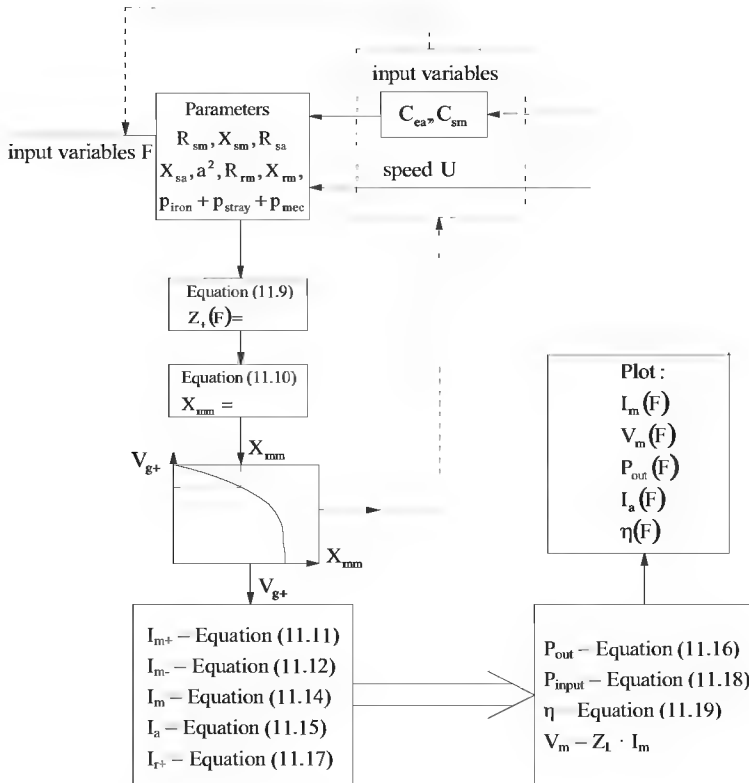


FIGURE 11.5 Performance computation algorithm.

The IG data obtained through tests are $P_n = 700$ W, $n_n = 3000$ rpm, $V_{Ln} = 230$ V, $f_{In} = 50$ Hz, $R_{sm} = 3.94 \Omega$, $R_{sl} = 4.39 \Omega$, $R_{rm} = 3.36 \Omega$, $X_{rm} = X_{sm} = 5.48 \Omega$, $X_{sl} = 7.5 \Omega$, unsaturated $X_{mm} = 70 \Omega$, $C_{ea} = 40 \mu\text{F}$, and $C_{sm} = 100 \mu\text{F}$ [1].

The magnetization curves $V_{gt}(I_m)$ have been obtained experimentally, in the synchronous bare rotor test. That is, before the rotor cage was located in the rotor slots, the IG was driven at synchronism, $n = 3000$ rpm ($f = 50$ Hz), and was a.c.-fed from a Variac in the main winding only. Alternatively, it may be calculated at standstill with d.c. excitation via FEM. In both cases, the auxiliary winding is kept open (for more details on testing of single-phase IMs, see Chapter 14, Vol. 2). The experimental results shown in Figure 11.6 [1] warrant a few remarks:

- The larger the speed, the larger the load voltage.
- The lower the speed, the larger the current for given load.
- Voltage regulation is very satisfactory: from 245 V at no load to 230 V at full load.
- The no-load voltage increases only slightly with C_{ea} (the capacitance) in the auxiliary winding.
- The higher the series capacitor (above $C_{sm} = 40 \mu\text{F}$), the larger the load voltage.
- It was also shown that the voltage waveform is rather sinusoidal up to rated load.
- The fundamental frequency at full load and 3000 rpm is $f_{In} = 48.4$ Hz, an indication of small slip.

A real gas engine (without speed regulation) would lose some speed when the generator is loaded. Still the speed (and additional frequency) reduction from no load to full load is small. So aggregated

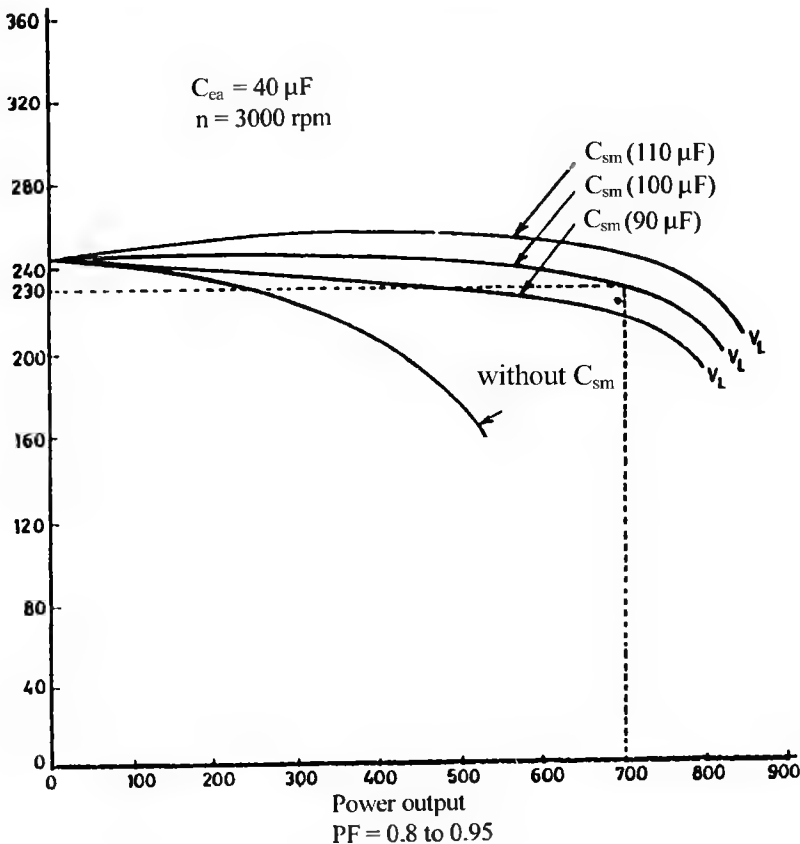


FIGURE 11.6 Steady-state performance of a self-excited self-regulated single-phase IG. (After Ref. [1].)

voltage regulation is, in these conditions, at full load, slightly larger but still below 8%, with a speed drop from 3000 to 2920 rpm [1].

11.3 THE d-q MODEL FOR TRANSIENTS

The transients may be treated directly via d-q model in stator coordinates with saturation included (as done for motoring).

$$V_{ds} = -V_{csm} - R_L I_{ds} - L_L \frac{dI_{ds}}{dt} \quad (11.20)$$

$$\frac{dV_{Csm}}{dt} = \frac{1}{C_{sm}} I_{ds}; \quad I_{ds} = I_m \quad (11.21)$$

$$V_q = -V_{cea} \quad (11.22)$$

$$\frac{dV_{cea}}{dt} = \frac{1}{a^2 C_{ea}} I_{qs}; \quad I_{qs} = I_a \cdot a; \quad a = \frac{W_a K_{wa}}{W_m K_{wm}} \quad (11.23)$$

The d-q model in Section 11.2 is

$$\begin{aligned} \frac{d\Psi_{ds}}{dt} &= V_{ds}(t) - R_{sm} I_{ds} \\ \frac{d\Psi_{qs}}{dt} &= -V_{cea} - \frac{R_{sa}}{a^2} I_{qs} \\ \frac{d\Psi_{dr}}{dt} &= I_{dr} R_{rm} - \omega_r \Psi_{qr} \\ \frac{d\Psi_{qr}}{dt} &= I_{qr} R_{rm} + \omega_r \Psi_{dr} \end{aligned} \quad (11.24)$$

$$\begin{aligned} \Psi_{ds} &= L_{sm} I_{ds} + \Psi_{dm}; \quad \Psi_{dm} = L_{mm} I_{dm} \\ \Psi_{dr} &= L_{rm} I_{dr} + \Psi_{qm}; \quad \Psi_{qm} = L_{mm} I_{qm} \\ \Psi_{qs} &= \frac{L_{sa}}{a^2} I_{qs} + \Psi_{qm}; \quad I_{dm} = I_{ds} + I_{dr} \\ \Psi_{qr} &= L_{rm} I_{qr} + \Psi_{qm}; \quad I_{qm} = I_{qs} + I_{qr} \\ \text{and: } \Psi_m &= L_{mm} (I_m) \cdot I_m; \quad I_m = \sqrt{I_{dm}^2 + I_{qm}^2} \end{aligned} \quad (11.25)$$

To complete the model, the motion equation is added

$$\frac{J}{p_l} \frac{d\omega_r}{dt} = T_{pmover} + T_e \quad (11.26)$$

$$T_e = p_l (\Psi_{ds} I_{qs} - \Psi_{qs} I_{ds}) < 0$$

The prime mover torque T_{pmover} may be dependent on speed or/and on the rotor position also. The equations of the prime mover speed governor (if any) may be added.

Equation (11.20) shows that when the load contains an inductance L_L (e.g. a single-phase IM), I_{ds} has to be a variable and thus the whole d-q model (Equation 11.24) has to be rearranged to accommodate this situation in the presence of magnetic saturation.

However, with resistive load (R_L)— $L_L = 0$ —the solution is straightforward with V_{csm} , V_{cea} , Ψ_{ds} , Ψ_{qs} , V_{qs} , Ψ_{dr} , Ψ_{qr} , and ω_r as variables.

If the speed ω_r is a given function of time, the motion Equation (11.26) is simply ignored. The self-excitation under no load, during prime mover start-up, load sudden variations, load dumping, or sudden short circuit are typical transients to be handled via the d-q model.

11.4 EXPANDING THE OPERATION RANGE WITH POWER ELECTRONICS

Power electronics can provide more freedom to the operation of single-phase IMs in terms of load voltage and frequency control [3]. An example is shown in Figure 11.7 [4].

The auxiliary winding is now a.c.-fed at the load frequency f_l , through a single-phase inverter, from a battery.

To filter out the double-frequency current produced by the converter, the L_r , C_r filter is used. C_a filters the d.c. voltage of the battery.

The main winding reactive power requirement may be reduced by the parallel capacitor C_m with or without a short- or a long-shunt series capacitor.

By adequate control in the inverter, it is possible to regulate the load frequency and voltage when the prime mover speed varies.

The inverter may provide more or less reactive power.

It is also possible that, when the load is large, the active power is contributed by the battery. On the other hand, when the load is low, the auxiliary winding can pump back active power to recharge the battery.

This potential infusion of active power from the battery to load may lead to the idea that, in principle, it is possible to operate as a generator even if the speed of the rotor ω_r is not greater than $\omega_l = 2\pi f_l$. However, as expected, more efficient operation occurs when $\omega_r > \omega_l$.

The auxiliary winding is 90° (electrical) ahead of the main winding, and thus, no pulsation-type interaction with the main winding exists. The interaction through the motion emfs is severely filtered for harmonics by the rotor currents. So the load voltage is practically sinusoidal. The investigation of this system may be performed through the d-q model, as presented in the previous section (for more details on this subject in [5,6], see Ref. [4]).

A series connected capacitor in the short shunt for the output winding has been proved to reduce drastically voltage regulation [7], even when the IG is self-excited [8].

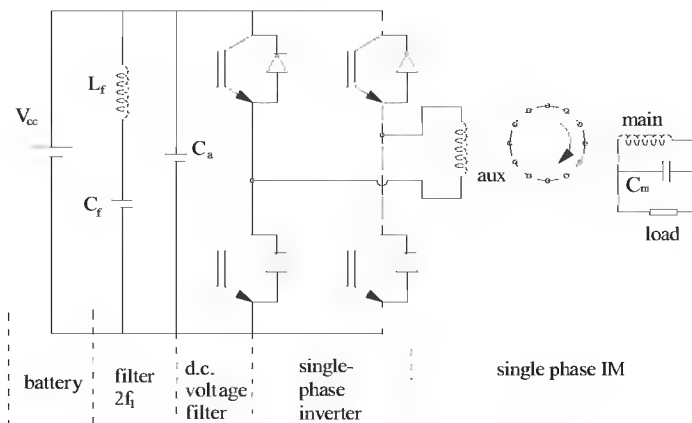


FIGURE 11.7 Single-phase IG with battery inverter-fed auxiliary winding.

11.5 SUMMARY

- The two winding IM may be used for low-power autonomous single-phase generators.
- Among many possible connections, it seems that one of the best connects the auxiliary winding upon an excitation capacitor C_{ea} , while the main winding (provided with a self-regulation series capacitors C_{sm}) supplies the load.
- The steady-state modelling may be done with the revolving theory (+, – or f, b) model.
- The saturation plays a key role in this self-excited self-regulated configuration.
- The magnetic saturation is related, in the model, to the direct (+) forward component.
- A rather simple computer program can provide the steady-state characteristics: output voltage, current, frequency versus output power for given speed, machine parameters, and magnetization curve.
- Good voltage regulation (<8%) has been reported.
- The sudden short circuit apparently does not threaten the IG integrity.
- The transients may be handled through the d-q model in stator coordinates via some additional terminal voltage relationships.
- More freedom in the operation of single-phase IG is brought by the use of a fractional rating battery-fed inverter to supply the auxiliary winding. Voltage and frequency control may be provided this way. Also, bi-directional power flow between inverter and battery can be performed. So the battery may be recharged when the IG load is low.
- A three-phase IM operating as single-phase IG with promising performance is described in Ref. [9].
- In the power range of 10–20 kW, the single-phase IG represents a strong potential competitor to existing gensets using synchronous generators.
- Using three-phase IGs with inverter excitation at load winding terminals to yield single-phase a.c. power generation may also produce a strong competitor to small-power synchronous generators [10].

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12 Linear Induction Motors

12.1 INTRODUCTION

For virtually every rotary electric machine, there is a linear motion counterpart. So is the case with induction machines. They are called linear induction machines.

Linear induction motors (LIMs) develop directly an electromagnetic force, called *thrust*, along the direction of the travelling field motion in the airgap.

The imaginary process of “cutting” and “unrolling” rotary counterpart is illustrated in Figure 12.1.

The *primary* contains in general a three-phase winding in the uniform slots of the laminated core.

The *secondary* is made of either a laminated core with a ladder cage in the slots or an aluminium (copper) sheet with (or without) a solid iron back core.

Apparently, the LIM operates as its rotary counterpart does, with *thrust* instead of torque and linear speed instead of angular speed, based on the principle of travelling field in the airgap.

In reality, there are quite a few differences between linear and rotary IMs such as the following [1–8]:

- The magnetic circuit is open at the two longitudinal ends (along the travelling field direction). As the flux law has to be observed, the airgap field will contain additional waves whose negative influence on performance with LIM in motion is called *dynamic longitudinal end effect* (Figure 12.2a).
- In short primaries (with two, four poles), there are current asymmetries between phases due to the fact that one phase has a position to the core longitudinal ends which is different from those of the other two. This is called static longitudinal effect (Figure 12.2b).
- Due to same limited primary core length, the back iron flux density tends to include an additional nontravelling (ac) component which should be considered when sizing the back iron of LIMs (Figure 12.2c).
- In the LIM shown in Figure 12.1 (called single sided, as there is only one primary along one side of secondary), there is a strong normal force (of attraction or repulsion type) between the primary and secondary. This normal force may be put to use to compensate for part of the weight of the moving primary and thus reduce the wheel wearing and noise level (Figure 12.2d).
- For secondaries with aluminium (copper) sheet with (without) solid back iron, the induced currents (in general at slip frequency Sf_1) have part of their closed paths contained in the active (primary core) zone (Figure 12.2e). They have additional – longitudinal (along OX axis) – components which produce additional losses in the secondary and a distortion in the airgap flux density along the transverse direction (OY). This is called the transverse edge effect.

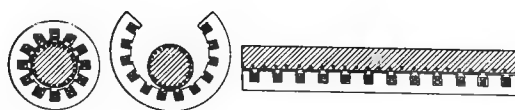


FIGURE 12.1 Imaginary process of obtaining a LIM from its rotary counterpart.

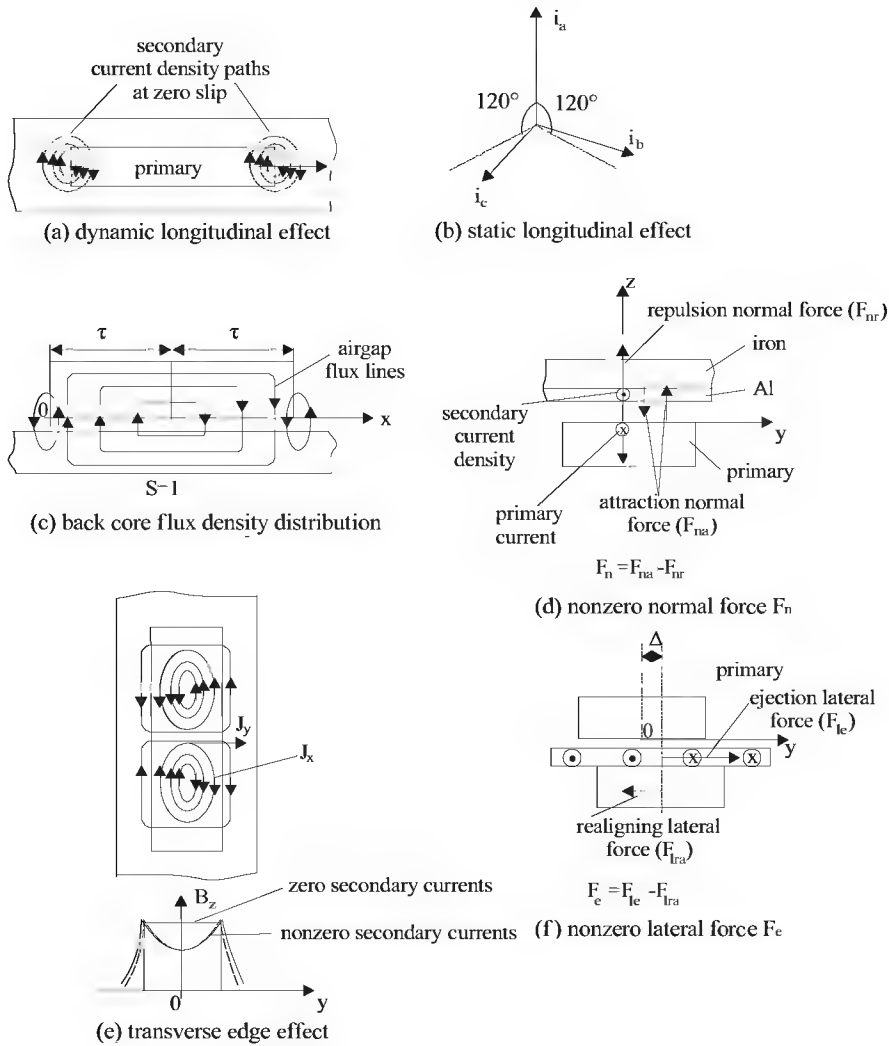


FIGURE 12.2 Panoramic view of main differences between LIMs and rotary IMs.

- When the primary is placed off centre along OY, the longitudinal components of the current density in the active zone produce an ejection-type lateral force. At the same time, the secondary back core tends to realign the primary along OY. So the resultant lateral force may be either decentralizing or centralizing in character (Figure 12.2f).

All these differences between linear and rotary IMs warrant a specialized investigation of field distribution and performance in order to limit the adverse effects (longitudinal end effects and back iron flux distortion, etc.) and exploit the desirable ones (normal and lateral forces, or transverse edge effects).

The same differences suggest the main merits and demerits of LIMs:

Merits

- Direct electromagnetic thrust propulsion (no mechanical transmission or wheel adhesion limitation for propulsion)
- Ruggedness; very low maintenance costs

- Easy topological adaptation to direct linear motion applications
- Precision linear positioning (no play (backlash) as with any mechanical transmission)
- Separate cooling of primary and secondary
- All advanced drive technologies for rotary IMs may be applied without notable changes to LIMs.

Demerits

- Due to large airgap-to-pole pitch (g/τ) ratios – $g/\tau > 1/250$ – the power factor and efficiency tend to be lower than with rotary IMs. However, the efficiency is to be compared with the combined efficiency of rotary motor + mechanical transmission counterpart. Larger mechanical clearance is required for speeds above 3 m/s. The aluminium sheet (if any) in the secondary contributes an additional (magnetic) airgap.
- Efficiency and power factor are further reduced by longitudinal end effects. Fortunately, these effects are notable only in high-speed low-pole count LIMs, and they may be somewhat limited by pertinent design measures.
- Additional noise and vibration due to uncompensated normal force, unless the latter is put to use to suspend the mover (partially or totally) by adequate close loop control.

As sample LIM applications have been presented in Chapter 1, Vol. 1, we may now proceed with the investigation of LIMs, starting with classifications and practical construction aspects.

12.2 CLASSIFICATIONS AND BASIC TOPOLOGIES

LIMs may be built single sided (Figure 12.1) or double sided (Figure 12.3a), with moving (short) primary (Figure 12.1) or moving (short) secondary (Figure 12.3b).

As single-sided LIMs are more rugged, they have been used in more applications. Figure 12.3b shows a double-sided practical short-moving secondary LIM for low-speed short-travel applications.

LIMs shown in Figures 12.1–12.3 are flat. For flat single-sided LIMs, the secondaries may be made of aluminium (copper) sheets on back solid iron (for low costs), ladder conductor in slots of laminated core (for better performance), and a pure conducting layer in electromagnetic metal stirrers (Figure 12.4a–c).

In double-sided LIMs, the secondary is made of an aluminium sheet (or structure) or from a liquid metal (sodium) as in flat LIM pumps.

Besides flat LIMs, tubular configurations may be obtained by rerolling the flat structures along the transverse (OY) direction (Figure 12.5). Tubular LIMs or tubular LIM pumps are in general single sided and have short fix primaries and moving limited length secondaries (except for liquid metal pumps).

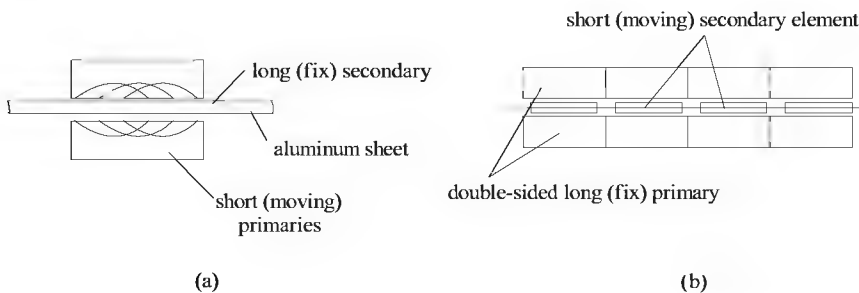


FIGURE 12.3 Double-sided flat LIMs (a) double-sided short (moving) primary LIM and (b) double-sided short (moving) secondary LIM for conveyors.

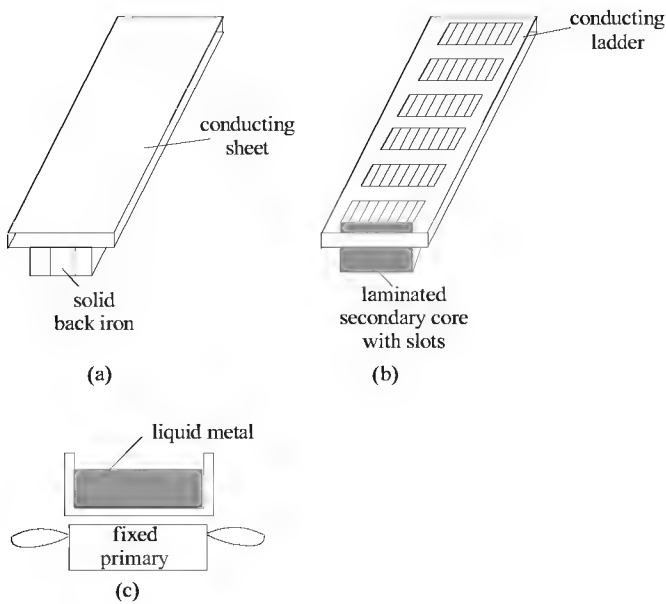


FIGURE 12.4 Flat single-sided secondaries (a) sheet on iron, (b) ladder conductor in slots, (c) liquid metal.

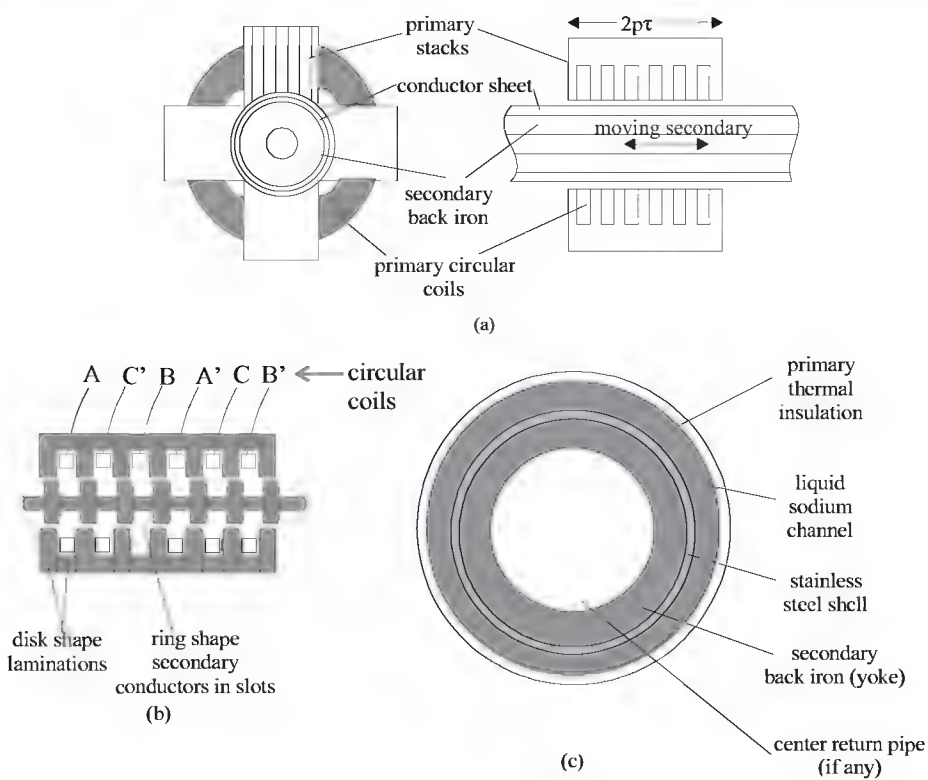


FIGURE 12.5 Tubular LIMs: (a) with longitudinal primary lamination stacks, (b) with disk-shaped laminations, and (c) liquid metal tubular LIM pump: the secondary.

The primary core may be made of a few straight stacks (Figure 12.5a) with laminations machined to circular stator bore shape. The secondary is typical aluminium (copper) sheet on iron. The stator coils have a ring shape.

While transverse edge effect is absent and coils appear to lack end connections, building a well-centred primary is not easy.

An easier-to-build solution is obtained with only two-size disk-shaped laminations on both primary and secondary (Figure 12.5b). The secondary ring-shaped conductors are thus placed also in slots.

Better performance is expected though the fact that, in the back cores, the magnetic field goes perpendicular to laminations, which tends to produce additional core losses. The interlamination insulation leads to an increased magnetization mmf and thus takes back part of this notable improvement.

The same rationale is valid for tubular LIM liquid metal pumps (Figure 12.5c) in terms of primary manufacturing process. Pumps allow for notably higher speeds ($u = 15$ m/s or more) when the fixed primaries may be longer and have more poles ($2p_1 = 8$ or more). The liquid metal (sodium) low electrical conductivity leads to a smaller dynamic longitudinal effect which at least has to be checked to see whether it is negligible.

12.3 PRIMARY WINDINGS

In general, three-phase windings are used as three-phase PWM converters are widely available for rotary induction motor drives. Special applications which require only $2p_1 = 2$ pole might benefit from two-phase windings as two-phase windings are both placed in the same position with respect to the magnetic core ends. Consequently, the phase currents are fully symmetric at very low speeds.

LIM windings are similar to those used for rotary IMs. and ideally, they produce a pure travelling mmf. However, as the magnetic circuit is open along the direction of motion, there are some particular aspects of LIM windings.

Among the possible winding configurations, we illustrate here a few:

- Single-layer full-pitch ($y = \tau$) windings with even number of poles $2p_1$ (Figure 12.6): three phase and two phase.
- Triple-layer chorded-coil ($y/\tau = 2/3$) winding with even number of poles $2p_1$ (Figure 12.7).
- Double-layer chorded-coil ($2/3 < y/\tau < 1$) coil winding with odd number of poles $2p_1 + 1$ (the two end poles have half-filled slots) (Figure 12.8).
- Fractionary winding for miniature LIMs (Figure 12.9).

A few remarks are in order

- The single-layer winding with an even number of poles makes better usage of primary magnetic core, but it shows rather large coil end connections. It is recommended for $2p_1 = 2, 4$.
- The triple-layer chorded-coil winding is easy to manufacture automatically; it has low end connections and also has a rather low winding (chording) factor $K_y = 0.867$.
- The double-layer chorded-coil winding with an odd total number of poles (Figure 12.8) has shorter end connections and is easier to build, but it makes a poorer use of primary magnetic core as the two end poles are half-wound. As the number of poles increases above seven, nine, the end poles influence becomes small. It is recommended thus for low-, medium-, and high-speed LIMs ($2p_1 + 1 > 5$).

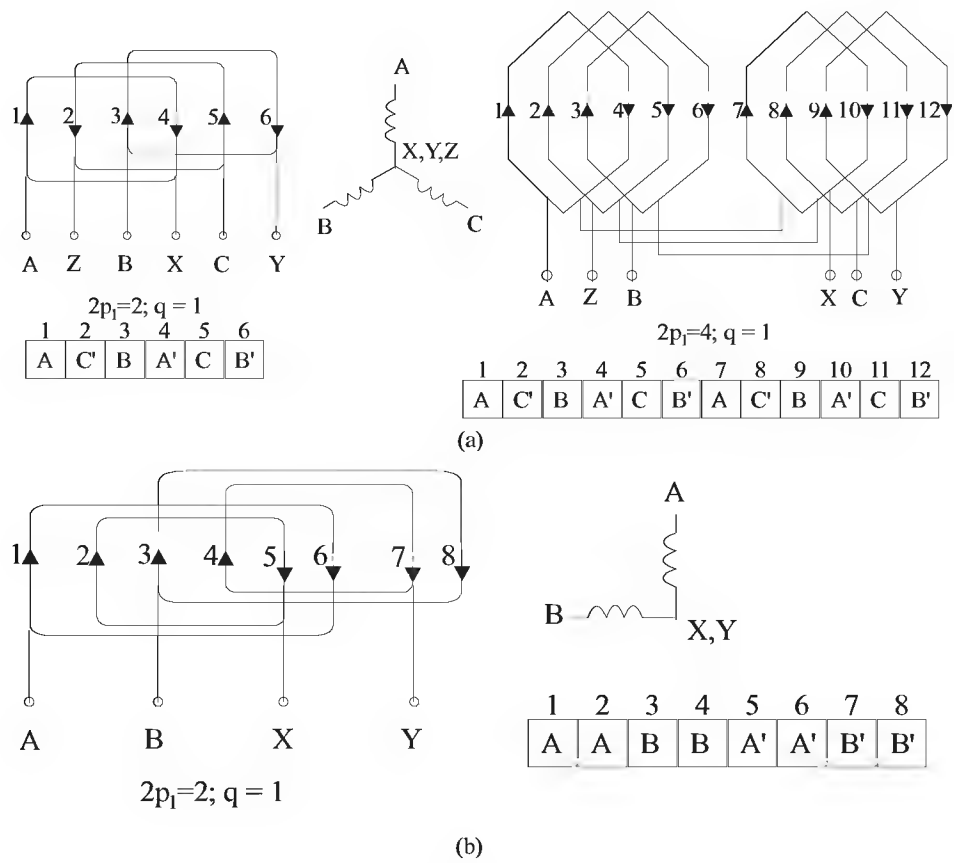


FIGURE 12.6 Single-layer windings: (a) three phase and (b) two phase.

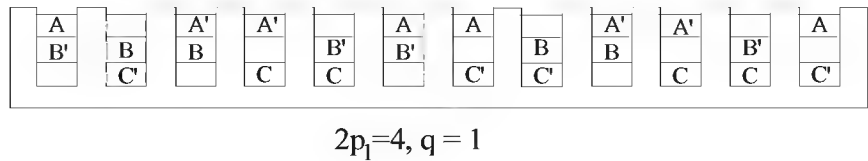


FIGURE 12.7 Triple-layer winding with chorded coils ($y/\tau = 2/3$) with very short end connections.

- The fractionary winding (Figure 12.9) – with $q = \frac{1}{2}$ in our case – is characterized by very short end connections, but the winding factor is low. It is recommended only in miniature LIMs where volume is crucial.
- When the number of poles is small, $2p_1 = 2$ especially, and phase current symmetry is crucial (low vibration and noise), the two-phase LIM may prove the adequate solution.
- Tubular LIMs are particularly suitable for single-layer even number of pole windings as the end connections are nonexistent with ring-shaped coils.
- In the introduction, we mentioned the transverse edge and longitudinal end effects as typical to LIMs. Let us now proceed with a separate analysis of transverse edge effect in double-sided and single-sided LIMs with sheet secondary.

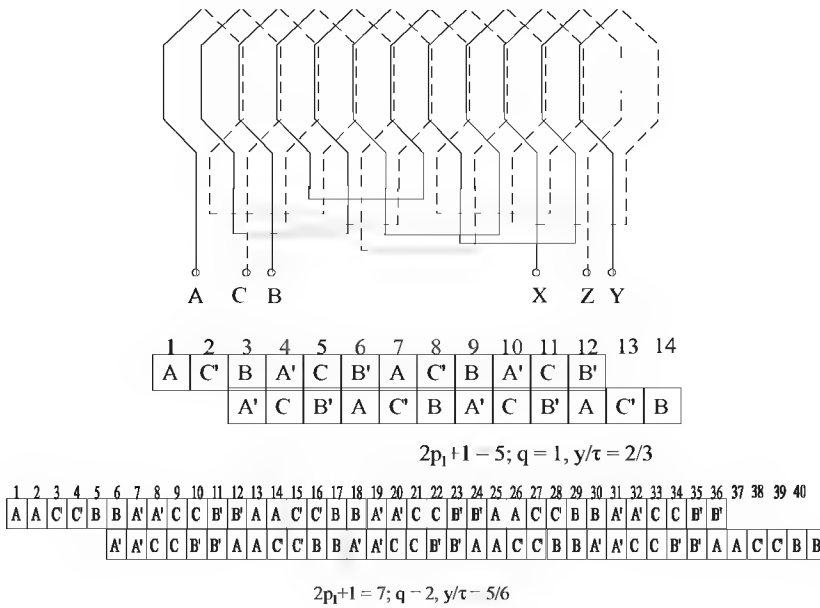


FIGURE 12.8 Double-layer chorded-coil windings with $2p_1 + 1$ poles.

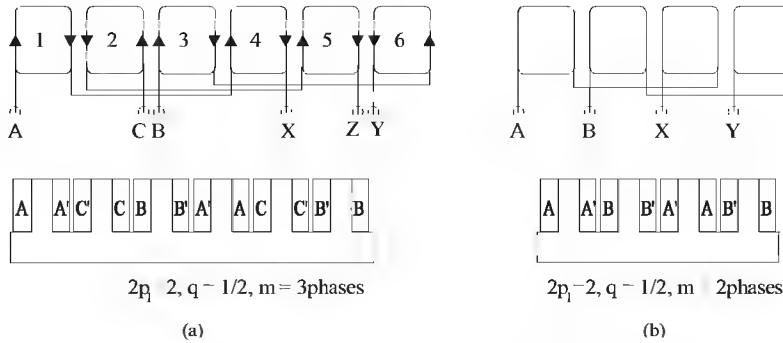


FIGURE 12.9 Fractionary single-layer winding (low end connections) (a) three phase and (b) two phase.

12.4 TRANSVERSE EDGE EFFECT IN DOUBLE-SIDED LIM

A simplified single-dimensional theory of transverse edge effect is presented here.

The main assumptions are

- The stator slotting is considered only through Carter coefficient K_c :

$$K_c = \frac{1}{1 - \gamma g / \tau_s}; \quad \gamma = \frac{\left(\frac{g}{b_{os}} \right)^2}{5 + \gamma \frac{g}{b_{os}}} \quad (12.1)$$

- The primary winding in slots is replaced by an infinitely thin current sheet travelling wave $J_1(x, t)$

$$J_1(x, t) = J_m e^{j\left(s\omega_1 t - \frac{\pi}{\tau}x\right)}; \quad J_m = \frac{3\sqrt{2}W_1 K_{w1} I_1}{p_1 \tau} \text{ in Aturns/m} \quad (12.2)$$

where S – slip, $2p_1$ – pole number, W_1 – turns/phase, K_{w1} – winding factor, τ – pole pitch, and I_1 – phase current (RMS). Coordinates are attached to secondary.

- The skin effect in the secondary conductor sheet is neglected or considered through the standard correction coefficient:

$$K_{\text{skin}} \approx \frac{d}{2d_s} \left[\frac{\sinh\left(\frac{d}{d_s}\right) + \sin\left(\frac{d}{d_s}\right)}{\cosh\left(\frac{d}{d_s}\right) - \cos\left(\frac{d}{d_s}\right)} \right] \geq 1 \quad (12.3)$$

$$\frac{1}{d_s} \approx \sqrt{\frac{\mu_0 \omega_1 S \sigma_{Al}}{2}} \quad (12.4)$$

For single-sided LIMs, d/d_s will replace $2d/d_s$ in (12.3); d_s – skin depth in the aluminium (copper) sheet layer. Consequently, the aluminium conductivity is corrected by $1/K_{\text{skin}}$:

$$\sigma_{Als} = \frac{\sigma_{Al}}{K_{\text{skin}}} \quad (12.5)$$

- For a large airgap between the two primaries, there is a kind of flux leakage which makes the airgap look larger g_1 [1].

$$g_1 = g K_c K_{\text{leakage}} \quad (12.6)$$

$$K_{\text{leakage}} = \frac{\sinh\left(\frac{g}{2\tau}\right)}{\frac{g}{2\tau}} > 1$$

- Only for large g/τ ratio, K_{leakage} is notably different from unity. The airgap flux density distribution in the absence of secondary shows the transverse fringing effect (Figure 12.10).

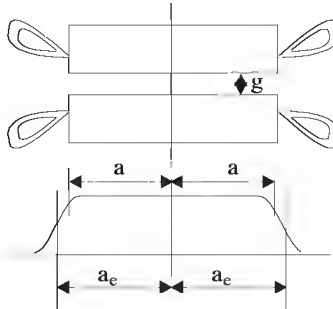


FIGURE 12.10 Fringing and end-connection flux considerations.

The transverse fringing effect can be calculated by introducing a larger (equivalent) stack width $2a_e$ instead of $2a$:

$$2a_e = 2a + (1.2 + 2.0)g \quad (12.7)$$

For large airgap in low-thrust LIMs this effect is notable.

As expected, the above approximations may be eliminated provided a 3D finite element modelling (FEM) was used. The amount of computation effort for a 3D-FEM model is so large that it is feasible mainly for special cases rather than for preliminary or optimization design.

- Finally, the longitudinal effects are neglected for the time being, and space variations along thrust direction and time variations are assumed to be sinusoidal.

In the active region ($|z| \leq a_e$), Ampere's law along contour 1 (Figure 12.11) yields

$$g_e \frac{\partial \underline{H}_y}{\partial z} = \underline{J}_{2x} d; \quad \underline{J}_1 = \underline{J}_m e^{j\left(\omega_1 t - \frac{\pi}{\tau} x\right)} \quad (12.8)$$

Figure 12.11b shows the active and overhang regions with current density along motion direction.

The same law applied along contour 2 (Figure 12.11b) in the longitudinal plane gives

$$-g_e \frac{\partial}{\partial x} (\underline{H}_x + \underline{H}_0) = \underline{J}_m + \underline{J}_{2z} d \quad (12.9)$$

Faraday's law also yields

$$\frac{\partial \underline{J}_{2z}}{\partial x} - \frac{\partial \underline{J}_{2x}}{\partial z} = -j\mu_0 S \omega_1 \sigma_{Als} (\underline{H}_y + \underline{H}_0) \quad (12.10)$$

Equations (12.8)–(12.10) are all in complex terms as sinusoidal time variation was assumed.

$\underline{H}_0$ is the airgap field in the absence of secondary:

$$\underline{H}_0 = \frac{j \underline{J}_m}{\pi \frac{g_e}{\tau}} \quad (12.11)$$

The three equations above combine to yield

$$\frac{\partial^2 \underline{H}_y}{\partial x^2} + \frac{\partial^2 \underline{H}_y}{\partial z^2} - j\mu_0 S \omega_1 \sigma_{Als} \frac{d}{g_e} \underline{H}_y = j\mu_0 S \omega_1 \sigma_{Als} \frac{d}{g_e} \underline{H}_0 \quad (12.12)$$

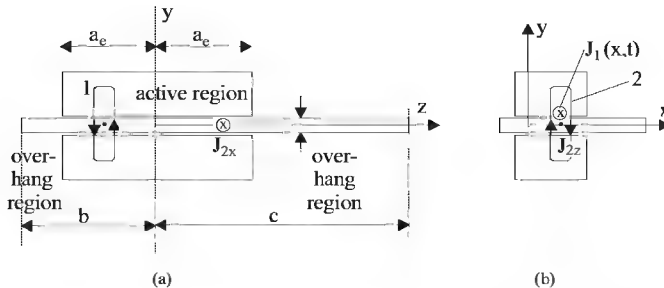


FIGURE 12.11 Transverse cross section of a double-sided LIM (a) and longitudinal view (b).

The solution of (12.12) becomes

$$\underline{H}_y = -\frac{j\underline{H}_0 \underline{S} G_e}{1 + j\underline{S} G_e} + A \cosh \underline{\alpha} z + B \sinh \underline{\alpha} z \quad (12.13)$$

with

$$G_i = \frac{\tau^2 \omega_l \mu_0 \sigma_{Als}}{\pi^2} \frac{d}{g_e} \quad (12.14)$$

$$\underline{\alpha}^2 = \left(\frac{\pi}{\tau} \right)^2 (1 + j\underline{S} G_e) \quad (12.15)$$

G_i is called the rather ideal goodness factor of LIM, a performance index we will return to frequently next.

In the overhang region ($|z| > a_e$), we assume that the total field is zero; that is, $\underline{J}_{2z}$ and $\underline{J}_{2x}$ satisfy Laplace's equation:

$$\frac{\partial \underline{J}_{2z}}{\partial x} - \frac{\partial \underline{J}_{2x}}{\partial z} = 0 \quad (12.16)$$

Consequently,

$$\underline{J}_{2xr} = \underline{C} \sinh \frac{\pi}{\tau} (c - z) \quad a_e < z \leq c \quad (12.17)$$

$$\underline{J}_{2xl} = j\underline{C} \cosh \frac{\pi}{\tau} (c - z)$$

$$\underline{J}_{2zl} = \underline{D} \sinh \frac{\pi}{\tau} (b + z) \quad -b < z < -a_e \quad (12.18)$$

$$\underline{J}_{2xl} = j\underline{D} \cosh \frac{\pi}{\tau} (b + z)$$

where r and l refer to right and left, respectively.

From the continuity boundary conditions at $z = \pm a_e$, we find

$$\underline{A} = \frac{\underline{H}_0 j\underline{S} G_i}{1 + j\underline{S} G_i} \left[\frac{(\underline{C}_1 + \underline{C}_2) \cosh(\underline{\alpha} a_e) + 2 \sinh(\underline{\alpha} a_e)}{(1 + \underline{C}_1 \underline{C}_2) \sinh(2\underline{\alpha} a_e) + (\underline{C}_1 + \underline{C}_2) \cosh(2\underline{\alpha} a_e)} \right] \quad (12.19)$$

$$\underline{B} = \frac{\underline{A} (\underline{C}_2 - \underline{C}_1) \sinh(\underline{\alpha} a_e)}{(\underline{C}_1 + \underline{C}_2) \cosh(\underline{\alpha} a_e) + 2 \sinh(\underline{\alpha} a_e)}$$

$$\underline{C}_1 = \frac{\alpha \tau}{\pi} \tanh \frac{\pi}{\tau} (c - a_e); \quad \underline{C}_2 = \frac{\alpha \tau}{\pi} \tanh \frac{\pi}{\tau} (b - a_e) \quad (12.20)$$

$$\underline{C} = \frac{-j g_e \underline{\alpha}}{d \cosh \frac{\pi}{\tau} (c - a_e)} \left[\underline{A} \sinh(\underline{\alpha} a_e) + \underline{B} \cosh(\underline{\alpha} a_e) \right] \quad (12.21)$$

$$\underline{D} = \frac{-j g_e \underline{\alpha}}{d \cosh \frac{\pi}{\tau} (b - a_e)} \left[-\underline{A} \sinh(\underline{\alpha} a_e) + \underline{B} \cosh(\underline{\alpha} a_e) \right] \quad (12.22)$$

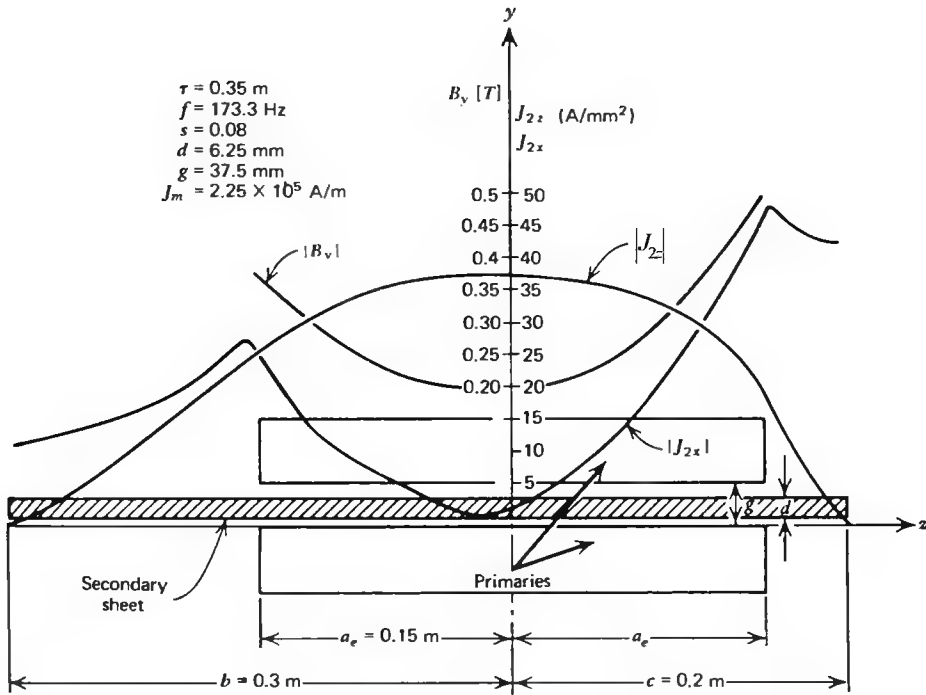


FIGURE 12.12 Transverse distribution of flux and secondary current densities. (After Ref. [5].)

Sample computation results of flux and current densities distributions for a rather high-speed LIM with the data – $\tau = 0.35 \text{ m}$, $f_1 = 173.3 \text{ Hz}$, $S = 0.08$, $d = 6.25 \text{ mm}$, $g = 37.5 \text{ mm}$, $J_m = 2.25 \cdot 10^5 \text{ A/m}$ – are shown in Figure 12.12 [5].

The transverse edge effect produces a “deep” in the airgap flux density transverse (along OZ) distribution. Also, if the secondary is placed off centre, along OZ, the distribution of both secondary current density components is nonsymmetric along OZ.

The main consequences of transverse edge effect are an apparent increase in the secondary equivalent resistance R'_2 and a decrease in the magnetization inductance (reactance X_m). Besides, when the secondary is off centre in the transverse (lateral) direction, a lateral decentralizing force F_{za} is produced:

$$F_{za} = \mu_0 d \tau p_l \operatorname{Re} \int_{-a_e}^{a_e} (\underline{J}_{2x} \underline{H}_y^*) dz \quad (12.23)$$

Again sample results for the same LIM as above are given in Figure 12.13.

The lateral force F_z decreases as a_e/τ decreases or the overhangs $c - a_e$, $b - a_e > \tau/\pi$. In fact, there is no use to extend the overhangs of secondary beyond τ/π as there are hardly any currents for $|z| > \tau/\pi + a_e$.

12.4.1 THE TRANSVERSE EDGE EFFECT CORRECTION COEFFICIENTS

In the absence of transverse edge effect, the magnetization reactance X_m has the conventional expression (for rotary induction machines):

$$X_m = \frac{6\mu_0 \omega_1 (W_l K_{w1})^2 \tau (2a_e)}{\pi^2 p_l g_l} \quad (12.24)$$

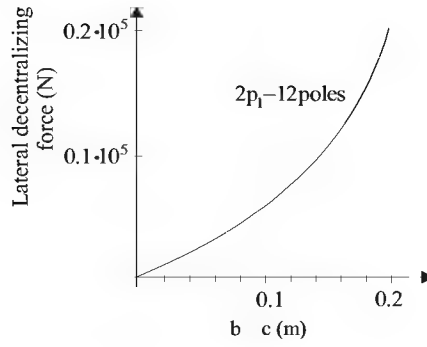


FIGURE 12.13 Decentralizing lateral force F_z .

The secondary resistance reduced to the primary R'_2 is

$$R'_2 = \frac{X_m}{G_i} = \frac{12(W_l K_{w1})^2 a_e}{\sigma_{Al} p_1 \tau d} \quad (12.25)$$

Because of the transverse edge effect, the secondary resistance is increased by $K_t > 1$ times and the magnetization inductance (reactance) is decreased by $K_m < 1$ times:

$$K_t = \frac{K_X^2}{K_R} \left[\frac{1 + S^2 G_i^2 K_R^2 / K_X^2}{1 + S^2 G_i^2} \right] \geq 1 \quad (12.26)$$

$$K_m = \frac{K_R K_t}{K_X} \leq 1 \quad (12.27)$$

For $b = c$ [9],

$$K_R = 1 - \operatorname{Re} \left[(1 - S G_i) \frac{\lambda}{\alpha a_e} \tanh(\alpha a_e) \right] \quad (12.28)$$

$$K_X = 1 + \operatorname{Re} \left[(j + S G_i) \frac{S G_i \lambda}{\alpha a_e} \tanh(\alpha a_e) \right] \quad (12.29)$$

$$\lambda = \frac{1}{1 + (1 + j S G_i)^{1/2} \tanh(\alpha a_e) \cdot \tanh \frac{\pi}{\tau} (c - a_e)} \quad (12.30)$$

For LIMs with narrow primaries ($2a_e/\tau < 0.3$) and at low slips, $K_m \approx 1$ and

$$K_t \approx \frac{1}{1 - \frac{\tanh \frac{\pi}{\tau} a_e}{\frac{\pi}{\tau} a_e} \left[1 + \tanh \left(\frac{\pi}{\tau} a_e \right) \cdot \tanh \frac{\pi}{\tau} (c - a_e) \right]^{-1}} \quad (12.31)$$

Transverse edge effect correction coefficients depend on the goodness factor G_i , slip, S , and the geometrical type factors a_e/τ , $(c - a_e)/\tau$.

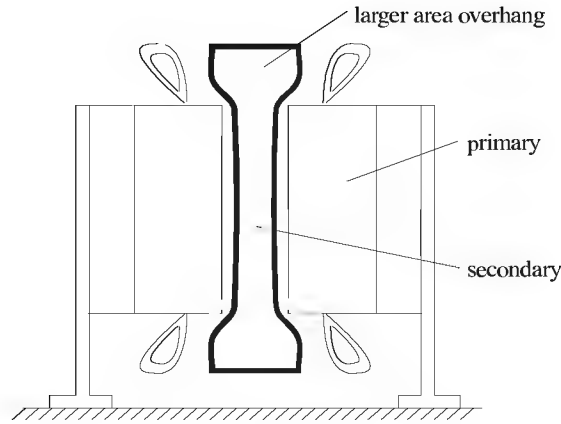


FIGURE 12.14 Reduced transverse edge effect secondary.

For $b \neq c$, the correction coefficients have slightly different expressions, but they may be eventually developed based on the flux and secondary current densities transverse distribution.

The transverse edge effect may be exploited for developing large thrusts with lower currents or may be reduced by large overhangs (up to τ/π) and an optimum a_c/τ ratio.

On the other hand, the transverse edge effect may be reduced, when needed, by making the overhangs of a larger cross section or of copper (Figure 12.14).

In general, the larger the value of SG_i , the larger the transverse edge effect for given a_c/τ and c/τ . In low-thrust (speed) LIMs, the pole pitch τ is small, so is the synchronous speed u_s ,

$$u_s = 2\tau f_1 = \tau \frac{\omega_1}{\pi} \quad (12.32)$$

Consequently, the goodness factor G_i is rather small, below, or slightly over unity.

Now, we may define an equivalent – realistic – goodness factor G_e as

$$G_e = G_i \frac{K_m}{K_t} < G_i \quad (12.33)$$

where the transverse edge effect is also considered.

Alternatively, the combined airgap leakage (K_{leak}), skin effect (K_{skin}), and transverse edge effect (K_m, K_t) may be considered as correction coefficients for an equivalent airgap g_e and secondary conductivity σ_e :

$$g_e = g \frac{K_c K_{leak}}{K_m} > g \quad (12.34)$$

$$\sigma_e = \frac{\sigma_{Al}}{K_{skin} K_t} < \sigma_{Al} \quad (12.35)$$

We notice that all these effects contribute to a reduction in the realistic goodness factor G_i .

12.5 TRANSVERSE EDGE EFFECT IN SINGLE-SIDED LIM

For the single-sided LIM with conductor sheet on iron secondary (Figure 12.15), a similar simplified theory has been developed where both the aluminium and saturated solid back iron contributions are considered [5,7].

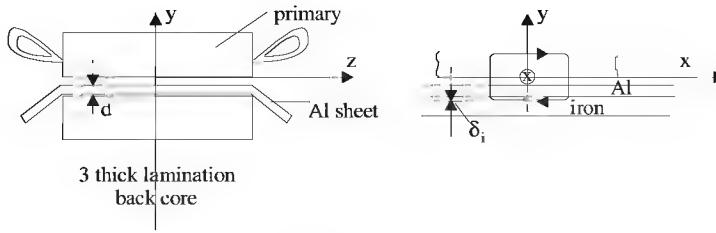


FIGURE 12.15 Single-side LIM with three thick lamination secondary back core.

The division of solid secondary back iron into, say, three ($i = 3$) pieces along transverse direction leads to a reduction of eddy currents. This is due to an increase in the “transverse edge effect” in the solid back iron.

As there are no overhangs for secondary back iron ($c - a_e = 0$), the transverse edge effect coefficient K_{ti} of (12.31) becomes

$$K_{ti} = \frac{1}{\tan \frac{\pi}{\tau} \frac{i}{a_e} - \frac{\pi}{\tau} \frac{i}{a_e}} \quad (12.36)$$

Now, an equivalent iron conductivity σ_{ti} may be defined as

$$\sigma_{ti} = \frac{\sigma_{iron}}{K_{ti}} \quad (12.37)$$

The depth of field penetration in the secondary back iron δ_i is thus

$$\delta_i^{-1} = \text{Re al} \left[\sqrt{\left(\frac{\pi}{\tau} \right)^2 + j S \omega_t \mu_{iron} \frac{\sigma_{iron}}{K_{ti}}} \right] \quad (12.38)$$

The iron permeability μ_{iron} depends mainly on the tangential (along OX) flux density B_{xi} , in fact, on its average over the penetration depth δ_i

$$B_{xi} = \frac{\tau}{\pi} \frac{B_g}{\delta_i} K_{pf}; \quad 1 \leq K_{pf} < 2 \quad (12.39)$$

where B_g is the given value of airgap flux density, and K_{pf} accounts for the increase in back-core maximum flux density in LIMs due to its open magnetic circuit along axis x [2,7]; $K_{pf} = 1$ for rotary IMs.

Now we may define an equivalent conductivity σ_e of the aluminium to account for the secondary back iron contribution

$$\sigma_e = \frac{\sigma_{Al}}{K_{skinal}} \left[\frac{1}{K_{ta}} + \frac{\sigma_{iron} \delta_i K_{skinal}}{\sigma_{Al} K_{ti} d} \right] \quad (12.40)$$

where K_{skinal} is the skin effect coefficient for aluminium (12.3), and K_{ta} the transverse edge effect coefficient for aluminium.

In a similar way, an equivalent airgap may be defined which accounts for the magnetic path in the secondary by a coefficient K_p :

$$g_e = (1 + K_p) g \frac{K_c K_{leak}}{K_{ma}} \quad (12.41)$$

$$K_p = \frac{\tau^2}{\pi^2} \frac{\mu_0}{2gK_c\delta_i\mu_{iron}} \quad (12.42)$$

K_p may not be neglected even if the secondary back iron is not heavily saturated.

Now the equivalent goodness factor G_e may be written as

$$G_e = \frac{\mu_0 \omega_1 \tau^2 \sigma_e d}{\pi^2 g_e} \quad (12.43)$$

The problem is that G_e depends on ω_1 , S , and μ_{iron} , for given machine geometry.

An iterative procedure is required to account for magnetic saturation in the back iron of secondary (μ_{iron}). The value of the resultant airgap flux density $B_g = \mu_0(H + H_0)$ can be obtained from (12.13) and (12.11) by neglecting the Z -dependent terms:

$$B_g \approx \frac{\mu_0 \tau}{\pi g_e} \frac{J_m}{(1 + jSG_e)}, \quad J_m \approx \frac{3\sqrt{2}I_1 W_1 k W_1}{p\tau} \quad (12.44)$$

For a given value of B_g , S , W_1 , K_t , K_{ma} , and K_{ti} are directly calculated. Then, from (12.38) to (12.39) and the iron magnetization curve, μ_{iron} (and B_{ti} , and δ_i) is iteratively calculated.

Then, σ_e and g_e are calculated from (12.40) to (12.42). Finally, G_e is determined. The primary phase current I_1 is computed from (12.44).

All above data are used to calculate the LIM thrust and other performance indices to be discussed in the next section using a technical longitudinal effect theory of LIMs.

12.6 A TECHNICAL THEORY OF LIM LONGITUDINAL END EFFECTS

Although we will consider the double-sided LIM, the results to be obtained here are also valid for single-sided LIMs with the same equivalent airgap g_e and secondary conductivity σ_e .

The technical theory as introduced here relies on a quasi-one-dimensional model attached to the short (moving) primary. Also, for simplicity, the primary core is considered infinitely long, but the primary winding is of finite length (Figure 12.16). All effects discussed in the previous section enter the values of σ_e , g_e and G_e , and the primary mmf is replaced by the travelling current sheet $\underline{J}_1$ (12.2). Complex variables are used as sinusoidal time variations are considered.

In the active zone ($0 \leq x \leq 2p_1\tau$), Ampere's law along $abcd$ (Figure 12.16) yields

$$g_e \frac{\partial H_t}{\partial x} = J_1 e^{j(\omega_1 t - \frac{\pi x}{\tau})} + J_2 d \quad (12.45)$$

where H_t is the resultant magnetic field in airgap. It varies only along OX axis. Also the secondary current density J_2 has only one component (along OZ).

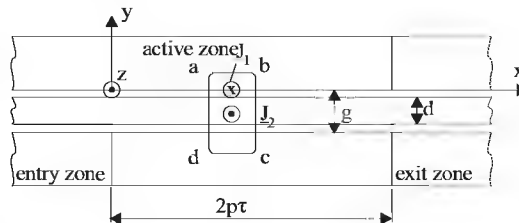


FIGURE 12.16 Double-sided LIM with infinitely long primary core.

Faraday's law is applied to moving bodies:

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \vec{u} \cdot \vec{B}; \quad \mathbf{J} = \sigma_e \mathbf{E} \quad (12.46)$$

yields

$$\frac{\partial J_2}{\partial x} = j\omega_l \mu_0 \sigma_e H_t + \mu_0 u \sigma_e \frac{\partial H_t}{\partial x} \quad (12.47)$$

where u is the relative speed between primary and secondary.

Equations (12.45) and (12.47) can be combined into

$$\frac{\partial^2 H_t}{\partial x^2} - \mu_0 u \sigma_e' \frac{\partial H_t}{\partial x} - j\omega_l \mu_0 \sigma_e' H_t = -j \frac{\pi}{\tau g_e} J_m e^{-j \frac{\pi}{\tau} x} \quad (12.48)$$

$$\sigma_e' = \sigma_e \frac{d}{g_e} \quad (12.49)$$

The characteristic equation of (12.48) becomes

$$\gamma^2 - \mu_0 \sigma_e' u \gamma - j\omega_l \mu_0 \sigma_e' = 0 \quad (12.50)$$

Its roots are

$$\gamma_{1,2} = \pm \frac{a_1}{2} \left[\sqrt{\frac{b_1 + 1}{2}} \pm 1 + j \sqrt{\frac{b_1 - 1}{2}} \right] = \gamma_{1,2r} \pm j \gamma_{1,2i} \quad (12.51)$$

with

$$a_1 = \mu_0 \sigma_e' u = \frac{\pi}{\tau} G_e (1 - S) \quad (12.52)$$

$$b_1 = \sqrt{1 + \left(\frac{4}{G_e (1 - S)^2} \right)^2} \quad (12.53)$$

The complete solution of H_t within active zone is

$$H_{ta}(x) = \underline{A}_e e^{\gamma_{1x}} + \underline{B}_e e^{\gamma_{2x}} + \underline{B}_n e^{-j \frac{\pi}{\tau} x} \quad (12.54)$$

with

$$\underline{B}_n = \frac{j \tau J_m}{\pi g_e (1 + S G_e)}; \quad S = \frac{u_s - u}{u_s} \quad (12.55)$$

The coefficient $e^{j\omega_l t}$ is to be added.

In the entry ($x < 0$) and exit ($x > 2p_1 \tau$) zones, there are no primary currents. Consequently,

$$H_{\text{entry}} = \underline{C}_e e^{\gamma_{1x}}; \quad x \leq 0 \quad (12.56)$$

$$H_{\text{exit}} = \underline{D}_e e^{\gamma_{2x}}; \quad x > 2p_1 \tau \quad (12.57)$$

At $x = 0$ and $x = 2p_1\tau$, the magnetic fields and the current densities are continuous:

$$(H_{\text{entry}})_{x=0} = (H_{\text{ta}})_{x=0} \quad (12.58)$$

$$(H_{\text{exit}})_{x=2p_1\tau} = (H_{\text{ta}})_{x=2p_1\tau} \quad (12.59)$$

$$\left(\frac{\partial H_{\text{entry}}}{\partial x}\right)_{x=0} = (J_2)_{x=0}; \quad \left(\frac{\partial H_{\text{exit}}}{\partial x}\right)_{x=2p_1\tau} = (J_2)_{x=2p_1\tau} \quad (12.60)$$

These conditions lead to

$$\underline{A}_t = -j \frac{\underline{J}_m}{\underline{D}} \left(\gamma_2 \frac{\tau}{\pi} + SG_e \right) e^{-2p_1\tau\gamma_1} \quad (12.61)$$

$$\underline{B}_t = j \frac{\underline{J}_m}{\underline{D}} \left(\gamma_1 \frac{\tau}{\pi} + SG_e \right) \quad (12.62)$$

$$\underline{D} = g_e (\gamma_2 - \gamma_1) (1 + jSG_e) \quad (12.63)$$

$$G_e = \frac{\tau^2}{\pi^2} \omega_l \mu_0 \sigma_e \frac{d}{g_e} \quad (12.64)$$

12.7 LONGITUDINAL END-EFFECT WAVES AND CONSEQUENCES

The above field analysis enables us to investigate the dynamic longitudinal effects.

Equation (12.54) reveals the fact that the airgap field H_t and its flux density B_t have, besides the conventional unattenuated wave, two more components: a forward and a backward travelling wave, because of dynamic longitudinal end effects. They are called end-effect waves:

$$\underline{B}_{\text{backward}} = -j\mu_0 \frac{\underline{J}_m}{\underline{D}} \left(\gamma_2 \frac{\tau}{\pi} + SG_e \right) e^{(\gamma_{1r} + j\gamma_{1i})(x - 2p_1\tau)} \quad (12.65)$$

$$\underline{B}_{\text{forward}} = j\mu_0 \frac{\underline{J}_m}{\underline{D}} \left(\gamma_1 \frac{\tau}{\pi} + SG_e \right) e^{(\gamma_{2r} - j\gamma_{2i})x} \quad (12.66)$$

They are called the exit and entry end-effect waves, respectively. The real parts of $\gamma_{1,2}$ (γ_{1r} , γ_{2r}) determine the attenuation of end-effect waves along the direction of motion, whereas the imaginary part $j\gamma_{1i}$ determine the synchronous speed (u_{se}) of end-effect waves:

$$u_{se} = \frac{\omega_l}{\gamma_i}; \quad \tau_e = \frac{\pi}{\gamma_i} \quad (12.67)$$

The values of $1/\gamma_{1r}$ and $1/\gamma_{2r}$ are called the depths of the end-effect waves penetration in the (along) the active zone.

As apparently from (12.51)

$$\frac{1}{\gamma_{1r}} \ll \frac{1}{\gamma_{2r}}; \quad \frac{1}{\gamma_{1r}} < (8 \div 10) \cdot 10^{-3} \text{ m} \quad (12.68)$$

Consequently, the effect of backward (exit) end-effect wave is negligible. Not so with the forward (entry) end-effect wave which attenuates slowly in the airgap along the direction of motion.

The higher the value of goodness factor G_e and the lower the slip S , the more important the end-effect waves. High G_e means implicitly high synchronous speeds.

The pole pitch ratio of end-effect waves (τ_e/τ) is

$$\frac{\tau_e}{\tau} = \frac{2\sqrt{2}}{G_e(1-S) \left(\sqrt{1 + \left(\frac{4}{G_e(1-S)^2} \right)^2} - 1 \right)} \geq 1 \quad (12.69)$$

It may be shown that $\tau_e/\tau \geq 1$ and is approaching unity (at $S = 0$) for large goodness factor values G_e .

The conventional thrust F_{xc} is

$$F_{xc} = \mu_0 a_e J_m \operatorname{Re} \left(\int_0^{2p_l \tau} B_n^* dx \right) \quad (12.70)$$

The end-effect force F_{xe} has a similar expression:

$$F_{xe} = \mu_0 a_e J_m \operatorname{Re} \left(\int_0^{2p_l \tau} B_t^* e^{\gamma_2^* x} e^{-j \frac{\pi}{\tau} x} dx \right) \quad (12.71)$$

The ratio f_e of these forces is a measure of end-effect influence:

$$f_e = \frac{F_{xe}}{F_{xc}} = \frac{\operatorname{Re} \left[B_t^* \frac{-1 + \exp 2p_l \tau \left(\gamma_2^* - j \frac{\pi}{\tau} \right)}{\left(\gamma_2^* - j \frac{\pi}{\tau} \right)} \right]}{\operatorname{Re} [B_n^* 2p_l \tau]} \quad (12.72)$$

Or, finally,

$$f_e = \frac{1 + S^2 G_e^2}{S G_e} \operatorname{Re} \left\{ \frac{j \left(\frac{\gamma_1 \tau}{\pi} + S G_e \right) \left(\exp \left[2p_l \tau \left(\gamma_2^* \frac{\tau}{\pi} - j \right) \right] - 1 \right)}{\frac{\tau}{\pi} (\gamma_2^* - \gamma_1^*) (1 - j S G_e) 2p_l \pi \left(\gamma_2^* \frac{\tau}{\pi} - j \right)} \right\} \quad (12.73)$$

As shown in (12.73), f_e depends only on the slip S , realistic goodness factor G_e , and the number of poles $2p_l$, $(2p_l + 1)$.

Quite general p.u. values $(F_{xe})_{p.u.}$ of F_{xe} may be expressed as

$$(F_{xe})_{p.u.} = F_{xc} \frac{\pi^2 G_e}{a_e \mu_0 J_m^2 \tau^2}; \quad \beta = \frac{\pi}{\tau} \quad (12.74)$$

$(F_{xe})_{p.u.}$ depends only on $2p_l$, S , and G_e and is depicted in Figure 12.17a–c.

The quite general results shown in Figure 12.17 suggest the following:

- The end-effect force at zero slip may be either propulsive (positive) (for low G_e values and/or large number of poles) or of braking character (negative) (for high G_e and/or smaller number of poles).

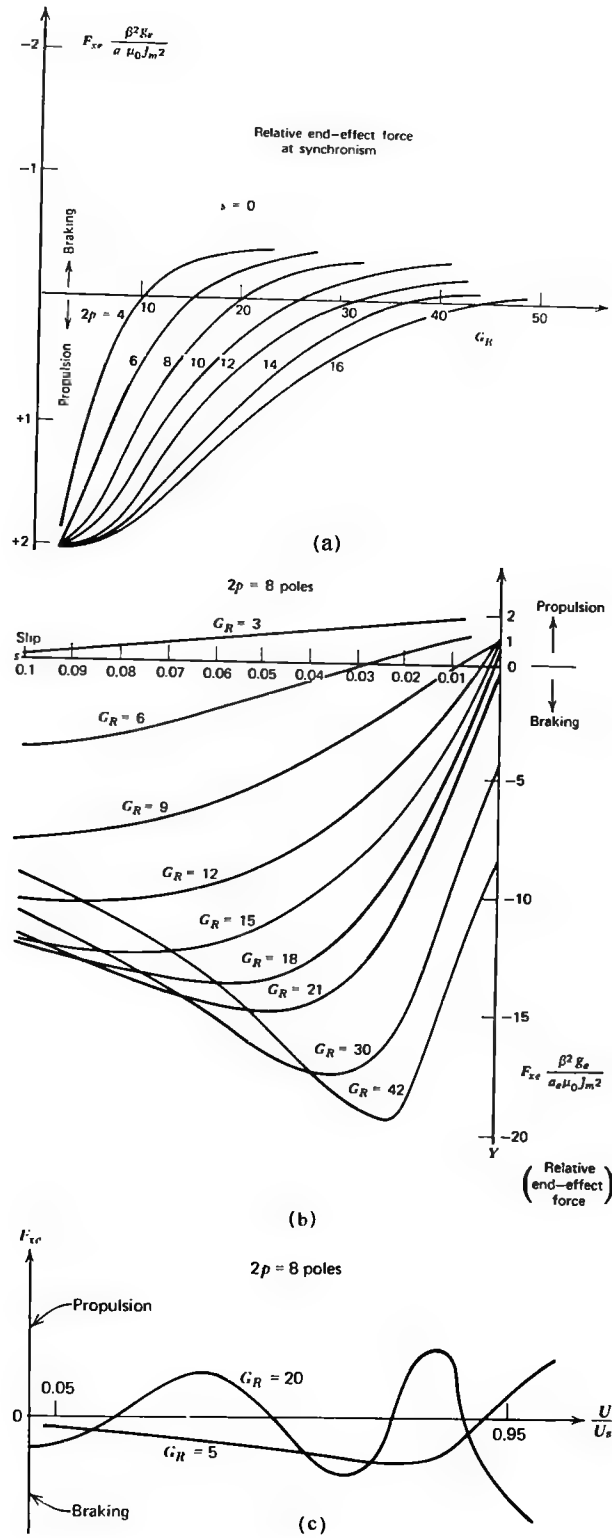


FIGURE 12.17 End-effect force F_{xe} in p.u.: (a) at zero slip (after Ref. [5]), (b) at small slips, and (c) versus normalized speed.

- For a given number of poles and zero slip, there is a certain value of the realistic goodness factor G_{eo} , for which the end-effect force is zero. This value of G_e is called here the optimum goodness factor [3].
- For large values of G_e , the end-effect force changes sign more than once as the slip varies from one to zero.
- The existence of the end-effect force at zero slip is a distinct manifestation of dynamic longitudinal end effect.

Further on, the airgap flux density $B_g = \mu_o H_{ia}$ (see (12.54)) has a nonuniform distribution along OX that accentuates if S is low, goodness factor G_e is high and the number of poles is low.

Typical qualitative distributions are shown in Figure 12.18.

This phenomenon produces nonuniform attraction forces between the two primaries along OX and nonuniformities in the back-core flux density B_{core} distribution:

$$B_{core} = \frac{1}{h_{core}} \int_0^x B_g dx \quad (12.75)$$

where h_{core} is the primary core height.

The problem is similar in single-sided LIMs, but there the saturation of secondary solid iron requires iterative computation procedures.

A nonuniform distribution shows also the secondary current density J_2 :

$$J_2 = \frac{g_e}{d} \left[\underline{\gamma}_1 A_t e^{\gamma_1 x} + \underline{\gamma}_2 B_t e^{\gamma_2 x} - \frac{j S G_e J_m e^{-j \frac{\pi}{\tau} x}}{g_e (1 + j S G_e)} \right] \quad (12.76)$$

Higher current density values are expected at the entry end ($x = 0$) and/or at the exit end at low values of slip for high goodness factor G_e and low number of poles $2p_1$ [5, p. 271].

Consequently, the secondary plate losses are distributed nonuniformly along the direction of motion in the airgap [5, p. 231].

Also, the propulsion force is not distributed uniformly along the core length. In the presence of large longitudinal end effects, the thrust at entry end goes down to zero, even to negative values [5, p. 232].

Similar aspects occur in relation to normal forces in double-sided and single-sided LIMs [5, p. 232].

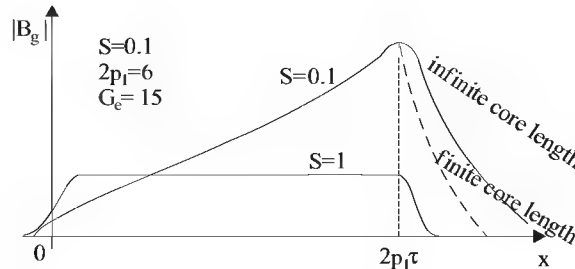


FIGURE 12.18 Airgap flux density distribution along OX.

12.8 SECONDARY POWER FACTOR AND EFFICIENCY

Based on the secondary current density $\underline{J}_2$ distribution (12.76), the power losses in the secondary P_2 are

$$P_2 = \frac{a_e d^2}{\sigma_e g_e} \int_{-\infty}^{\infty} (\underline{J}_2 \underline{J}_2^*) dx \quad (12.77)$$

Similarly, the reactive power Q_2 in the airgap is

$$Q_2 = a_e \omega_l \mu_0 g_e \int_{-\infty}^{\infty} (\underline{H}_{ta} \underline{H}_{ta}^*) dx \quad (12.78)$$

The secondary efficiency η_2 is

$$\eta_2 = \frac{u F_x}{u F_x + P_2}; \quad F_x = F_{xc} + F_{xe} \quad (12.79)$$

The secondary power factor $\cos \phi_2$ is

$$\cos \phi_2 = \frac{(u F_x + P_2)}{\sqrt{(u F_x + P_2)^2 + Q_2^2}} \quad (12.80)$$

Longitudinal end effects deteriorate both the secondary efficiency and power factor. Typical numerical results for a super-high-speed LIM are shown in Figure 12.19.

So far, we considered the primary core as infinitely long. In reality, this is not the case. Consequently, the field in the exit zone decreases more rapidly (Figure 12.18), and thus, the total secondary power losses are in fact smaller than calculated above.

However, due to the same reason, at exit end there will be an additional reluctance small force [3, pp. 74–79].

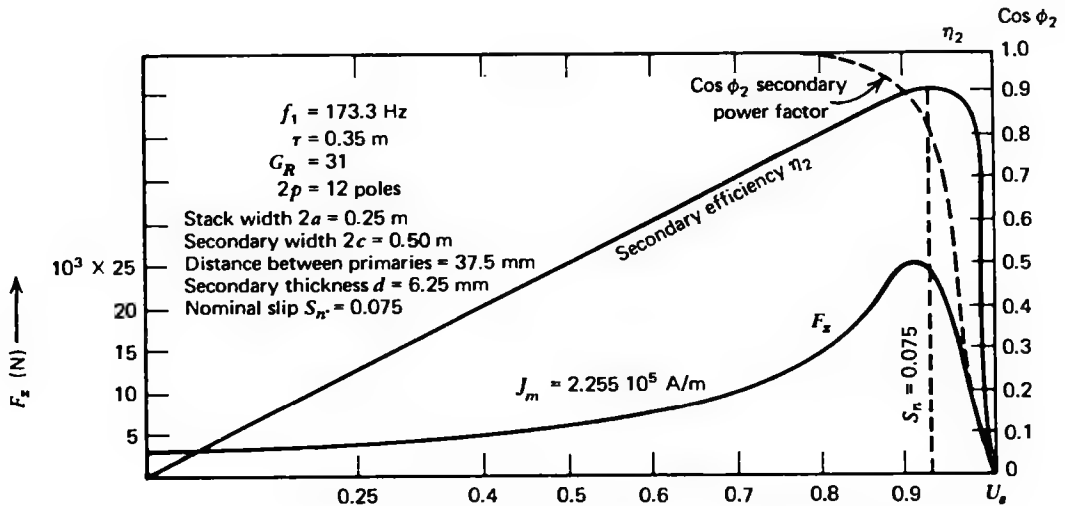


FIGURE 12.19 LIM secondary efficiency η_2 and power factor $\cos \phi_2$. (After Refs. [5,10].)

Numerical methods such as FEM would be suitable for a precise estimation of field distribution in the active, entry, and exit zones. However, to account for transverse edge effect also, 3D-FEM is mandatory.

Alternatively, 3D multilayer analytical methods have been applied successfully to single-sided LIMs with solid saturated and conducting secondary back iron [11–13] for reasonable computation time.

12.9 THE OPTIMUM GOODNESS FACTOR

As we already noticed, the forward end-effect wave has a longitudinal penetration depth of $\delta_{\text{end}} = 1/\gamma_{2r}$. We may assume that if

$$\frac{(\delta_{\text{end}})_{s=0}}{2p_1\tau} \leq 0.1 \quad (12.81)$$

the longitudinal end-effect consequences are negligible for $2p_1 \geq 4$. Condition (12.81) involves only the realistic goodness factor, G_o , and the number of poles.

Indirectly condition (12.81) is related to frequency, pole pitch (synchronous speed), secondary sheet thickness, conductivity, total airgap, etc.

Consequently, two LIMs of quite different speeds and powers may have the same longitudinal effect relative consequences if G_e and $2p_1$ and slip S are the same [14].

End-effect compensation schemes have been introduced earlier [2,3] and recently [15], but they did not prove to be better in terms of overall (global) advantages compared to well-designed LIMs.

By well-designed LIMs for high speed, we mean here those designed for zero longitudinal end-effect force at zero slip, that is designs at optimum goodness factor G_{eo} [5. p. 238] which is solely dependent on the number of poles $2p_1$ (Figure 12.20).

G_{eo} is a rather intuitive compromise as higher G_e leads to both conventional performance enhancement and increase in the longitudinal effect adverse influence on performance.

LIMs where the dynamic longitudinal end effect may be neglected are called low-speed LIMs or linear induction actuators, whereas the rest of them are called high-speed LIMs.

High-speed LIMs are used for transportation – urban and interurban. In urban (suburban) transportation, the speed hardly goes above 20(30) m/s, but this is enough to make the longitudinal end effects worth considering, at least by global thrust correction coefficients [16].

12.10 LINEAR FLAT INDUCTION ACTUATORS (NO LONGITUDINAL END EFFECT)

Again, we mean by linear induction actuators (LIAs) low-speed short travel, linear induction motor drives for which the dynamic longitudinal end effect may be neglected (12.81).

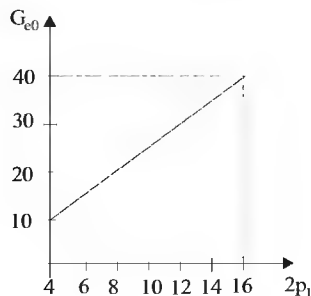


FIGURE 12.20 The optimum goodness factor.

Most LIAs are single sided (flat and tubular) with short primary and long conductor sheet-iron or ladder secondary in a laminated slotted core. For double-sided LIA, the long primary and short (moving) secondary configuration is of practical interest.

12.10.1 THE EQUIVALENT CIRCUIT

All specific effects – airgap leakage, aluminium plate skin effect and transverse edge effects – have been considered and their effects lumped into equivalent airgap g_e (12.41) and aluminium sheet conductivity σ_e (12.40).

These expressions account also for the solid secondary back iron contribution in eddy currents and magnetic saturation. The secondary resistance R'_2 reduced to the primary and the magnetizing reactance X_m (the secondary leakage reactance is neglected) can be adapted from (12.24) to (12.25) as

$$X_m = \frac{\mu_0 \omega_l (W_l K_{wl})^2 \tau (2a_e)}{\pi^2 p_l g_e (S \omega_l, I_l)} = K_m W_l^2 \quad (12.82)$$

$$R'_2 = \frac{X_m}{G_e} = \frac{12 (W_l K_{wl})^2 a_e}{p_l \tau d \sigma_e (S \omega_l, I_l)} = K_{R2} W_l^2 \quad (12.83)$$

So, in fact, both X_m and R'_2 vary with slip frequency $S \omega_l$ and the stator current due to skin effect and transverse edge effect in both, the conducting sheet and the back iron (if it is made of solid iron).

The equivalent circuit of rotary IM is now valid for LIAs, but with the variable parameters X_m and R'_2 (Figure 12.21).

To fully exploit the equivalent circuit, we still need the expressions of primary resistance R_l and leakage reactance $X_{l\sigma}$.

$$R_l = \frac{1}{\sigma_{Co}} \frac{(4a + 2l_{ec})}{W_l I_l} W_l^2 j_{con} = K_{R1} W_l^2 \quad (12.84)$$

where l_{ec} is the coil end-connection length, j_{con} rated current density, and σ_{Co} copper electrical conductivity.

$$X_{l\sigma} = \frac{2\mu_0 \omega_l}{P_l q} [(\lambda_s + \lambda_d) 2a + \lambda_r l_{ec}] W_l^2 = K_{l\sigma} W_l^2 \quad (12.85)$$

$$\lambda_s \approx \frac{1}{12} \frac{h_s}{b_s} (1 + 3\beta'); \quad \beta' = \frac{y}{\tau} (\text{coil span}) \quad (12.86)$$

$$\lambda_d = \frac{5 \frac{g}{b_s}}{5 + 4 \frac{g}{b_s}} \quad (12.87)$$

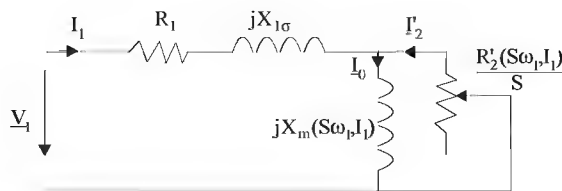


FIGURE 12.21 Linear induction actuator equivalent circuit.

where b_s and h_s are open slot width and height, respectively.

$$\lambda_f \approx 0.3q(3\beta - 1); \quad l_{ec} \approx K_f \tau; \quad K_f = 1.3 \div 1.6 \quad (12.88)$$

where λ_s , λ_d , and λ_f are slot, differential, and end connection-specific geometrical permeances.

The primary phase mmf $W_1 I_1$ (RMS) is

$$W_1 I_1 = j_{con} \left(\frac{\tau}{3q} \right)^2 K_d^2 \frac{h_s}{b_s} K_{fill} \quad (12.89)$$

where $K_d = \frac{3qb_s}{\tau}$, the open slot width/slot pitch, is 0.5–0.7 for LIAs; h_s/b_s is the slot aspect ratio and varies between 3 and 6(7) for LIAs; and K_{fill} is the slot fill factor and varies in the interval $K_{fill} = 0.35$ –0.45 for round wire random wound coils and is 0.6–0.7 for preformed rectangular wire coils.

12.10.2 PERFORMANCE COMPUTATION

Making use of the equivalent circuit (Figure 12.21), with all parameters already calculated, we can simply determine the thrust F_x as

$$F_x = \frac{3I_2'^2 R_2'}{S \cdot 2\tau f_1} = \frac{3I_1^2 R_2'}{S \cdot 2\tau f_1 \left[\left(\frac{1}{SG_e} \right)^2 + 1 \right]} \quad (12.90)$$

The efficiency η_1 and power factor $\cos \varphi_1$ are

$$\eta_1 = \frac{2\tau f_1 (1-S) F_x}{2\tau f_1 F_x + 3I_1^2 R_1} \quad (12.91)$$

$$\cos \varphi_1 = \frac{2\tau f_1 F_x + 3I_1^2 R_1}{3V_{1f} I_1} \quad (12.92)$$

As expected, the realistic goodness factor G_e plays an important role in thrust production (for given stator current I_1) and in performance (Figure 12.22).

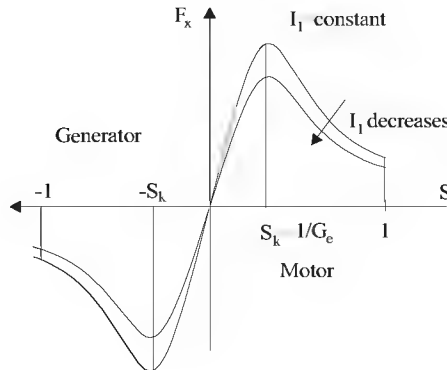


FIGURE 12.22 Thrust versus slip.

The peak thrust for given stator current follows from (12.90) with

$$\frac{\partial F_x}{\partial S} = 0 \rightarrow S_k G_e = \pm 1; \quad F_{xk} = \frac{\pm 3I_1^2 R'_2 G_e}{2\tau f_1 \cdot 2} \quad (12.93)$$

Evidently (12.93) is valid only if R'_2 , X_m , and G_e are constant.

In many low-speed applications, the realistic goodness is around unity. If peak thrust/current is needed at standstill, then $S_k = 1$ and thus the design goodness factor at start is

$$G_e = \omega_1 \sigma_e \mu_0 \frac{d}{g_e} \frac{\tau^2}{\pi^2} = 1 \quad \text{for} \quad S_k = 1 \quad (12.94)$$

Here, $\omega_1 = 2\pi f_1$ is the primary frequency at start. If a variable frequency converter is used, the starting frequency is reduced.

The thrust/secondary losses is, from (12.90),

$$\frac{F_x}{P_2} = \frac{F_x}{3I_2^2 R'_2} = \frac{1}{2\tau f_1 \cdot S} \quad (12.95)$$

For $S = 1$ the smaller the frequency f_1 at start, for given τ , the larger the thrust/secondary conductor losses.

However as the value of τ decreases, to reduce the back-core height, for given airgap flux density, the starting frequency does not fall below (5–6) Hz for practical cases.

12.10.3 NORMAL FORCE IN SINGLE-SIDED CONFIGURATIONS

The normal force between single-sided LIAs primary and secondary has two components:

- An attraction force F_{na} between the primary core and the secondary back iron produced by the normal airgap flux density component.
- A repulsion force F_{nr} between the primary and secondary current mmfs.

As the longitudinal end effect is absent, F_{na} is

$$F_{na} = \frac{2a_e \left| \underline{B}_g \right|^2 2p_1 \tau}{2\mu_0} \quad (12.96)$$

Substituting $\underline{B}_g$ from (12.44) in (12.96), F_{na} becomes

$$F_{na} \approx \frac{a_e \mu_0 J_{1m}^2 \tau^2 2p_1 \tau}{\pi^2 g_e^2 (1 + S^2 G_e^2)} \quad (12.97)$$

To calculate the repulsive force F_{nr} , the tangential component (along OX) of airgap magnetic field H_x is needed. The value of H_x along the primary surface is

$$H_x = J_m; \quad B_x = \mu_0 J_m \quad (12.98)$$

$$F_{nr} = 4 \frac{a_e}{2} \tau p_1 d \mu_0 \operatorname{Re} \left[\underline{J}_2^* \underline{J}_m \right] \quad (12.99)$$

$$\text{But } \underline{J}_2 = S \omega_1 \sigma_e \frac{\pi}{\tau} \underline{B}_g \quad (12.100)$$

Finally,

$$F_{nr} \approx -2a_e \tau S \omega_l a_e d p_l \frac{\tau^2 (\mu_0 J_m)^2 S G_e}{\pi^2 g_e (1 + S^2 G_e^2)} \quad (12.101)$$

The net normal force F_n is

$$F_n = F_{na} + F_{nr} = 2a_e p_l \tau \frac{\mu_0 J_m^2 \tau^2}{\pi^2 g_e^2 (1 + S^2 G_e^2)} \left[1 - \left(\frac{g_e \pi}{\tau} \right)^2 S^2 G_e^2 \right] \quad (12.102)$$

From (12.102), the net normal force becomes repulsive if

$$S G_e > \frac{\tau}{g_e \pi} \quad (12.103)$$

which is not the case in most low-speed LIMs (LIAs) even at zero speed ($S = 1$).

Core losses in the primary core have been so far neglected, but they may be added as in rotary IMs. Anyway, they tend to be relatively smaller as the airgap flux density is only $B_{gn} = (0.2\text{--}0.45)$ T, because of the rather large magnetic airgap (air-plus-conductor sheet thickness).

12.10.4 A NUMERICAL EXAMPLE

Let us consider a sheet on iron secondary (single-sided) LIA. The initial data are pole pitch $\tau = 0.084$ m, $2a_e = 0.08$ m, the number of secondary back iron “solid” laminations is $i = 3$, number of poles $2p_l = 6$, $q = 2$, $W_l = 480$ turns/phase, $g = 0.012$ m (out of which $d_{Al} = 0.006$ m), aluminium plate width $2c = 0.12$ m, $\sigma_{Al} = 3.5 \cdot 10^7 (\Omega\text{m})^{-1}$, $\sigma_{iron} = 3.55 \cdot 10^6 (\Omega\text{m})^{-1}$, slot depth $h_s = 0.045$ m, slot width $b_s = 0.009$ m, and $\beta' = y/\tau = 5/6$ (coil span ratio).

The equivalent conductivity and airgap ratios versus frequency at start ($S = 1$) and the thrust and goodness factor for given (constant) stator current $I_l = 3$ A (RMS) are shown in Figure 12.23 [5].

Figure 12.23 shows the influence of skin effect, saturation, and eddy currents in the secondary back iron.

As the frequency increases, G_e slowly deteriorates (decreases) because the skin depth δ_i in iron decreases and so the back iron contribution to the equivalent airgap increases.

The airgap ratio shows a minimum. It is the σ_e/g_e ratio that counts in the goodness factor. For our case, approaching $f_l = 10$ Hz at start seems a good choice.

12.10.5 DESIGN METHODOLOGY BY EXAMPLE

In a single-sided LIA, the back iron in the secondary is necessary to provide a low magnetic reluctance path for the magnetic flux which crosses the airgap. Usage of a solid back iron is imposed by economic reasons. If a laminated secondary core is to be used to step-up performance, then a ladder secondary in slots seems the adequate choice. This case will be investigated in a separate section.

For the time being, we consider the case of secondary solid back iron. Due to larger equivalent airgap per pole pitch ratio (g_e/τ) both, the efficiency and power factor, are not particularly high: they do not seem the best design criteria to start with.

We also need some data from past experience: mechanical gap $g\text{--}d = 1\text{--}4$ mm, airgap flux density $B_g = 0.15$ to 0.35 T, aluminium sheet thickness $d = 2\text{--}4$ mm, slot depth/slot width, $K_{slot} = h_s/b_s = 3\text{--}6$.

The number of poles should be $2p_l = 4, 6, 8$. The case of $2p_l = 2$ should be used only in very small thrust applications. The secondary overhangs ($c\text{--}a$) should not surpass τ/π . Let us consider $c\text{--}a = \tau/\pi$.

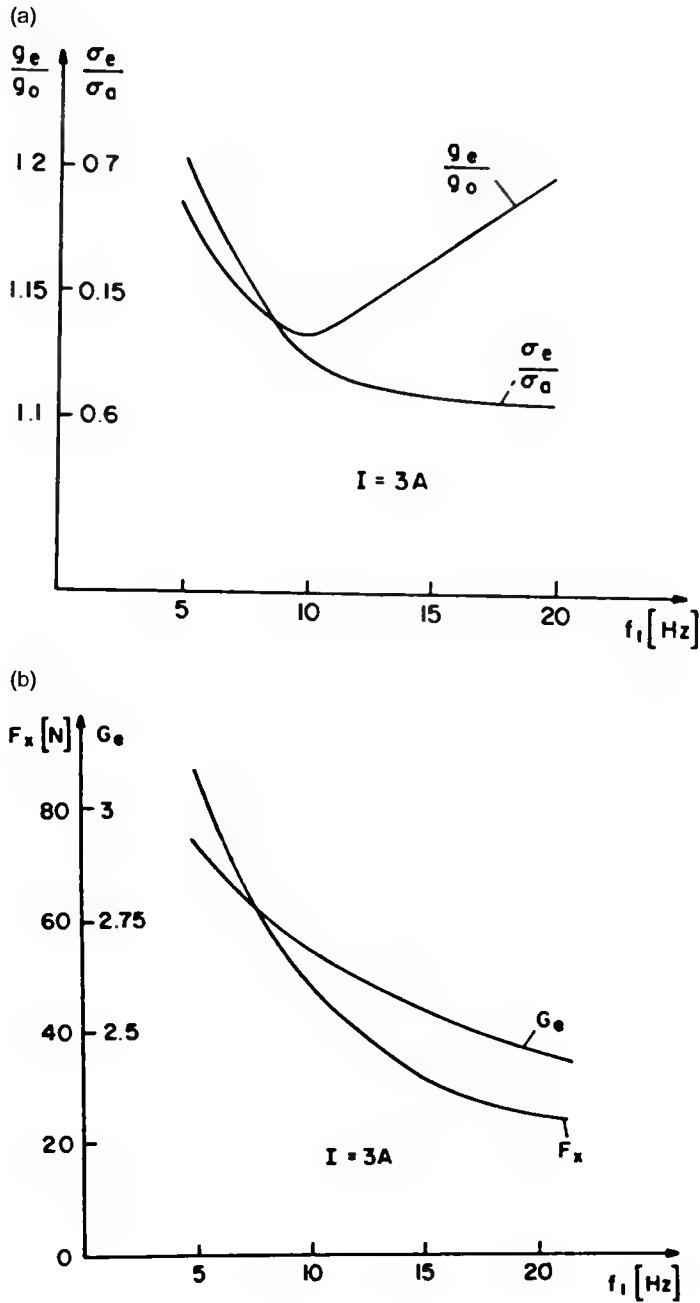


FIGURE 12.23 Equivalent airgap (g_e/g) and secondary conductivity (σ_e/σ_{Al}) thrust F_x and goodness factor G_e versus frequency at standstill. (After Ref. [5].)

The basic design criterion is chosen as maximum thrust/given stator current (12.93) which yields

$$S_k G_e = 1 \quad (12.104)$$

This case corresponds approximately to maximum thrust per conductor losses.

Design specifications are considered here to be

- Peak thrust up to rated speed $F_{xk} = 1500 \text{ N}$
- Rated speed: $u_n = 6 \text{ m/s}$
- Rated frequency: $f_{ln} = 50 \text{ Hz}$
- Rated phase voltage: $V_l = 220 \text{ V (RMS)}$ star connection
- Variable frequency converter supply
- Excursion length $L_l = 50 \text{ m}$.

The design algorithm unfolds as follows:

The peak thrust F_{xk} (from (12.90) with R'_2 from (12.83)) is

$$F_{xn} = (F_{xk})_{S_k G_e = 1} = \frac{3I_l^2 (W_l K_{wl})^2 12a_e}{d_{Al} \tau p_l \sigma_e (2\tau f_l - u_n)} \frac{1}{2} = 1500 \text{ N} \quad (12.105)$$

Also from (12.104) with G_e from (12.43)

$$S_k G_e = 1 = 2\mu_0 \frac{\tau^2 f_l}{\pi g_e} S_k d_a \sigma_e \quad (12.106)$$

The rated speed

$$u_n = 2f_l \tau (1 - S_k) \quad (12.107)$$

Also from (12.43), the airgap flux density for $S_k G_e = 1$ is

$$B_{gn} = (B_g)_{S_k G_e = 1} = \frac{3\sqrt{2} W_l K_{wl} \mu_0 I_l}{g_e p_l \pi \sqrt{2}}; \quad B_{gn} = 0.3 \text{ T} \quad (12.108)$$

The secondary back iron tangential flux density B_{xin} should also be limited to a given value, say $B_{xin} = 1.7 \text{ T}$ (even higher values may be accepted). From (12.39), the secondary iron penetration depth required δ_{in} is

$$\delta_{in} = \frac{\tau B_{gn} K_{pf}}{\pi B_{xin}}; \quad K_{pf} = 1.6 \quad (12.109)$$

We may also choose the rated current density $j_{con} = 4 \text{ A/mm}^2$ (for no cooling), the number of slots per pole and phase $q = 2$, slot aspect ratio $K_{slot} = (3-6)$ and slot/pole pitch area ratio $K_d = 3qb/\tau = 0.5-0.7$, and slot fill factor $K_{fill} = 0.35-0.45$. With these data, the phase mmf $W_l I_{ln}$ is

$$W_l I_{ln} = C_s p_l \tau^2; \quad C_s = \frac{1}{9q} K_d^2 K_{slot} K_{fill} j_{con} \quad (12.110)$$

with

$$K_d = \frac{3b_s q}{\tau} = 1 - \frac{B_{gn}}{B_{tn}} \quad (12.111)$$

where B_{tn} is the primary tooth design flux density.

The unknowns of the problem as defined so far are τ , a_e/τ , p_l , $S_k \omega_l$, and d_{Al} , and only an iterative procedure can lead to solutions.

However, by choosing a_e/τ as a parameter the problem simplifies considerably.

From (12.109) and (12.38), we get

$$\tau^2 S_k \omega_l = \frac{K_{ti} \pi^2}{\mu_0 \sigma_i} \sqrt{\left(\frac{B_{xin}}{B_{gn}} \right)^4 - 4} = c_\tau \quad (12.112)$$

The thrust F_{xk} , from (12.105) with (12.108) and (12.106), is

$$F_{xn} = 6C_s B_{gn} K_{wl} p_l \tau^3 \left(\frac{a_e}{\tau} \right) \quad (12.113)$$

As the primary frequency f_l is given, we may choose also the number of poles as a parameter $2p_l = 4, 6 \dots$. In this case from (12.113), the pole pitch τ can be calculated as

$$\tau = \sqrt[3]{\frac{F_{xn}}{6C_s B_{gn} K_{wl} p_l \left(\frac{a_e}{\tau} \right)}} \quad (12.114)$$

Now, $S_k \omega_l$ is

$$S_k \omega_l = 2\pi f_l - u_n \frac{\pi}{\tau} \quad (12.115)$$

Alternatively, we may have used (12.112) to find $S_k \omega_l$ in which case even f_l from (12.115) could have been calculated. This would be the case with f_{ln} not specified. Anyways, (12.112) is to be checked.

Now, from (12.105) to (12.108)

$$\sigma_e d_{Al} = \frac{3C_s \tau \pi}{S_k \omega_l B_{gn}} \quad (12.116)$$

From (12.40), the value $\sigma_{Alskin} d_a$ is

$$\sigma_{Alskin} d_{Al} = \left(\frac{3C_s \tau \pi}{S_k \omega_l B_g} - \frac{\sigma_i \delta_i}{K_{ti}} \right) K_{ta} \quad (12.117)$$

As K_{ta} comes from Equations (12.14), (12.26) to (12.30), where G_e (which contains $\sigma_{Alskin} d_a$) is included, the computation of $\sigma_{Alskin} d_{Al}$ may be done only iteratively from these equations.

With $\sigma_{Alskin} d_{Al}$ known, again, iteratively, d_{Al} may be determined (the skin effect depends on d_{Al}). A valid solution is found if $d_{Al} < g_e$.

With the above procedure, the equivalent parameters entering the equivalent circuit, X_m , R_2 , R_1 , and $X_{l\sigma}$ from (12.92) to (12.88), for rated speed u_n and thrust F_{xn} , can be calculated. The only unknown is now the number of turns per phase W_l

$$W_l = \frac{V_l}{W_l I_{ln} \left[\left(K_{R1} + \frac{K'_{R2}}{S_k (1 + (1/S_k G_e)^2)} \right)^2 + \left(K_{X1\sigma} + \frac{K_m}{(1 + (S_k G_e)^2)} \right)^2 \right]} \quad (12.118)$$

with $W_l I_{ln}$ from (12.110), (12.118) provides a unique value for W_l .

To choose the adequate a_c/τ ratio, which will lead to the best design, the cost C_t of both primary (C_p) and secondary (C_{sec}) is evaluated:

$$C_t = C_p + C_{sec} \quad (12.119)$$

$$C_p = \frac{6W_l I_{ln}}{J_{con}} (2a + K_l \tau) \gamma_{Co} C_{Co} + \left[2P_l \tau (1 - K_d) \cdot 2a K_{slot} \frac{K_d \tau}{3q} + 3\delta_i 2P_l \tau \cdot 2a \right] \gamma_i C_{ii} \quad (12.120)$$

where γ_{Co} , γ_i , C_{Co} , and C_{ii} are the specific weights and specific prices of copper and core, respectively. C_{sec} is given as

$$C_{sec} = \left[\left(2a + \frac{2\tau}{\pi} \right) d_{Al} \gamma_{Al} C_{Al} + 3\delta_i 2a \gamma_i C_i \right] L_l \quad (12.121)$$

where γ_{Al} and C_{Al} are aluminium specific weight and specific cost, respectively, and L_l is the total LIA secondary length (excursion length + primary length).

When $L_l \gg 2p_l \tau$ the secondary cost becomes an important cost factor. Thus, a high a_c/τ ratio leads to a smaller length primary but also to a wider (large cost) secondary. The final choice is left to the designer. The apparent power S_l may be used as an indicator of converter costs.

For our numerical case, the results in Figure 12.24 are obtained. The total cost/iron specific cost (C_t/C_{ii}) has a minimum for $a_c/\tau = 0.7$, but this corresponds to a rather high S_l (KVA). A high S_l means a higher cost frequency converter. A good compromise would be $a_c/\tau = 0.9$, $S_k \omega_l = 93 \text{ rad/s}$, $\tau = 0.085 \text{ m}$, $d_{Al} = 1.8 \cdot 10^{-3} \text{ m}$, $W_l = 384 \text{ turns/phase}$, $I_{ln} = 32.757 \text{ A}$, $S_l = 21.92 \text{ KVA}$.

Now that the LIA dimensions are all known, the performance may be calculated iteratively for every (V_l , f_l) pair and speed U .

For a double-sided LIA with aluminium sheet secondary, the same design procedure may be used, after its drastic simplification due to the absence of solid iron in the secondary. Also the two-sided windings may be lumped into an equivalent one.

12.10.6 THE LADDER SECONDARY

Lowering the airgap to $g = 1 \text{ mm}$ or even less in special applications with LIAs may be feasible. In such cases, the primary slots should be semiclosed to reduce Carter's coefficient value and additional losses. The secondary may then be built from a laminated core with, again, semiclosed slots, for same reasons as above (Figure 12.25).

To calculate the performance, the conventional equivalent circuit is used. The secondary parameter expressions are identical to those derived for the rotary IM. The adequate number of combinations of primary/secondary slots (per primary length) is the same as for rotary IM. In fact, the whole design process is the same as for the rotary IM.

The only difference is that the electromagnetic power is

$$P_{elm} = F_x 2\tau f_l = \frac{3R'_2 I_2'^2}{S} \quad (12.122)$$

instead of

$$P_{elm} = T_e \frac{\omega_l}{p_l} = \frac{3R'_2 I_2'^2}{S} \quad (12.123)$$

for rotary IMs.

For short excursion (less than a few metres) and $g \approx 1 \text{ mm}$, quite good performance may be obtained with ladder secondary.

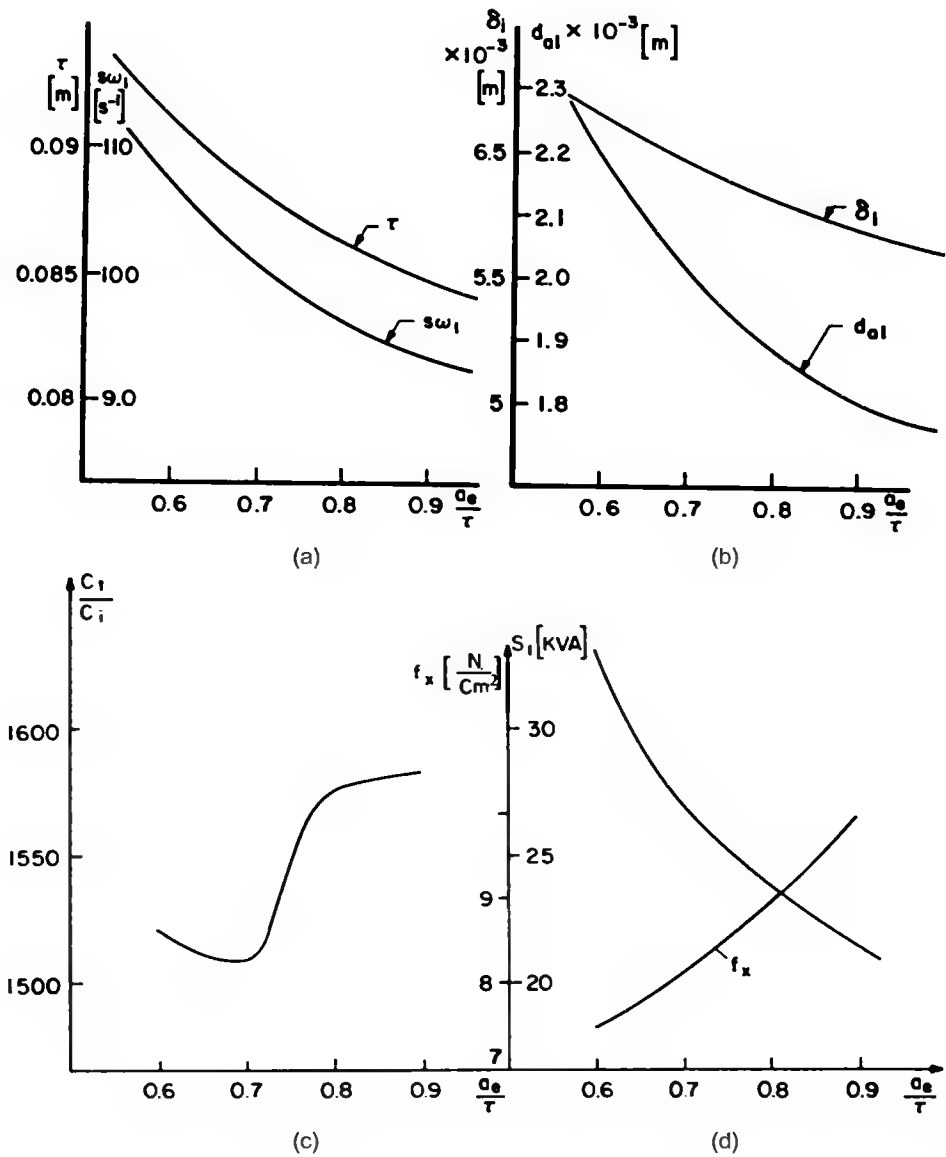


FIGURE 12.24 Design numerical results: (a) $S_k \omega_1$, (b) iron penetration depth δ_i , aluminium thickness d_{Al} , (c) cost versus a_e/τ , and (d) S_1 (KVA), specific thrust f_x (N/cm²). (Adapted from Ref. [10].)

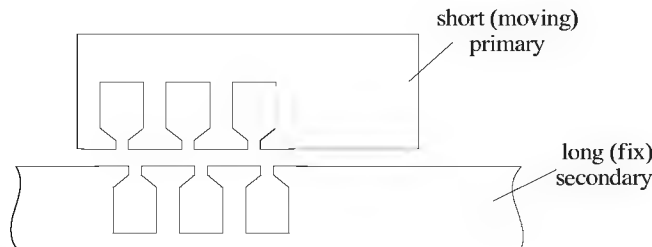


FIGURE 12.25 LIA with ladder secondary.

12.11 TUBULAR LIAS

There are two main tubular LIA configurations: one with longitudinal primary laminations stack and the other with ring-shaped laminations and secondary ring-shaped conductors in slots (Figure 12.26).

Due to manufacturing advantages, we consider here – except for liquid pump applications – disk-shaped laminations (Figure 12.26).

The main problem of this configuration is that it has open slots on both sides of the airgap.

To limit the airgap flux harmonics (with all their consequences in “parasitic” losses and forces), for an airgap g ($g \approx 1$ mm) the primary slot width b_s should not be larger than $(4-6)g$. Fortunately, this suffices for many practical cases.

The secondary ring width b_r should not be larger than $(3-4)g$. Another secondary effect originates in the fact that the back-core flux path goes perpendicular to laminations.

Consequently, the total airgap is increased at least by the insulation total thickness along say $\approx \tau/3$. Fortunately, this increase by g_{ad} is not very large:

$$g_{ad} = 0.03 \frac{1}{3} \tau \approx 0.01 \tau \quad (12.124)$$

For a pole pitch, $\tau = 30$ mm, $g_{ad} = 0.3-0.4$ mm, which is acceptable. A very good stacking factor of 0.97 has been assumed in (12.124). Also eddy current core losses will be increased notably in the back iron due to the perpendicular field (Figure 12.27).

Performing slits in the back-core disk-shaped laminations during stamping leads to a notable reduction of eddy current losses. Finally, the secondary is to be coated with mechanically resilient, nonconducting, nonmagnetic material for allowing the use of linear bearings.

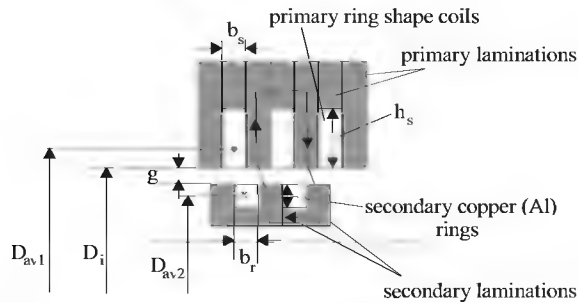


FIGURE 12.26 Tubular LIA with secondary conductor rings.

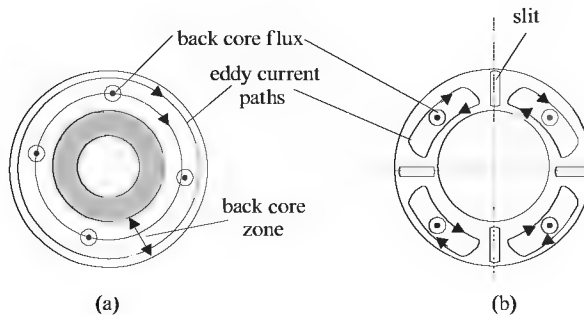


FIGURE 12.27 Tubular LIA back-core disk-shaped lamination (a) and its slitting (b) to reduce eddy current losses.

Once these peculiarities are taken care of, the tubular LIM performance computation runs smoothly. The absence of end connections, in both the primary and secondary coils, is a definite advantage of tubular configurations.

So no transverse edge effect occurs in the secondary. The primary back iron is not likely to saturate, but the secondary back iron may do so as its area for the half-pole flux is much smaller than in the primary.

The equivalent circuit parameters are

$$R_{1t} = \frac{1}{\sigma_{Co}} \frac{\pi D_{av1} J_{con}}{W_l I_{ln}} W_l^2 \quad (12.125)$$

$$X_{1t} = 2\mu_0 \omega_l \frac{\pi D_{av1}}{p_l q} (\lambda_{s1} + \lambda_{d1}) W_l^2 \quad (12.126)$$

We have just “marked” in (12.125) and (12.126) that the primary coil average length is πD_{av1} . For the secondary with copper rings,

$$X'_{2r} = 2\mu_0 \omega_l \pi D_{av2} \cdot 12 \frac{(W_l K_{wl})^2}{N_{s2}} (\lambda_{s2} + \lambda_{d2}) \quad (12.127)$$

$$R'_{2r} = \frac{1}{\sigma_{Co}} \frac{\pi D_{av2}}{A_{ring}} \cdot 12 \frac{(W_l K_{wl})^2}{N_{s2}} \quad (12.128)$$

where λ_s and λ_d are defined in (12.86) and (12.87), N_{s2} is the number of secondary slots along primary length, and A_{ring} is the copper ring cross section.

The magnetization reactance X_m is

$$X_m = \frac{6\mu_0 \omega_l}{\pi^2} (W_l K_{wl})^2 \frac{\pi D_i \tau}{p_l g K_c \left(1 + 2 \frac{g_{ad}}{g} + K_{sat} \right)} \quad (12.129)$$

where K_{sat} , iron saturation coefficient, should be kept (in general) below 0.4–0.5.

For an aluminium sheet secondary on laminated core,

$$R'_2 = \frac{6\pi (D_i - 2g - d_{Al}) (W_l K_{wl})^2}{p_l \tau d_{Al} \sigma_{Al}}; \quad X'_2 = 0 \quad (12.130)$$

The equivalent circuit (Figure 12.28) now contains also the secondary leakage reactance which is however small as the slots are open and not very deep.

Based on the equivalent circuit, all performance may be obtained at ease.

The tubular LIA is used for short stroke applications which implicitly leads to maximum speeds below 2–3 m/s. Consequently, the design of tubular LIAs, in terms of energy conversion, should aim to produce the maximum thrust at standstill per unit of apparent power, if possible.

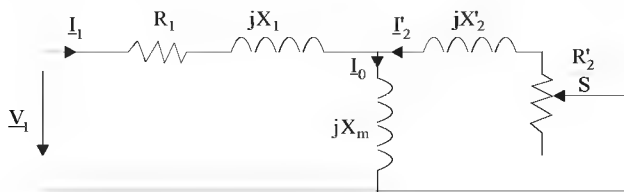


FIGURE 12.28 Equivalent circuit of tubular LIA with secondary copper rings in slots.

The primary frequency may be varied (and controlled) through a static power converter.

In terms of costs, both the LIA and power converter costs are to be considered. Hereby, however, we illustrate the criterion of maximum thrust at standstill.

The electromagnetic power P_{elm} is

$$P_{elm} = F_x 2\tau f_l = 3R_e I_l^2 \quad (12.131)$$

$$R_e = \frac{\frac{R'_2}{S} X_m^2}{\left(\frac{R'_2}{S}\right)^2 + (X_m + X'_2)^2} \quad (12.132)$$

or with

$$G_e = \frac{X_m}{R'_2}, \quad R_e = \frac{R'_2}{S \left[\left(\frac{1}{S G_e} \right)^2 + \left(1 + \frac{X'_2}{X_m} \right)^2 \right]} \quad (12.133)$$

Now, the maximum thrust is obtained for

$$(S_k G_e)_{opt} = \frac{1}{1 + \frac{X'_2}{X_m}} \quad (12.134)$$

For standstill, $S_k = 1$. When initiating the design process, the ratio X'_2/X_m is not known and can only be assigned a value to be adjusted iteratively later on. A good start would be $X'_2/X_m = 0.2$.

From now on, the design methodology developed for the flat LIA may be used here.

12.11.1 A NUMERICAL EXAMPLE

Let us consider a tubular LIA with copper ring secondary and the data primary bore diameter $D_i = 0.15$ m, primary external diameter $D_e = 0.27$ m, pole pitch $\tau = 0.06$ m, $q = 1$ slots/pole/phase, primary slot depth $h_s = 0.04$ m, primary slot width $b_s = 0.0125$ m, number of poles $2p_1 = 8$, the secondary slot pitch $\tau_{s2} = 10^{-2}$ m, secondary slot width $b_{s2} = 6 \cdot 10^{-3}$ m, secondary slot depth $h_{s2} = 4 \cdot 10^{-3}$ m, airgap $g = 2 \cdot 10^{-3}$ m, current density $j_{con} = 3 \cdot 10^6$ A/m², and slot fill factor $K_{fill} = 0.6$.

Let us determine the frequency f_2 for maximum thrust at standstill and the corresponding thrust, apparent power, and conductor losses.

First, with the available data, we may calculate from (12.127) the secondary leakage reactance as a function of $f_l W_l K_{wl}$:

$$X'_2 = 4\mu_0 \pi D_{av2} \cdot \frac{12}{N_{s2}} (\lambda_{s2} + \lambda_{d2}) f_l W_l^2 K_{wl}^2 = 1.97 \cdot 10^{-6} f_l W_l^2 K_{wl}^2 \quad (12.135)$$

with

$$\lambda_{s2} = \frac{h_{s2}}{3b_{s2}} = \frac{4}{3 \cdot 6} = 0.33; \quad \lambda_{d2} = \frac{\frac{5g}{b_{s2}}}{5 + \frac{4g_0}{b_{s2}}} = \frac{\frac{5 \cdot 3}{6}}{5 + \frac{4 \cdot 3}{6}} = 0.57 \quad (12.136)$$

The secondary resistance (12.128) is

$$R'_2 = \frac{1}{\sigma_{Co}} \frac{\pi D_{av2}}{h_{s2} b_{s2}} \cdot 12 (W_1 K_{w1})^2 = 0.876 \cdot 10^{-4} (W_1 K_{w1})^2 \quad (12.137)$$

The magnetic reactance X_m (12.129) becomes

$$X_m = \frac{6\mu_0 \omega_l}{\pi^2} (W_1 K_{w1})^2 \frac{\pi D_i \tau}{P_l g \left(1 + \frac{g_{ad}}{g} + K_{sat} \right)} = 10^{-5} f_l (W_1 K_{w1})^2 \quad (12.138)$$

Let us assign $\frac{g_{ad}}{g} + K_{sat} = 0.5$.

Now, the optimum goodness factor is (from (12.134))

$$G_0 = \frac{X_m}{R'_2} = 0.114 f_{ls} = \frac{1}{1 + \frac{X'_2}{R'_2}} = \frac{1}{1 + 2.24 \cdot 10^{-2} f_{ls}} \quad (12.139)$$

From (12.139), the primary frequency at standstill f_{ls} is $f_{ls} \approx 7.5$ Hz. The primary phase mmf $W_1 I_{ln}$ is

$$W_1 I_{ln} = p q n_s I_{ln} = p_l q (h_s b_s K_{fill} J_{con}) = 4 \cdot 4 \cdot 10^{-2} \cdot 1.2 \cdot 10^{-2} \cdot 0.6 \cdot 3 \cdot 10^{-6} = 3456 \text{ Atums} \quad (12.140)$$

Hence, the primary phase resistance and leakage reactance R_{lt} , X_{lt} are (12.125) and (12.126)

$$R_{lt} = \frac{1}{\sigma_{Co}} \frac{\pi D_{av1} J_{con}}{W_1 I_{ln}} W_l^2 = 1.035 \cdot 10^{-5} W_l^2 \quad (12.141)$$

with $D_{av1} = D_i + h_s = 0.15 + 0.04 = 0.19$ m.

$$X_{lt} = 2\mu_0 \omega_l \frac{\pi D_{av1}}{p_l q} (\lambda_{s1} + \lambda_{d1}) W_l^2 = 2.483 \cdot 10^{-5} W_l^2 \quad (12.142)$$

The number of turns W_l may be determined once the rated current I_{ln} is assigned a value. For $I_{ln} = 20$ A, from (12.140), $W_l = 3456/20 \approx 172$ turns/phase.

The apparent power S_l does not depend on voltage (or on W_l) and is

$$(S_l)_{S=1} = 3I_l^2 \left| R_{lt} + jX_{lt} + \frac{jX_m (R'_2 + jX'_2)}{j(X_m + X'_2) + R'_2} \right| \quad (12.143)$$

With $W_l I_{ln}$ and R_{lt} , X_{lt} , X_m , R'_2 , X'_2 proportional to W_l^2 and with $K_{w1} = 1.00$ ($q = 1$), we obtain

$$S_l \approx 3000 \text{ VA}; \quad \cos \varphi_l \approx 0.52 \quad (12.144)$$

The corresponding phase voltage at start is

$$V_l = \frac{S_l}{3I_{ln}} = \frac{3000}{3 \cdot 20} = 50.00 \text{ V(RMS)} \quad (12.145)$$

The rated input power $P_l = S_l \cos \varphi_l = 3000 \cdot 0.52 = 1560$ W.

The primary conductor loss p_{cos} is

$$p_{cos} = 3R_l I_{ln}^2 = 3 \cdot 0.135 \cdot 10^{-4} (3456)^2 = 370.86 \text{ W} \quad (12.146)$$

The electromagnetic power P_{elm} can be expressed as

$$P_{elm} = P_1 - p_{cos} = 1560 - 370.86 = 1189 \text{ W} \quad (12.147)$$

The thrust at standstill F_{xk} is

$$F_{xk} = \frac{P_{elm}}{2\tau f_1} = \frac{1189}{2 \cdot 0.06 \cdot 7.5} = 1321.26 \text{ N} \quad (12.147')$$

Thus, the peak thrust at standstill per watt is

$$f_{xw} = \frac{F_{xk}}{P_1} = \frac{1321.26}{1560} = 0.847 \text{ N/W} \quad (12.148)$$

Also

$$f_{xVA} = \frac{F_{xk}}{S_1} = \frac{1321.26}{3000} = 0.44 \text{ N/VA} \quad (12.149)$$

Such specific thrust values are typical for tubular LIAs.

12.12 SHORT-SECONDARY DOUBLE-SIDED LIAS

In some industrial applications such as special conveyors, the vehicle is provided with an aluminium (light-weight) sheet which travels guided between two-layer primaries (Figure 12.29).

The primary units may be switched on and off as required by the presence of a vehicle in the nearby unit.

The fact that we deal with a secondary which is shorter than the primary leads to a redistribution of secondary current density along the direction of motion. Let us consider the simplified case of a single short vehicle inside a single longer primary.

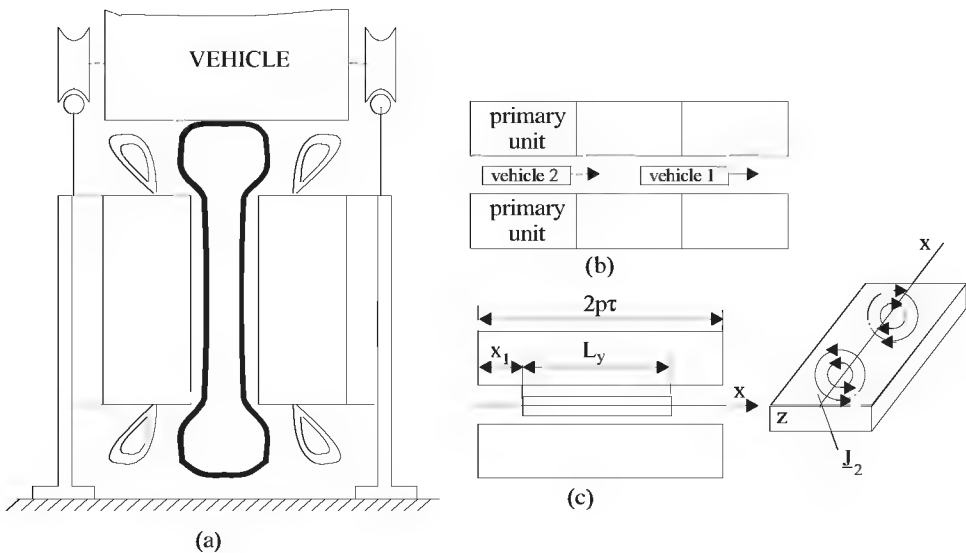


FIGURE 12.29 Short-secondary double-sided LIA: (a) cross section, (b) multiple vehicle system, and (c) single vehicle.

The second current density paths (Figure 12.29c) reveal the transverse edge effect which may be accounted for as in the previous section. Also, the skin effect may be considered as done before in the chapter.

The dynamic longitudinal end effect is considered negligible. So the secondary current density in the secondary is simply having the conventional travelling component and a constant one which provides for the condition

$$\int_0^{L_v} (J_{2z} dx) = 0 \quad (12.150)$$

J_{2z} (from (12.76) with $A_t = B_t = 0$) is

$$J_{2z} = \frac{-jSG_e J_m e^{-j\frac{\pi}{\tau}x}}{d_{Al}(1+jSG_e)} + A \quad (12.151)$$

From (12.150) to (12.151), A is

$$A = \frac{-jSG_e J_m}{d_{Al}(1+jSG_e)} \frac{\tau}{j\pi} \frac{\left(1 - e^{j\frac{\pi}{\tau}L_v}\right)}{L_v} \quad (12.152)$$

In fact, A represents an alternative current density which produces additional losses and a pulsation in thrust. Its frequency is the slip frequency Sf_1 , of the entire J_{2z} .

As expected when $L_v = 2K\tau$, $A = 0$.

Care must be exercised when defining the equivalent parameters of the equivalent circuit, as part of the primary “works” on empty airgap while only the part facing the vehicle is active. Three-dimensional FEM approaches seem most adequate to solve this case.

Linear induction pumps are also complex systems characterized by various phenomena (for details, see Reference [5], Chapter 5).

12.13 LINEAR INDUCTION MOTORS FOR URBAN TRANSPORTATION

LIMs when longitudinal effects have to be considered are typically applied to people movers on airports (Chicago, Dallas–Fortworth, Tokyo), in metropolitan areas (Toronto, Vancouver, Detroit), or even for interurban transportation (S. Korea, China). Only single-sided configurations have been applied so far.

As in the previous section, we did analyse all special effects in LIMs, including the optimal goodness factor definition corresponding to zero longitudinal end-effect force at zero slip (Figure 12.20), we will restrict ourselves here to discuss only some design aspects through a rather practical numerical example. More elaborated optimization design methods including some based on FEM are reported in [17–19]. However, they seem to ignore the transverse edge effect and saturation and eddy currents in the secondary solid back iron.

12.13.1 SPECIFICATIONS

- Peak thrust at standstill, $F_{xk} = 12.0$ kN
- Rated speed, $U_n = 34$ m/s
- Rated thrust, at U_n $F_{xn} = 3.0$ kN;
- Pole pitch, $\tau = 0.25$ m (from 0.2 to 0.3 m)
- Mechanical gap, $g_m = 10^{-2}$ m
- Starting primary current, $I_K = 400$ –500 A.

12.13.2 DATA FROM PAST EXPERIENCE

- The average airgap flux density $B_{gn} = 0.25\text{--}0.40\text{ T}$ for conductor sheet on solid iron secondary and $B_{gn} = 0.35\text{--}0.45\text{ T}$ for ladder-type secondary.
- The primary current sheet fundamental $J_{mk} = 1.5\text{--}2.5 \cdot 10^5\text{ A/m}$.
- The effective thickness of aluminium (copper) sheet of the secondary $d_{Al} = (4\text{--}6)10^{-3}\text{ m}$.
- The pole pitch $\tau = 0.2\text{--}0.3\text{ m}$ to limit the secondary back iron depth and reduce end connections of coils in the primary winding.
- The frequency at standstill and peak thrust should be larger than $f_{isc} = 4\text{--}5\text{ Hz}$ to avoid large vibration and noise during starting.
- The primary stack width/pole pitch: $2a/\tau = 0.75\text{--}1$. The lower limit is required to reduce the too large influence of end-connection losses and the upper to secure reasonable secondary costs.

12.13.3 OBJECTIVE FUNCTIONS

Typical objective functions to minimize are

- Inverse efficiency $F_1 = 1/\eta$
- Secondary costs $F_2 = C_{sec}$
- Minimum primary weight $F_3 = G_1$
- Primary KVA $F_4 = S_{lk}$
- Capitalized cost of losses $F_c = (P_{loss})_{av}$.

A combination of these objectives may be used to obtain a reasonable compromise between energy conversion performance and investment and loss costs [17].

12.13.4 TYPICAL CONSTRAINTS

- Primary temperature $T_1 < 120^\circ$ (with forced cooling)
- Secondary temperature in stations during peak traffic hours T_2
- Core flux densities $< (1.7\text{--}1.9)\text{ T}$
- Mechanical gap $g_m \geq 10^{-2}\text{ m}$.

Some of the unused objective functions may be taken as constraints.

12.13.5 TYPICAL VARIABLES

Integer variables

- Number of poles $2p_1$
- Slots per pole per phase q
- Number of turns per coils n_c ($W_1 = 2p_1 q n_c$).

Real variables

- Pole pitch τ
- Airgap g_m
- Aluminium thickness d_{Al}
- Stack width $2a$
- Primary slot height h_s
- Primary teeth flux density B_{tt}
- Back iron tangential flux density B_{xin}
- Rated frequency f_1 .

12.13.6 THE ANALYSIS MODEL

The analysis model has to make the connection between the variables, the objective functions, and the constraints. So far in this chapter, we have developed a rather complete analytical model which accounts for all specific phenomena in LIMs such as transverse edge effect, saturation and eddy currents in the secondary back iron, airgap leakage, and skin effect in the aluminium sheet of secondary and longitudinal end effect.

So, from this point of view, it would seem reasonable to start with a feasible variable-vector and then calculate the objective function(s) and constraints and use a direct search method to change the variables until sufficient convergence of the objective function is obtained (see the Hooke–Jeeves method presented in Chapter 8).

To avoid the risk of being trapped in a local minimum, a few different starting variable-vectors should be tried and the best design is chosen. Finding a good starting variable-vector is a key factor in such approaches.

On the other hand, genetic algorithms are known for being able to reach global optima as they start with a population of variable-vectors (chromosomes); FEM approaches have been presented in [18].

To find a good initial design, the methodologies developed for rotary IM still hold, but the thrust expression includes an additional term to account for longitudinal effect.

Following the analysis model developed in this chapter, the following results have been obtained for the above initial data $\tau = 0.25$ m, $2p_1 = 10$, $W_1 = 90$, $I_{sc} = 420$ A (for peak thrust), $f_{sc} = 6$ Hz, $2a = 0.27$ m, $V_{sc} = 220$ V (at start), phase voltage at 20 m/s and rated thrust is $V_{in} = 200$ V, efficiency at rated speed and thrust $\eta_1 = 0.8$, rated power factor $\cos \phi_1 = 0.55$. Performance characteristics are shown in Figures 12.30–12.32.

12.13.7 DISCUSSION OF NUMERICAL RESULTS

- Above 80% of rated speed (34 m/s), the efficiency is quite high: 0.8.
- The power factor remains around 0.5 above 50% of rated speed.
- The thrust versus speed curves satisfy typical urban traction requirements.
- Regenerative braking, provided the energy retrieval is accepted through the static converter, is quite good down to 25% of maximum speed. More regenerative power can be produced at high speeds by adequate control (full flux in the machine) but at higher voltages in the D.C. link.
- Though not shown, a substantial net attraction force is developed (two to three times or more than the thrust) which may produce noise and vibration if not kept under control.
- These results may be treated as an educated starting point in the optimization design. Minimum cost of secondary objective function would lead to a not so wide stack

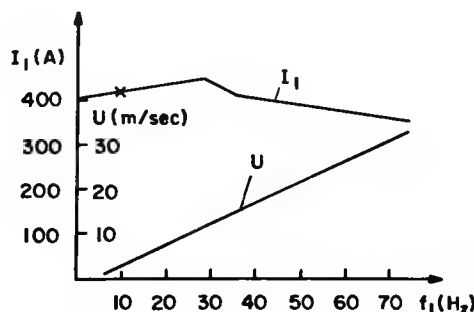


FIGURE 12.30 Imposed current and speed dependence of frequency.

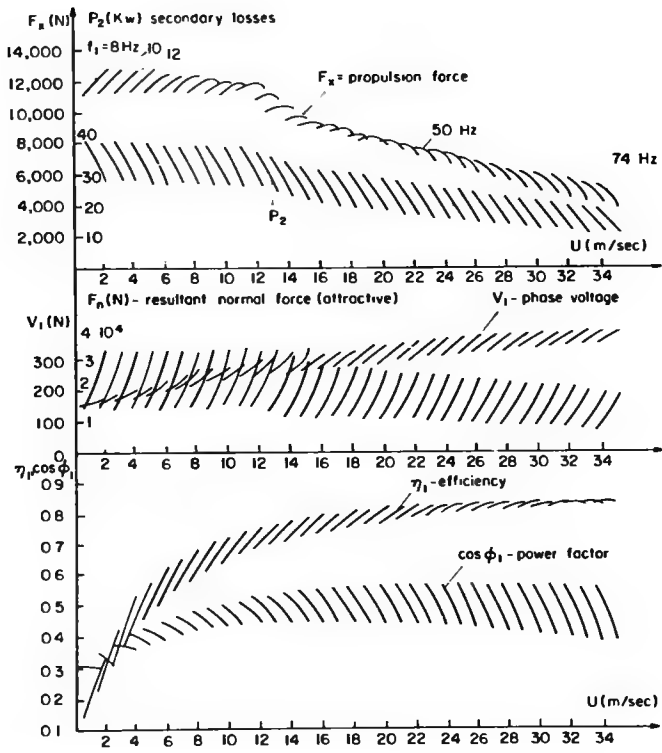


FIGURE 12.31 Motoring performance. (After Refs. [5,10].)

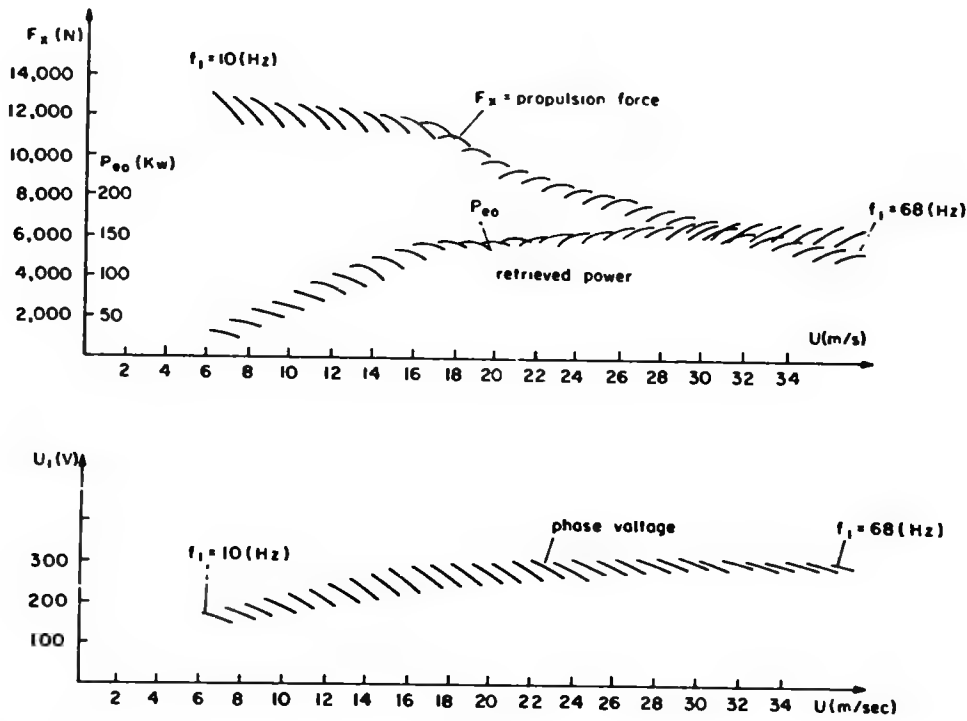


FIGURE 12.32 Generator braking performance.

($2a < 0.27$ m) with a lower efficiency in the motor. The motor tends to be larger. The pole pitch may be slightly smaller [17].

- The thermal design of secondary, especially around the stop stations, during heavy traffic, is a particular constraint which needs a special treatment [5, pp. 248–250].

12.14 TRANSIENTS AND CONTROL OF LIMs

The modelling for transients, as for steady state, given the complex influences of magnetic saturation and skin effects for essentially 3D configurations, should be approached through 3D-FEM or 3D analytical models.

However, for control system design purposes, a circuit model is needed. The methods of LIM control follow the path for rotary induction motors, but constant (or controlled) slip frequency control and stator current control have gained practical acclaim for urban transportation systems.

Such a generic system is shown in Figure 12.33.

The slip frequency $(Sf_1)^*$ versus speed U is imposed a priori. The speed regulator (a robust one: sliding mode type, for example) produces the reference thrust. Now, with speed U , $(Sf_1)^*$ and F_x^* known, we need to calculate the reference current I_s^* . If FEM extended data are available, it is possible to produce, by optimization, both $(Sf_1)^*$ and I_s^* for given F_x^* and speed U , based on, say, max thrust per losses or per stator current.

Adopting for optimization initialization a given constant slip frequency value, constant, or slightly increasing with speed (positive for motoring and negative for regenerative braking) is a practical solution.

However, if these data are not available, it would be practical to have an analytical tool model to calculate the stator current for given thrust, slip frequency, and speed from an equivalent circuit that accounts for, mainly, longitudinal effect.

As shown in Ref. [3] the longitudinal effect deteriorates the thrust, increases the losses in the secondary and deteriorates the power factor.

Starting from the equivalent circuit of rotary IMs, good for linear induction actuators (when longitudinal and effect is negligible) a rather complete parameter equivalent circuit has been introduced for LIMs (Figure 12.34).

The single-sided LIM with aluminium on solid or on secondary is characterized by a sizeable leakage inductance L_{rl} besides resistor R_r .

Based on power balance (calculated by technical field theories or FEM) or by equivalent circuit fitting for a few representative measured speeds U , slip frequencies, and current values, the

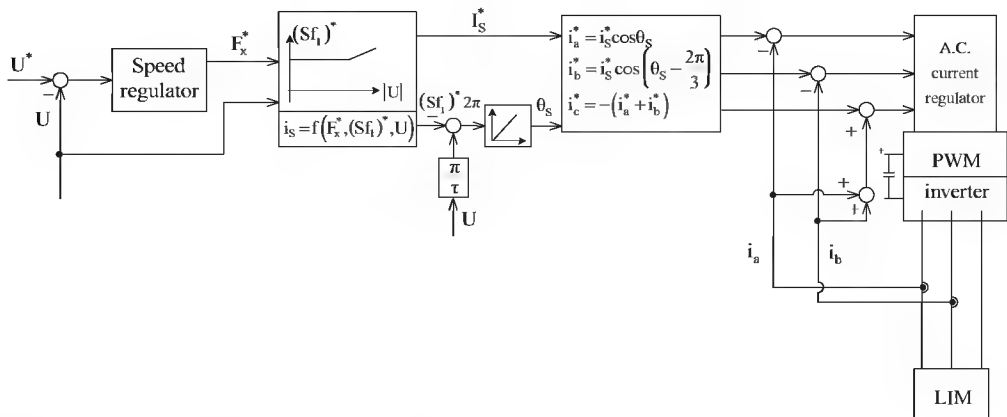


FIGURE 12.33 Generic slip frequency (Sf_1) control of LIM for propulsion only.

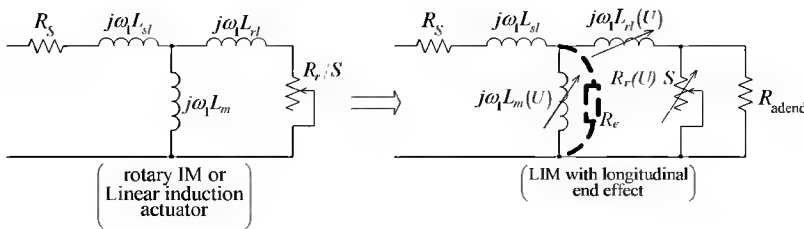


FIGURE 12.34 Complete variable equivalent circuit for LIMs; magnetic saturation is neglected.

magnetizing inductance L_m decreases with increasing speed, the secondary resistance R_r increases with speed, while the secondary leakage inductance remains rather constant, for a 200 km/h urban LIM system as shown in Figure 12.35.

An additional resistance R_{adend} is required to fit both the secondary losses and thrust up to the largest speed when longitudinal end effect is most important.

The main problem to sort out with this equivalent circuit is the design of R_{adend} when the end effect thrust at zero slip, S , changes sign from braking (for large longitudinal effect LIMs, $R_{adend} > 0$) to perhaps a negative value of R_{adend} when the end-effect thrust at zero slip is motoring (for lower longitudinal effect LIMs).

Finally, for LIMs designed for optimal goodness factor (Figure 12.20) when $(F_{x\text{end}})_{S=0} = 0$, apparently R_{adend} should be perhaps discarded.

If combined propulsion and suspension control is pursued, then the stator current control may be used for suspension control, while slip frequency may be used for propulsion control (Figure 12.36). Low limitations on slip frequency have to be eliminated in this case. Alternatively, direct thrust and suspension control may be used as for rotary IMs [20,21].

The control in Figure 12.36 is basically of feedforward A.C. current vector type.

Again, either the equivalent circuit or the FE method may be used to find a reasonable slip frequency reference $(Sf_1)^*$.

Motion sensorless control may be approached as for rotary IMs but with extreme care in urban transportation system applications when notable longitudinal end effect occurs.

Finally, robust control [22] with self-commissioning and online parameter detuning elimination, when magnetic saturation, skin effect, temperatures of stator and mover, and the airgap vary, is still due as urban (and suburban) people movers keep spreading around megacities of the future.

In the absence of longitudinal end effects, the theory of transients as developed for rotary induction motors may be used. The influence of transverse edge effect and secondary eddy currents may

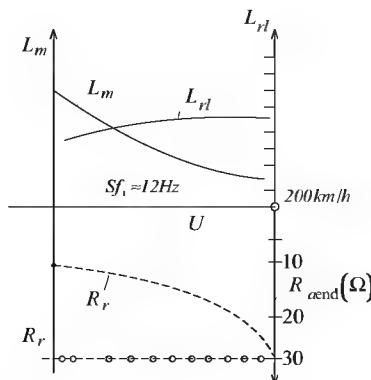


FIGURE 12.35 Parameters' variation with speed at constant slip frequency for a typical urban LIM system.

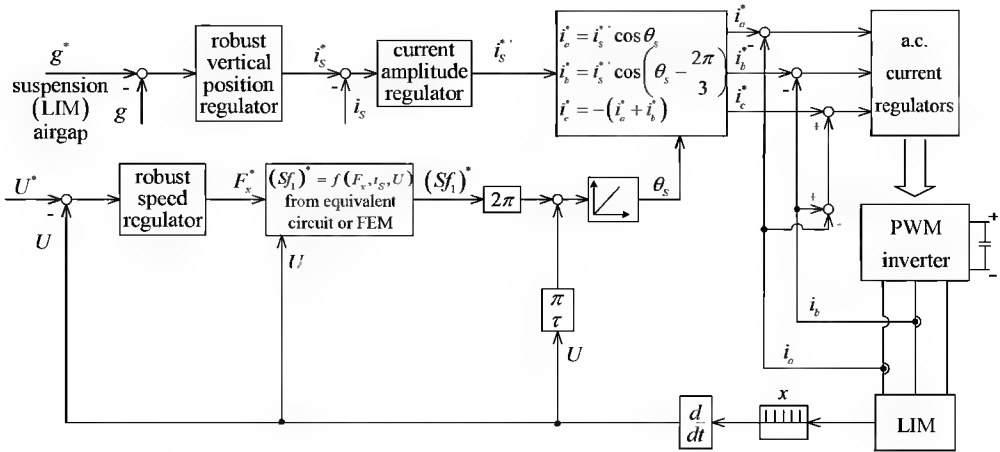


FIGURE 12.36 Combined generic suspension and propulsion LIM control.

be modelled by an equivalent double cage (Chapter 1). The ladder secondary with broken bars may need a bar-to-bar simulation as done for rotary IMs (Chapter 1).

A fictitious ladder [23] (or d-q pole by pole [24]) model of transients may be used to account for longitudinal end effects.

Also FEM-circuit-coupled models have been introduced for the scope [19]. The control of LIMs by static power converters is very similar to that presented in Chapter 8, Vol. 1, for rotary IMs. Early control systems in use today apply slip frequency control in PWM-voltage source inverters such as in the UTDC2 system [25].

Flux control may be used to tame the normal force while still providing for adequate flux and thrust combinations for good efficiency over the entire speed range [26].

Advanced vector control techniques – such as direct torque and flux control (DTFC) – have been proposed for combined levitation–propulsion of a small vehicle for indoor transport in a clean room [20].

12.15 LIM CONTROL WITH DYNAMIC LONGITUDINAL END EFFECT

The rather complete equivalent circuit shown in Figure 12.34 cannot be used in a practical control strategy for high-speed (large dynamic longitudinal end effect) LIMs.

For such purposes, a simplified equivalent circuit [10,17] has been proposed where the dynamic end effect is represented by an additional secondary resistance R_e (in parallel to the main flux branch), flowed by a current $\bar{i}_e$:

$$R_e = \frac{(1 - K_e(U))R'_2}{K_e(U)} \quad (12.153)$$

where R'_2 is the secondary equivalent resistance, and $K_e(U)$ takes into account the dynamic longitudinal end effect:

$$K_e(U) \approx K_1 U + K_2 U^2 \quad (12.154)$$

The coefficients K_1 and K_2 may be obtained by detailed analytical or 3D-FEM calculations. $K_e(U)$ increases monotonically with speed.

In the same time, the airgap flux $\bar{\Psi}_m$ is

$$\bar{\Psi}_m = L_m (1 - K_e(U)) \bar{i}_m \quad (12.155)$$

The stator flux vector $\bar{\Psi}_1$ is

$$\bar{\Psi}_1 = L_{11} \cdot \bar{i}_1 + \bar{\Psi}_m \quad (12.156)$$

The dynamic longitudinal end effect is represented by R_e through a fictitious winding:

$$\bar{\Psi}_1 = L_{11} \cdot \bar{i}_1 + \bar{\Psi}_m \quad (12.157)$$

The thrust $\hat{F}_x$ is thus

$$\hat{F}_x = \frac{3}{2} \frac{\pi}{\tau} \text{Imag} \left[(\bar{\Psi}_1 \cdot \bar{i}_1^*) - (\bar{\Psi}_m \cdot \bar{i}_e^*) \right] \quad (12.158)$$

The secondary circuit in stator coordinates is

$$0 = R'_2 \bar{i}'_2 + p \bar{\Psi}'_2 + j \frac{\pi}{\tau} U \bar{\Psi}'_2 \quad (12.159)$$

with

$$\bar{\Psi}'_2 = L'_{21} \bar{i}'_2 + \bar{\Psi}_m; \quad \bar{i}_e + \bar{i}_m = \bar{i}_1 + \bar{i}'_2 \quad (12.160)$$

Based on this model, the stator flux $\bar{\Psi}_1$ can be estimated from a voltage model:

$$\bar{\Psi}_1 = \int (\bar{V}_1^* - R_1 \bar{i}_1) dt \approx \frac{T_e}{1 + T_e \cdot p} (\bar{V}_1^* - R_1 \bar{i}_1) \quad (12.161)$$

where T_e is a control design time constant to reduce integral's error above say, 1 Hz.

In (12.161) $\bar{V}_1^*$ is the reference voltage vector. Then, from (12.156) the airgap flux $\bar{\Psi}_m$ is estimated. Finally, the current $\bar{i}_e$ of dynamic longitudinal end effect is calculated:

$$\bar{i}_e = \frac{R'_2 (\bar{i}_1 - \bar{\Psi}_m / L_m) - j \frac{\pi}{\tau} U \bar{\Psi}_m}{R'_1 + R_e} \quad (12.162)$$

Now from (12.158), the thrust $\hat{F}_x$ is estimated. Based on this model, a DTFC strategy has been developed (Figure 12.37).

A typical result of such a DTFC for a LIM is shown in Figure 12.38 [10].

Note. Similar dynamic longitudinal correction coefficients as in [10] may be adopted without R_e concept but with $L_m = L_{m0}(1 - f(Q))$; $R'_2 = R'_{20} \cdot f(Q)$; $f(Q) = (1 - e^{-Q})/Q$; $Q = I_{\text{primary}} / (U \cdot L'_2 / R'_2)$.

12.16 ELECTROMAGNETIC INDUCTION LAUNCHERS

The principle of electromagnetic induction may be used to launch by repulsion an electrically conducting armature at a high speed by quickly injecting current in a primary coil placed in its vicinity (Figure 12.39).

A multicoil arrangement is also feasible for launching a large weight vehicle.

The principle is rather simple. The “air core” stator coil gets a fast injection of current i_1 from a high-voltage capacitor through a very fast high-voltage switch. The time varying field thus created produces an emf and thus a current i_2 in the secondary (mover) coil(s). The current in the mover coil i_2 decreases slowly enough in time to produce a high repulsion force on the mover coil. That is to launch it.

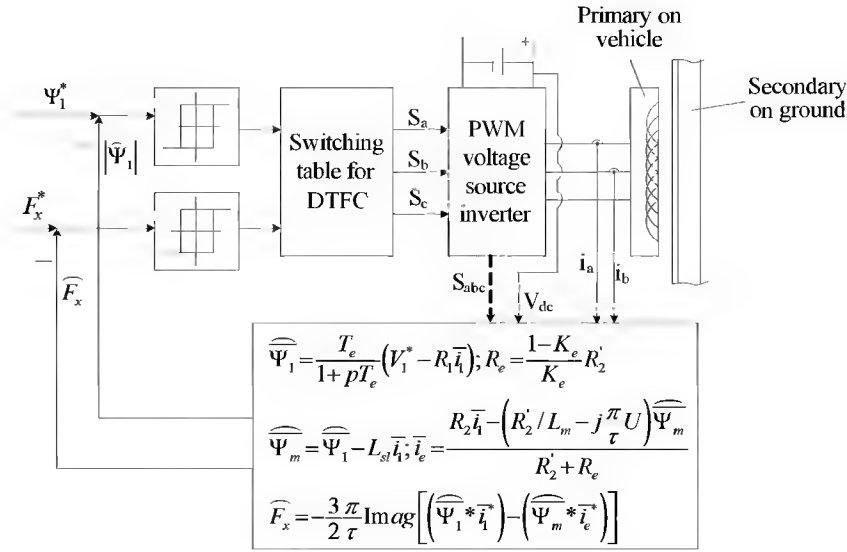


FIGURE 12.37 Direct thrust and flux control of LIMs with dynamic longitudinal end effect.

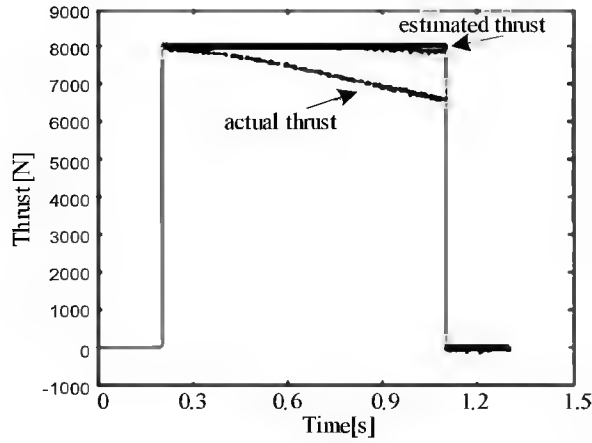


FIGURE 12.38 Step thrust response in the DTFC of an urban LIM without and with dynamic end-effect consideration. (After Ref. [10].)

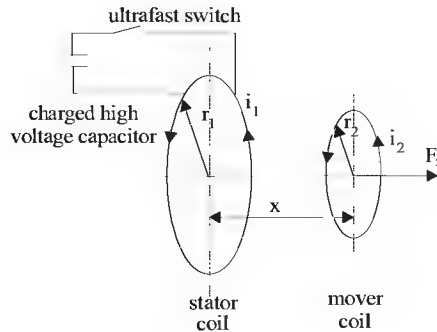


FIGURE 12.39 Coaxial filamentary coil structure.

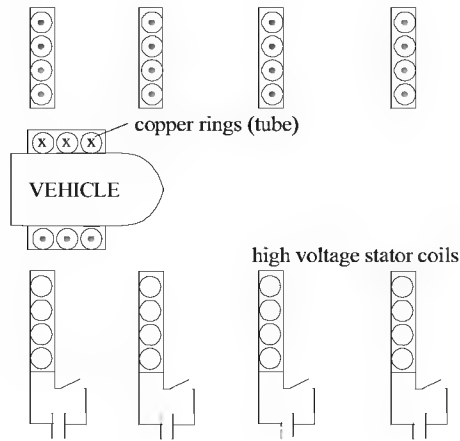


FIGURE 12.40 Multiple coil structure.

A complete study of this problem requests a 3D-FEM eddy current approach. As this is time-consuming, some analytical methods have been introduced for preliminary design purposes.

The main assumption is that the two currents (mmfs) are constant in time during the launching process and equal to each other $W_1 I_1 = -W_2 I_2$, as in an ideal short-circuited transformer.

In this case, the force F_x in the mover may be calculated from the known formula.

$$F_x = i_1 i_2 \frac{\partial M(x)}{\partial x} \quad (12.163)$$

Now, the problem retorts to the computation of the mutual inductance $M(x)$ between stator and mover coil versus position.

For concentric filamentary coils, analytical expressions for $M(x)$ making use of elliptic functions exist [27]. For more involved configurations (Figure 12.40), again FEM may be used to calculate the mutual inductance M and, for given constant currents, the force F_x , and even the force along radial direction which is related to motion stability. With known (given) currents, the FEM computation effort is reasonably low.

The design of such a system faces two limitations:

- The numerical maximum stress on the stator coils, in the radial direction mainly, σ_s
- The mover coil maximum admitted temperature T_m .

For given stator coil base diameter D_i , mover coil weight m_α (total mover weight is $m(1 + \alpha)$), and barrel length L , the maximum speed of the mover is pretty much determined for given σ_s and T_m constraints [27].

There is a rich literature on this subject (see *IEEE Transactions on Magnetics*, No. 1, 2007, 2005, 2003, 2001, 1999, 1997, 1995 – the symposium on electromagnetic launch technology (EML)).

The radial force in concentric coils may be used to produce “magnetic” compression of magnetic powders, for example, with higher permeability, to be used in permanent magnetic electric motor with complex geometry.

12.17 SUMMARY

- LIMs develop directly an electromagnetic force along the travelling field motion in the airgap.

- Cutting and unrolling a rotary IM leads to a single-sided LIM with now a ladder-type secondary.
- A three-phase winding produces basically a travelling field in the airgap at the speed $U_s = 2\tau f_1$ (τ – pole pitch and f_1 – primary frequency).
- Due to the open character of the magnetic circuit along the travelling field direction, additional currents are induced at entry and exit ends.
- They die out along the active part of LIM producing additional secondary losses, thrust, power factor, and efficiency deterioration. All these are known as dynamic longitudinal end effects.
- The longitudinal end effects may be neglected in the so-called low-speed LIMs called linear induction actuators.
- When the secondary ladder is replaced by a conducting sheet on solid iron, a lower cost secondary is obtained. This time, the secondary current density has longitudinal components under the stack zone. This is called transverse edge effect which leads to an equivalent reduction of conductivity and an apparent increase in the airgap. Also the transverse airgap flux density is nonuniform, with a minimum around the middle position.
- The ratio between the magnetization reactance X_m and secondary equivalent resistance R'_2 is called the goodness factor $G_e = X_m/R'_2$. Airgap leakage, skin effect, and transverse edge effects are accounted for in G_e .
- The longitudinal end effect is proven to depend only on G_e , number of poles $2p_1$, and the value of slip S .
- The longitudinal effect introduces a backward and a forward travelling field attenuated wave in the airgap. Only the forward wave dies out slowly and, when this happens along a distance shorter than 10% of primary length, the longitudinal end effect may be neglected.
- Compensation windings to destroy longitudinal effect have not proved yet practical. Instead, designing the LIM with an optimum goodness factor G_0 produces good results. G_0 has been defined such as the longitudinal end-effect force at zero slip be zero. G_0 increases rather linearly with the number of poles.
- In designing LIAs for speed short travel applications, a good design criterion is to produce maximum thrust per conductor losses (N/W) at zero speed. This condition leads to $(G_e)_{S=1}$, and it may be met with a rather low primary frequency $f_{1sc} = 6\text{--}15$ Hz for most applications.
- In designing LIA systems, the costs of primary + secondary + power converter tend to be a more pressing criterion than energy conversion ratings which are rather low generally for very low speeds ($\eta_1 \cos \phi_1 < 0.45$).
- The ladder secondary leads to magnetic airgaps of 1 mm or so, for short travels (in the metre range) and thus better energy conversion performance is to be expected ($\eta_1 \cos \phi_1$ up to 0.55).
- Tubular LIMs with disk shape laminations and secondary copper rings in slots are easy to build and produce satisfactory energy conversion ($\eta_1 \cos \phi_1$ up to 0.5) for short travels (below 1 m long) at rather good thrust densities (up to 1 N/W and 1.5 N/cm²).
- LIMs for urban transportation have been in operation for more than two decades. Their design specifications are similar to a constant torque + constant power torque speed envelope rotary IM drive. The peak thrust at standstill is obtained at $G_e = 1$ for a frequency in general larger than (5–6) Hz to avoid large vibrational noise during starting.
- For base speed (continuous thrust at full voltage) and up to a maximum speed (at constant power), energy conversion (η , KVA) or minimum costs of primary and secondary or primary weight objective functions are combined for optimization design. All deterministic and stochastic optimization methods presented in Chapter 8, for rotary IMs, are also suitable for LIMs.
- Electromagnetic induction launchers based on the principle of repulsion force between opposite sign currents in special single and multistator ring-shaped stator coils fed from

precharged high-voltage capacitors and concentric copper rings may have numerous practical applications. They may be considered as peculiar configurations of air core linear induction actuators [27,28]. Also they may be used to produce huge compression stresses for various applications.

- New aspects of LIMs design and control have been given special attention in the recent years [10,15,21,22,29–34], as dynamic longitudinal end effect has to be considered in urban and interurban transport by LIM propulsion control [26,35].

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13 Testing of Three-Phase IMs

Experimental investigation or testing of induction machines (IMs) at the manufacturer's and user's site may be considered an engineering art in itself.

It is also an indispensable tool in research and development of newly fabricated and new IMs in terms of new materials, sizing, topologies, control, or power supply and application requirements.

There are national and international standards on the testing of IMs of low and high-power with cage or wound rotor, fed from sinusoidal or PWM converter, and working in various environments.

We mention here the International Electrotechnical Committee (IEC) and the National Electrical Manufacturers Association (NEMA) with their standards on IMs (IEC-34 series and NEMA MG1-1993 for large IMs).

Temperature, losses and efficiency, starting, unbalanced operation, overload, dielectric limits, cooling, noise, surge capabilities, and electromagnetic compatibility tests are all standardized.

A description of standard tests is not considered here as the reader may study the standards for himself; the space required would be too large and the diversity of different standards prescriptions is so pronounced that it may create confusion for a newcomer in the field.

Instead, we decided to present here the most widely accepted tests and a few nonstandardized ones which have been promoted recently, with strong international vigour.

They refer to

- Loss segregation/power- and temperature-based methods
- Load testing/direct and indirect approaches
- Machine parameters estimation methods
- Noise testing methodologies.

13.1 LOSS SEGREGATION TESTS

Let us first recall here the loss breakdown (Figure 13.1) in the IM as presented in detail in Chapter 11, Vol. 1.

For sinusoidal (power grid) supply, the time harmonics are neglected. Their additional losses in the stator and rotor windings and cores are considered zero.

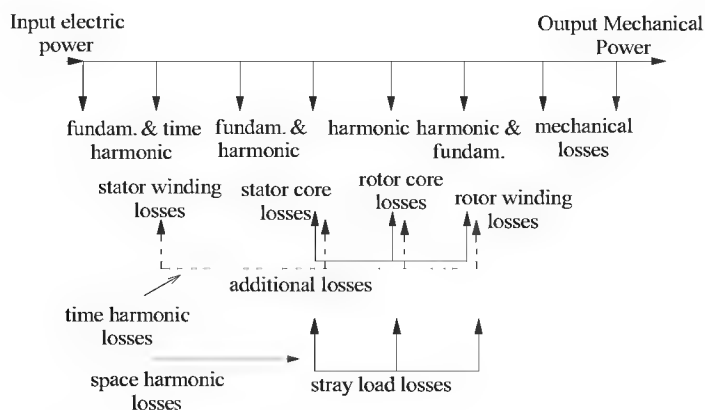


FIGURE 13.1 Loss breakdown in IMs.

However, besides the fundamental stator core and stator and rotor winding losses, additional losses occur. There are additional core losses (rotor and stator surface and tooth pulsation losses) due to slotting, slot openings for different stator and rotor number of slots. Saturation adds new losses. Also, there are space harmonics produced time harmonic rotor current losses which tend to be smaller in skewed rotors where surface iron additional losses are larger. The lack of insulation between the rotor cage bars and the rotor core allows for inter-bar currents and additional losses which are not negligible for high-frequency rotor current harmonics.

All this non-fundamental losses are called either additional or stray load losses.

In fact, the correct term would be “additional” or “non-fundamental” as they exist, to some extent, even under no mechanical load. They accentuate with load and are, in general, considered proportional with current (or torque) squared.

On top of that, time harmonics additional losses (Figure 13.1), in both copper and iron, occur with nonsinusoidal voltage (current) power supplies such as PWM power electronics converters.

As a rather detailed analysis of such a complex loss composition has been carried out in Chapter 11, Vol. 1, here we present only one sequence of testing for loss segregation, believed to be coherent and practical. A few additional methods are merely suggested. This line of testing contains only the standard no-load test at variable voltage, but extended well above rated voltage, and the stall rotor test.

The no-load and short-circuit (stall rotor) variable voltage tests are known to allow for the segregation of mechanical losses, p_{mec} , the no-load core losses, and the fundamental copper losses.

The extension of no-load test well above rated voltage (as suggested in [1]) is used as basis to derive an expression for the stray load losses considered proportional to current squared. The same tests are recommended for the PWM power electronics converter IM drives when the inverter is used in all tests.

13.1.1 THE NO-LOAD MOTOR TEST

A variable-voltage transformer, with symmetric phase voltages, supplies an induction motor whose rotor is free at shaft. A data acquisition system acquires three currents and three voltages and, if available, a power analyser, measures the power per each phase.

The method of 2 Wm leads to larger errors as the power factor on no load is low and the total power is obtained by subtracting two large numbers.

It is generally accepted that at no-load motoring, up to rated voltage, the loss composition is approximately

$$P_0 = 3R_1 I_{10}^2 + p_{iron} + p_{mec} \quad (13.1)$$

If hysteresis losses are neglected or measured as the jump in input power when the motor on no load is driven through the synchronous speed, the iron losses may be assimilated with eddy current losses which are known to be proportional to flux and frequency squared. This is to say that p_{iron} is proportional to voltage squared (Figure 13.2):

$$p_{iron} = K_{iron} V_1^2 \quad (13.2)$$

When reducing the voltage down to 25%–30% of rated value, the speed decreases very little so the mechanical losses are independent of voltage V_1 . The voltage reduction is stopped when the stator current starts rising.

Consequently,

$$P_0 - 3R_1 I_{10}^2 = K_{iron} V_1^2 + p_{mec} \quad (13.3)$$

The stator resistance may be measured through a D.C. voltage test with two phases in series.

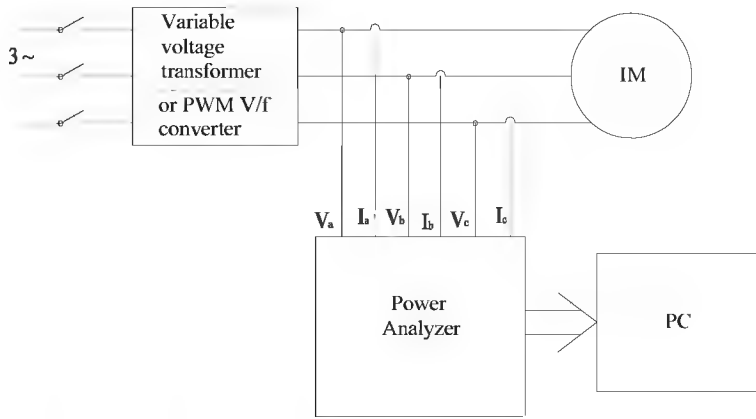


FIGURE 13.2 No-load motor test arrangement.

Alternatively, when all six terminals are available, the A.C. test with all phases in series is preferable as the airgap field is very low, so the core loss is negligible (Figure 13.3).

With voltage, current, and power measured, the stator resistance R_s and homopolar reactance X_0 (lower or equal to stator leakage reactance) are

$$R_{l-} = \frac{V_-}{3I_-} \quad (13.4)$$

$$X_0 \leq X_{ll} = \sqrt{\left(\frac{V_-}{3I_-}\right)^2 - R_l^2} \quad (13.5)$$

With full pitch coil windings $X_0 = X_{ll}$. However, $X_0 < X_{ll}$ for chorded windings. Low voltage is required to avoid overcurrents in this test.

For large machines with skin effect in the stator, even at fundamental frequency, R_{l-} is required.

The same is valid with IMs fed from PWM power converters. In the latter case, the Variac is replaced by the PWM converter, triggered for two power switches only, with a low modulation index.

The graphical representation of (13.3) is shown in Figure 13.4.

The intercept of the graph on the vertical axis is the mechanical losses.

The fundamental iron losses p_{iron}^l for various values of voltage are graphically represented in Figure 13.4.

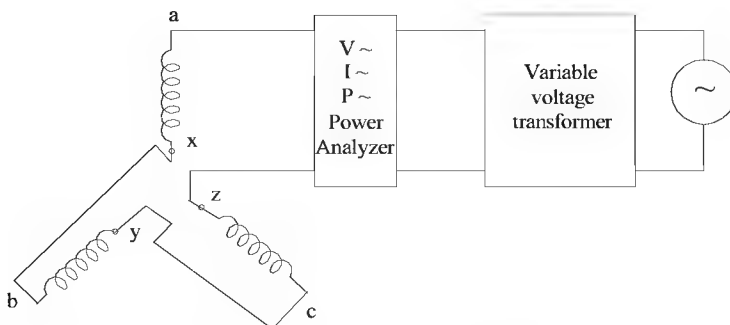


FIGURE 13.3 AC resistance measurement when six terminals are available.

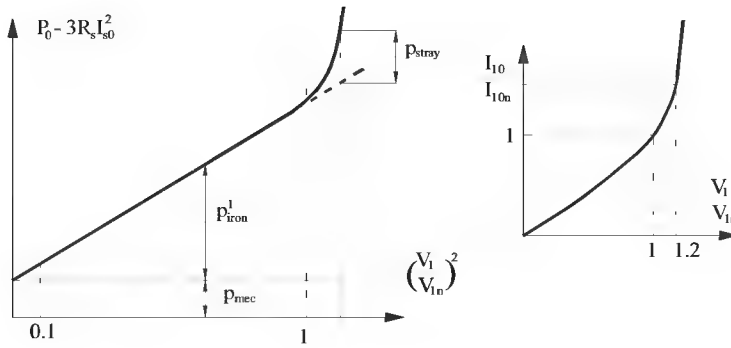


FIGURE 13.4 No-load input (less stator copper loss) versus voltage squared.

We may infer that in most IMs, the stator impedance voltage drop is small so the fundamental core loss determined at no load is valid as well for on-load conditions.

In reality, under load conditions the value of p_{iron}^l slightly decreases as the emf does the same.

13.1.2 STRAY LOSSES FROM NO-LOAD OVERVOLTAGE TEST

When increasing the voltage over its rated (design) value, Equation (13.3) deviates from a straight line (Figure 13.4), at least for low-power IMs [1].

Apparently in this case, the iron saturates so the third flux harmonic, due to saturation, causes more losses. This is not so in most IMs as explained in Section 5.4.4, (Vol. 1). However, the stator current increases notably above rated no-load current. Space harmonics-induced currents in the rotor will also increase. Not so for the fundamental rotor current. So the stator core surface and tooth pulsation losses are not notable. Fortunately, they are not large even under load conditions.

All in all, we can use this extension of no-load test to find the stray loss coefficient as

$$K_{\text{stray}} = \frac{P_{\text{stray}}}{3I_{10}^2} \quad (13.6)$$

With the voltage values <115%–120%, the difference in power shown in Figure 13.4 is calculated, together with the respective current I_{10} measured. A few readings are taken, and an average value for the stray load coefficient K_{stray} is obtained.

Test results on three low-power IMs (250 W, 550 W, and 4.2 kW) have resulted in stray losses at a rated current of 2%, 0.9%, and 2.5% [1]. It is recognized that for stray load losses, generalizations have to be made with extreme caution.

On-load tests are required to verify the practicality of this rather simple method for wide power ranges. It is today recognized that stray load losses are much larger than 0.5% or 1% as stipulated in some national and still in IEC standards [2], although standard changes are underway. Their computation from on-load tests, as in IEEE–112B standard, seems a more realistic approach. As the on-load testing is rather costly, other simpler methods are considered.

Historically, the reverse rotation test at low voltage and rated current has been considered as an acceptable way of segregating stray load losses [3].

13.1.3 STRAY LOAD LOSSES FROM THE REVERSE ROTATION TEST

Basically, the IM is rotated in the opposite direction of its stator travelling field (Figure 13.5). The stator is fed at low voltage through a Variac.

With the speed $n = -f_1/p$, the value of slip $S = 1 - np_1/f_1 = 2$, and thus, the frequency of rotor currents is $2f_1$.

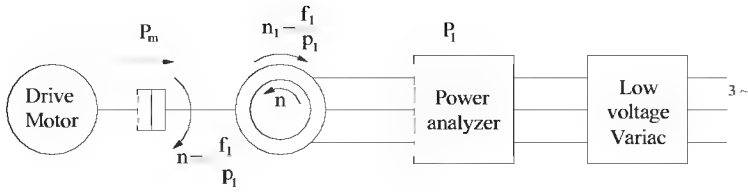


FIGURE 13.5 Reverse rotation test.

The mechanical input will cover the mechanical losses p_{mec} , the rotor stray losses plus the term due to $3I_2^2R_2(1-S)/S$. For $S = 2$, this term becomes $-3I_2^2R_2/2$. Consequently, the difference between the stator electric input P_1 and the mechanical input P_m is

$$P_1 - P_m = 3R_1I_1^2 + P_{iron} + 3\frac{I_2^2R_2}{2} + P_{stray} - \frac{3I_2^2R_2}{2} - P_{mec} \quad (13.7)$$

The rotor winding loss terms gets cancelled in (13.7).

The fundamental iron losses in P_{iron} are different from those for rated power motoring as they tend to be proportional to voltage squared. Also P_{iron} contains the stator flux pulsation losses, which again may be considered to depend on voltage squared.

So,

$$P_{iron} \approx P_{(iron)rated} \cdot \left(\frac{V_1}{V_{In}} \right)^2 \quad (13.8)$$

The method has additional precision problems, as detailed in [4], besides the need for a drive with measurable shaft torque (power).

By comparison, the extension of no-load test above rated voltage is much more practical. But is it a satisfactory method? Only time will tell as many other attempts to segregate the stray load losses did not yet get either universal acceptance.

13.1.4 THE STALL ROTOR TEST

Traditionally, the stall rotor (short-circuit) test is carried out with a three-phase supply and mechanical blockage to stall the rotor. With single-phase supply, however, the torque is zero and thus the rotor remains at standstill by itself (Figure 13.6a and b).

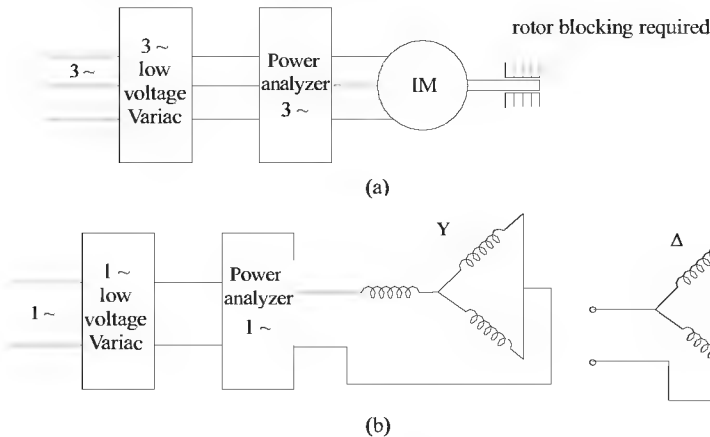


FIGURE 13.6 Stall rotor tests (a) three-phase supply and (b) single-phase supply.

The tests are carried out for low voltages until the current reaches its rated value. If the test is carried out at rated frequency f_{ln} , the skin effect in the rotor is pronounced and thus the rotor resistance is notably larger than that in load operation when the slip frequency $Sf_{ln} \ll f_{ln}$. Only for low-power IMs (in the kW range), of general design (moderate starting torque), the assumption of low rotor skin effect is true.

It is true also for all wound rotor IMs.

With voltage, current, and power measured, the stall rotor (short-circuit) resistance R_{sc} and reactance X_{sc} are determined as follows:

$$\begin{aligned} R_1 + R_2^s &= R_{sc}^s = \frac{P_{sc3}}{3I_{sc3}^2}; \\ X_{sc}^s &= \sqrt{\left(\frac{V_{sc3}}{I_{sc3}}\right)^2 - R_{sc}^2} = X_{11}^s + X_{12}^s, \text{ respectively;} \\ R_1^s + R_2^s &= R_{sc}^s = 2 \frac{P_{sc1}}{3I_{sc1}^2}; \\ X_{sc}^s &= \sqrt{\frac{2}{3} \left(\frac{V_{sc1}}{I_{sc1}}\right)^2 - R_{sc}^2} = X_{11}^s + X_{12}^s \end{aligned} \quad (13.9)$$

We use the superscript “s” to emphasize that these values have been obtained at stall. The values of R_{sc}^s and X_{sc}^s are less practical than they seem for load conditions. Skin effect and leakage saturation, at high values of stall currents, for rated voltage, lead to different values of R_{sc}^s , X_{sc}^s at stall.

The short-circuit test is supposed to provide also for fundamental winding losses computation at rated current:

$$P_{Co} = P_{co1n} + P_{co2n} = 3R_{sc}^s \cdot I_{ln}^2 \quad (13.10)$$

that is, to segregate the winding losses for rated currents. It manages to do so correctly only for IMs with no skin effect at rated frequency in the rotor ($Sf_1 = f_1$) and in wound rotor IMs.

However, if a low-frequency low-voltage power source is available (even 5% of rated frequency will do), the test will provide correct data of rated copper losses (and $R_1 + R_2$, $X_{11} + X_{12}$) to be used in fundamental winding losses for efficiency calculation attempts.

13.1.5 NO-LOAD AND STALL ROTOR TESTS WITH PWM CONVERTER SUPPLY

The availability of PWM converters for IM drives raises the question if their use in no-load and stall tests as variable voltage and frequency power sources is not the way of the future.

It is evident that for IMs destined for variable-speed applications, with PWM converter supplies, the no-load and stall tests are to be performed with PWM converter rather than with Variac (transformer) supplies.

As the voltage supplied to the motor has time harmonics, the stator and rotor currents have time harmonics. So, in the first place, even if $S \approx 0$, the time harmonics produced rotor currents are non-zero as their slip is $S_v \approx 1$. The space harmonics produced rotor harmonics currents, existing also with sinusoidal voltage supply, are augmented by the presence of stator voltage time harmonics. They are, however, load independent ($S_v \approx 1$).

So the loss breakdown with PWM converter supply at no load is

$$P'_0 = 3R'_1 I_{10}^2 + 3R'_2 I_{20}^2 + p'_{iron} + p_{mec} \quad (13.11)$$

It has been shown that despite the fact that the stator and rotor no-load currents show time harmonics due to PWM converters, the airgap emf is quasi-sinusoidal [5]. And so is the magnetization current I_m .

So, performing the no-load test for same fundamental voltage and frequency, once with sinusoidal power source and once with PWM converter, the current relationships are

$$\begin{aligned} I_{10}^2 &= I_m^2 \\ I_{10}^{\prime 2} &\approx I_{20}^2 + I_m^2 \end{aligned} \quad (13.12)$$

Thus, the rotor current under no-load I_{20} for PWM converter supply is

$$I_{20} = \sqrt{I_{10}^{\prime 2} - I_{10}^2} \quad (13.12')$$

The stator and rotor resistances R_1' and R_2' valid for the given current harmonics spectrum are not known.

However, with the rotor absent, a slightly overestimated value of R_1' may be obtained. From a stall rotor test at low fundamental frequency (5%), $R_1' + R_2'$ may be found.

From Equation (13.11), we may represent graphically $p'_{\text{iron}} + p_{\text{mec}}$ as a function of fundamental voltage squared for various fundamental frequencies. Finally, we obtain $P_{\text{iron}}(f_1, (V_1/V_{1n})^2)$ and $p_{\text{mec}}(f_1)$.

In general, for PWM converter supply, both the no-load losses and the stall rotor losses are larger than those for sinusoidal supply with same fundamental voltage and frequency.

With high switching frequency PWM converters, the time harmonics content of voltage and current is less important and thus smaller no-load and stall rotor additional losses occur [5] (Figures 13.7 and 13.8). For the cage rotor IM stall tests, the difference in losses is negligible, although the current (RMS) is larger (Figure 13.8).

It may be argued that for given A.C. power grid parameters, in general, PWM converters cannot produce the rated voltage fundamental to the motor.

A 3%–5% voltage fundamental reduction due to converter limits is accepted. So either correction of all losses, proportional to voltage ratio squared, is applied to PWM converter tests or the power grid is set to provide 5% more than rated voltage fundamental.

As for the extension of the voltage beyond rated voltage by 10%–15%, for the no-load test, to calculate the stray load loss coefficient mentioned in a previous section, a voltage up/down transformer is required. Alternatively, the ratio V_{1n}/f_1 may be increased by decreasing the frequency.

All these being said, it appears that the no-load and stall rotor tests for loss segregation may be extended to PWM converter supplied induction motors.

Pertinent data acquisition and processing systems are required with nonsinusoidal voltages and currents to calculate fundamentals and losses.

In parallel with the electrical methods to segregate the losses in IM, presented so far, temperature/time and calorimetric methods have also been developed to determine the various losses in the IM.

Temperature-time methods require numerous temperature sensors to be planted in key points within the IM. If the loss distribution is known and the IM is disconnected from the source and kept at constant speed, the temperature/time derivative, at the time of disappearance of the loss source, is

$$\frac{dT}{dt} = -\frac{1}{C} \cdot Q \quad (13.13)$$

where C is the thermal capacity of the body volume considered and Q the respective whole losses in that region.

The tests is carried out on both load and no load [6,7]. The intrusive character of the method and the requirement of knowing the thermal capacity of various parts of IM body seem to limit the use of this method to prototyping.

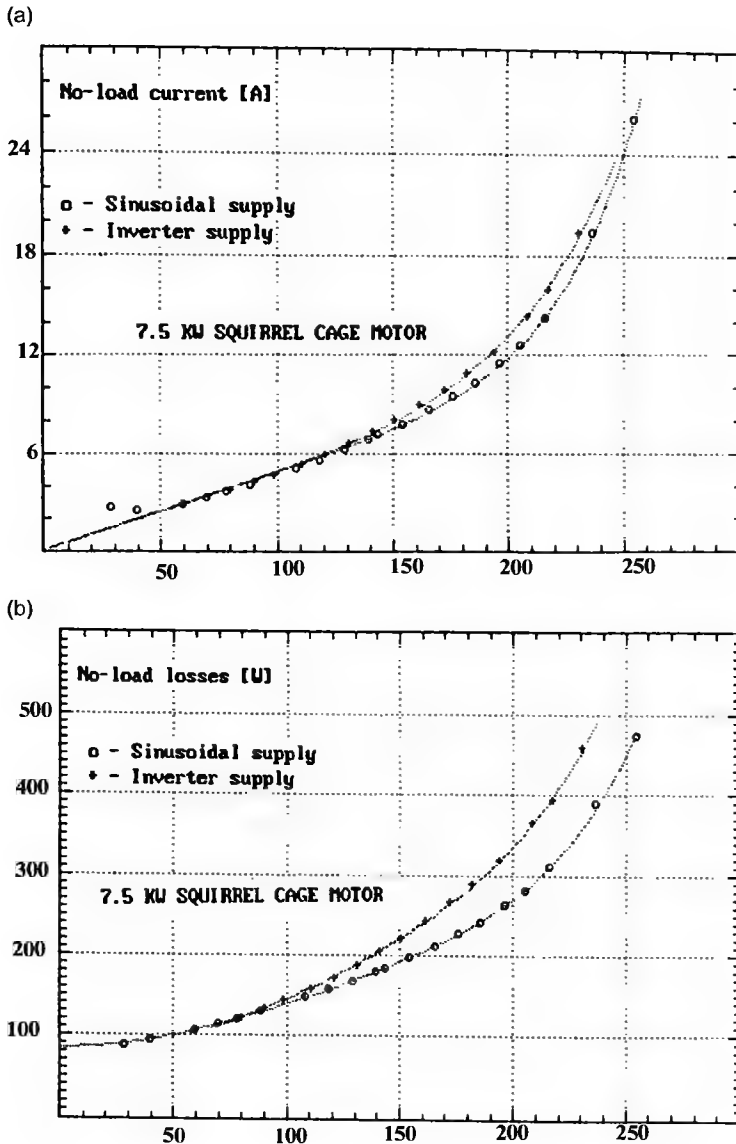


FIGURE 13.7 No-load testing (after Ref. [5]) (a) current and (b) losses.

13.1.6 LOSS MEASUREMENT BY CALORIMETRIC METHODS

The calorimetric method [8] is based on the principle that the temperature rise in a wall-insulated chamber is proportional to the losses dissipated inside.

For steady state (thermal equilibrium), the rate of heat transfer by the air coolant, Q , which in fact represents the IM losses, is

$$Q = MC_p \Delta T \quad (13.14)$$

where $M(\text{kg/s})$ is the mass flow rate of coolant, $C_p(\text{J/KgK})$ the specific heat of the air, and ΔT the temperature rise (K) of coolant at exit with respect to entrance. For better precision, the dual chamber calorimetric approach has been introduced [9] (Figure 13.9).

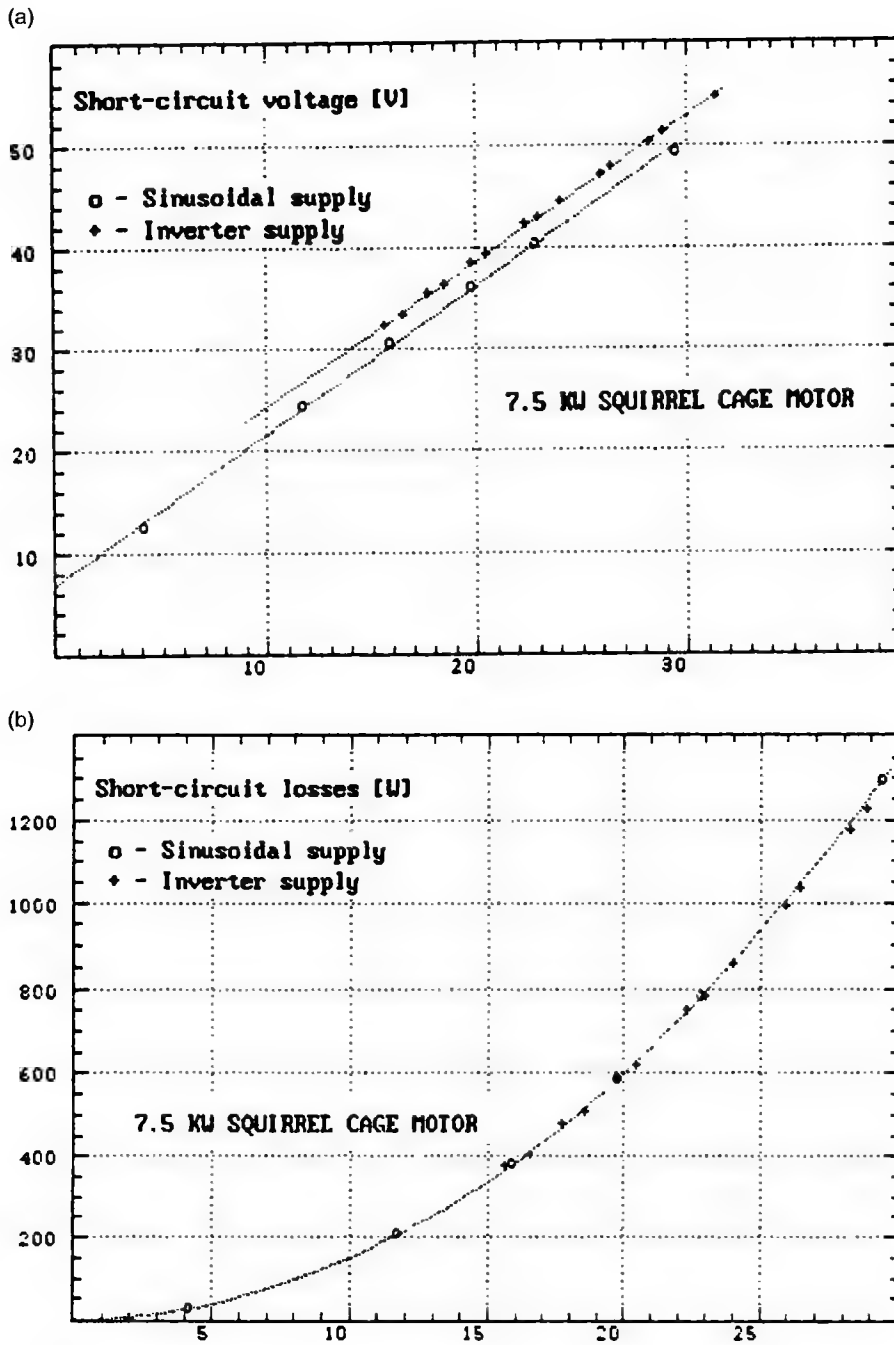


FIGURE 13.8 Stall rotor test (after Ref. [5]) (a) current and (b) losses.

In the first chamber, the IM is placed. In the second chamber, a known power heater is located. This way, the motor losses Σp_{IM} are

$$\Sigma p_{IM} = P_{heater} \frac{\Delta T_1}{\Delta T_2} \quad (13.15)$$

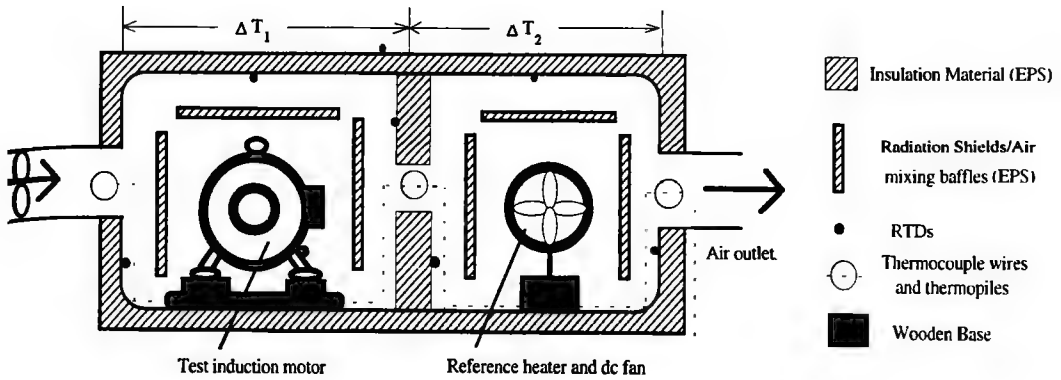


FIGURE 13.9 The dual chamber calorimeter method.

The air properties, not easy to find with variable temperature, are not required. Also the errors of temperature sensors tend to cancel each other, as we need only $\Delta T_1/\Delta T_2$.

There is some heat leakage from the calorimeter. This loss may be determined with a precision of ± 0.5 W [9]. Also, as an order of magnitudes, 60–100 L/s of air is required for loss measurements of 0.2–1 kW. A proper temperature rise per chamber should be around 10°C .

The overall uncertainty of this method is about ± 15 W. The friction losses due to stuffing the motor shaft through the chamber walls have to be considered too.

For IMs with losses above 150 W, an error in the loss measurements $<10\%$ is expected. The method works for any load levels and with any kind of IM supply. However, it takes time – until thermal steady state is reached – and implies notable costs. For prototyping, however, it seems appropriate.

13.2 EFFICIENCY MEASUREMENTS

Due to its impact on energy costs, efficiency is the single most important parameter index in electric machines.

As induction motors are fabricated and used worldwide, national and international standards for efficiency measurements have been introduced. We mention here some of them which are deemed to be highly representative

- IEEE Standard 112–1996 [10]
- IEC–34–2 and IEC–34–2A [11]
- JEC 37.

NEMA MG1–1993 and the Canadian standard C390 correspond to IEEE–112, whereas most European countries abide by IEC standard. JEC holds in Japan, mainly.

The definition of efficiency η is basically unique

$$\eta = \frac{\text{Output power}}{\text{Input power}} = 1 - \frac{\text{Overall losses}}{\text{Input power}} \quad (13.16)$$

However, the main difference between the above standards involves how the stray load losses are defined and treated as part of overall losses.

Direct measurement of stray losses is obtained from

$$P_{\text{stray}} = P_{\text{input}} - (P_{\text{output}} + p_{\text{iron}} + 3R_1 I_1^2 + 3R_2 I_2^2 + p_{\text{mec}}) \quad (13.17)$$

with: $I_2 \approx \sqrt{I_1^2 - I_m^2}$

where I_m , the magnetization current, is equal to the no-load current at the respective voltage.

The various losses in (13.17) are obtained through the loss segregation methods. The input and output powers are measured directly.

Let us now summarize how the efficiency and stray load losses are handled in the IEEE–112 and IEC–34–2 standards.

13.2.1 IEEE STANDARD 112–1996

This rather complete standard consists of five methods to determine efficiency. They are called methods A, B, C, E, and F.

Method A is, in fact, a direct method where the input and output powers are measured directly.

Method B measures directly the stray load losses.

Linear regression is used to reduce measurement errors. The stray load losses are considered proportional to torque squared.

$$(P_{\text{stray}})_{\text{corrected}} = AT_{\text{shaft}}^2 \quad (13.18)$$

The correlation coefficient of linear regression has to be larger than 0.9 to secure good measurements.

Method C introduces a back-to-back (motor/generator) test. The stray load losses are obtained by loss segregation (13.17), but the measured power is, in fact, the difference between the input and output of the two identical machines. So the total losses in the two machines are measured. Consequently, better precision is expected. The total stray load losses thus segregated are divided between the motor and generator considering that they are proportional to current squared.

Methods E and E_1 are indirect methods. So the output power is not measured directly.

In fact, in method E, the stray load losses are measured via the reverse rotation test (see Section 13.1.3). In method E_1 , the stray load losses are simply assumed to have a certain value (Table 13.1).

Methods F and F_1 make use of the equivalent circuit with stray losses directly measured (F) or assigned a certain value (F_1).

13.2.2 IEC STANDARD 34–2

The efficiency is estimated by determining all loss components of the IM.

The losses are to be determined by the loss segregation methods or from the measurement of overall losses:

- Direct load test with torque measurement
- Calibrated torque load machine
- Mechanical back-to-back test (as for IEEE–112C)
- Electrical back-to-back test.

TABLE 13.1

Assumed Stray Load Losses/Rated Output/IEEE–112 Method E_1

Rated Power (kW)	Stray Load Losses (%)
0.75–90	1.8
91–375	1.5
376–1800	1.2
≥1800	0.9

The preferred method in the standard is however the segregation of losses with stray load losses having a fixed value of 0.5% of rated power. The Japanese standard JEC still apparently neglects the stray load losses altogether.

Stray load losses in Table 13.1 (IEEE–112E1) are notably higher than the 0.5% in IEC–34–2 standard and the zero value in JEC standard. Consequently, the same induction motor would be labelled with the highest efficiency in the JEC standard and then in the IEC–34–2, and finally the lowest, and most realistic, in the IEEE–112E1 standard. IEC prepared recently a new efficiency standard close to IEEE–112E1 standard.

13.2.3 EFFICIENCY TEST COMPARISONS

Typical tests according to the three standards run on the same induction motor of 75 kW are shown in Table 13.2 [12].

A few remarks are in order

- The first test, the direct method, involves direct measurement of input electrical power, shaft torque, and speed with a total possible error in efficiency of maximum 1%.
- IEEE–112B tests imply that linear regression is used on the measured stray losses, considered proportional to torque squared.
- For IEC–34–2, the stray load losses are considered 0.5% of rated output and proportional to torque squared.

The main conclusion is that differences in efficiency, with respect to direct measurements of more than 2%, may be encountered with the IEC–34–2 and JEC, whereas IEEE–112B is much closer to reality.

The evident suggestion then is to use the IEEE–112B method as often as possible whenever the direct method is feasible. The problem is that the direct method requires almost one day testing time per motor, notable man power, and energy.

Providing and mounting a brushless torque-meter with integrated speed sensor of both high precision (<1%) and 1rpm speed error, is not an easy task as torque (power) increases. Also the power grid rating increases with the power of the tested IM.

13.2.4 THE MOTOR/GENERATOR SLIP EFFICIENCY METHOD

Difficulties related to the torque-meter acquisition, mounting and frequent calibration, and excessive energy consumption may be tamed by eliminating the torque-meter and loading the IM with another IM fed from the now available bidirectional PWM voltage-source converter supply used for high-performance variable-speed drives (Figure 13.10). This method may be seen as an extension of IEEE–112C back-to-back method. A very precise speed sensor or a slip frequency measuring device is still required [13].

In principle, the IM is tested for ± 0.25 , ± 0.5 , ± 0.75 , ± 1.0 , ± 1.25 , and ± 1.5 times rated slip speeds as a motor and as a generator with the voltages, currents, and input power measured via a power analyser.

It is admitted that the power level is proportional to slip so the 25%, 50%, 75%, 100%, 125%, and 150% loads are approximately obtained. Alternatively, the slip is adjusted to the required power. The power balance equations for the motor and generator modes and equal (or known) slip values are

$$P_{in/m} = 3R_1 I_{1m}^2 + p_{iron/m} + K_{stray} \cdot I_{1m}^2 + 3R_2 \frac{I_{2m}^2}{S_m} \quad (13.19)$$

$$P_{out/g} = 3 \frac{R_2 I_{2g}^2}{|S_g|} - K_{stray} I_{1g}^2 - p_{iron/g} - 3R_1 I_{1g}^2 \quad (13.20)$$

TABLE 13.2

Efficiency Testing of a 75 kW Standard IM according to Direct Measurements: IEEE–112B, IEC–34–2, and JEC [12]

Load	25%	50%	75%	100%
U_{average} (V)	407.8	404.7	400.2	391.3
I_{average} (A)	71.4	91.2	118.1	149.6
P_{el} (W)	22270	41820	61850	81360
Power factor (-)	0.44	0.65	0.76	0.80
T (Nm)	182.2	364.0	548.3	723.3
n (rpm)	997	995	994	991
P_{shaft} (W)	19021	37924	57078	75064
Slip (%)	0.3	0.5	0.6	0.9
P_{loss} measured directly (W)	3249	3896	4772	6296
$P_{R1,1}^2$ (W)	354	578	968	1553
P_{core} (W)	1774	1728	1660	1527
$P_{R1,2}^2$ (W)	60.4	198	355	705
$P_{w,fr}$ (W)	910	910	910	910
P_{stray} (W)	151	482	879	1601
$P_{\text{shaft}}/P_{\text{rated}}$ (%)	25.4	50.6	76.1	100.1
Efficiency Direct (%)	85.4	90.7	92.3	92.3
Efficiency Using IEEE–112				
Linear Regression Coefficient. Slope: $2.802 \cdot 10^{-3}$ Correlation: 0.99				
$P_{\text{stray corr.}}$ (W)	93	371	843	1466
$P_{\text{loss core}}$ (W)	3191	3785	4736	6161
$P_{\text{shaft core}}$ (W)	19079	38035	57114	75199
$P_{\text{shaft}}/P_{\text{rated}}$ (%)	25.4	50.7	76.2	100.3
Efficiency IEEE–112 (%)	85.7	90.9	92.3	92.4
Efficiency Using IEC–34–2				
P_{stray} (W)	93	151	253	407
P_{loss} (W)	31.91	3565	4146	5102
P_{shaft} (W)	19079	38255	57704	76258
$P_{\text{shaft}}/P_{\text{rated}}$ (%)	25.4	51.0	76.9	101.7
Efficiency IEC–34–2 (%)	85.7	91.5	93.3	93.7
Efficiency Using JEC				
P_{stray} (W)	0	0	0	0
P_{loss} (W)	3098	3414	3893	4695
P_{shaft} (W)	19172	38406	57957	76665
$P_{\text{shaft}}/P_{\text{rated}}$ (%)	25.6	51.2	77.3	102.2
Efficiency JEC (%)	86.1	91.8	93.7	94.2

For the 25%–150% load range, the rotor circuit is highly resistive and thus

$$I_{2m}^2 = I_{1m}^2 - I_m^2 \quad (13.21)$$

$$I_{2g}^2 = I_{1g}^2 - I_m^2 \quad (13.22)$$

With I_m equal to the no-load current at rated voltage, the rotor currents for motor and generator modes, I_{2m} and I_{2g} , are obtained.

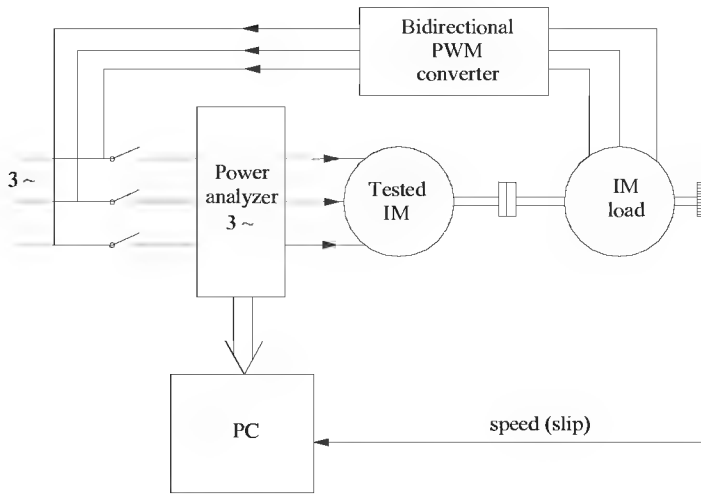


FIGURE 13.10 Motor/generator slip efficiency method [13].

The core losses are considered with their value at no-load and rated voltage, the same for motor and generator,

$$P_{\text{iron/m}} = P_{\text{iron/g}} \quad (13.23)$$

Now in Equations (13.19)–(13.20), we have only two unknowns: R_2 —rotor resistance and K_{stray} (stray load coefficient). As the equations are linear, the solution is straightforward.

It may be argued that while R_2 enters large-power components, K_{stray} enters loss (small) power components. While this is true, the direct method again subtracts output from input to compute efficiency. Good but costly instrumentation should deal with this properly.

A few remarks on this method are as follows:

- No torque-meter is required.
- Motor/generator slip loading is performed.
- The motor/generator slip levels may be rather equal, but exactness is not a must.
- The core losses and the no-load current at rated voltage are required.
- The mechanical losses p_{mec} , as determined from variable-voltage no-load test, are required to calculate the efficiency:

$$\eta = 1 - (3R_1 I_{1m} + p_{\text{iron/m}} + 3K_{\text{stray}} I_{1m}^2 + 3R_2 I_{2m}^2 + p_{\text{mec}}) / P_{\text{in/m}} \quad (13.24)$$

Efficiency can be also calculated from generator mode test results with

$$p_{\text{mecg}} = p_{\text{mecm}} \cdot (1 + |S|)^2 \quad (13.25)$$

The rotor resistance R_2 at low (slip) frequency may be determined eventually only at 25% rated slip and considered constant over the loading tests at 50%, 75%, 100%, 125%, and 150% as the temperature also has to stabilize for each test. This way for all loads except for 25% only, the difference between Equations (13.19) and (13.20) will be used. This should enhance precision notably. This estimation of R_2 , independent of skin effect, should be preferred to that from stall rotor tests at rated frequency. To reduce further the testing time, we may acquire the temperature of the stator frame at the time of every load level testing and correct the stator and rotor resistances accordingly. The stator resistance is to be found from a D.C. test at a known temperature.

If time permits, the machine may be stopped quickly, by fast braking it with the load machine which is supplied by a bi-directional power flow converter, for the stator resistance to be D.C. (by milliohm meter) measured.

Test results in [13] proved satisfactory when compared with IEEE–112B tests.

Still, the coupling of a loading machine at the IM shaft is required. For vertical shaft IMs as well as large-power IMs, this may be incurring too high costs.

Recently, the MF method, stemming from the classical two-frequency method [14], has been revived by using a static power converter to supply the IM [15,16]. The key problem in this artificial loading test is the equivalence of losses with those of direct load testing.

13.2.5 THE PWM MIXED-FREQUENCY TEMPERATURE RISE AND EFFICIENCY TESTS (ARTIFICIAL LOADING)

The two-frequency test relies on the principle of supplying the IM with voltages V_1 and V_2 of two different frequencies f_1 and f_2 :

$$V(t) = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t \quad (13.26)$$

The rated frequency f_1 differs from f_2 such that the IM speed oscillates very little. Consequently, the IM works as a motor for f_1 and as a generator for f_2 . The RMS value of stator current depends on the amplitude and frequency of second voltage: V_2 and f_2 .

Traditionally, the test uses a synchronous generator of frequency $f_2 \approx (0.8–0.85)f_1$ and $V_2 < V_1$, connected in series with the power grid voltage $V_1(f_1)$, to supply the IM. V_2 and f_2 are modified until the RMS value of stator current has the desired value (around or equal to rated value in general).

The test has been traditionally used to determine the temperature rise transients and steady-state value, to replicate direct load loss conditions. The exceptional advantage is that no loading machine is attached to the shaft. Consequently, the testing costs and time are drastically reduced, especially for vertical shaft IMs.

However, due to the lack of correct loss equivalence such tests were reported to yield, in general, higher temperature ($6^\circ\text{C}–10^\circ\text{C}$) than direct load tests for the same RMS current.

This is one reason why the MF traditional tests did not get yet enough acceptance.

The availability of PWM converters should change the picture to the point that such tests could be applied, at least for IMs destined for variable speed drives, to determine the efficiency. There are quite a few ways to control the PWM converter to obtain artificial loading.

Among them, we mention here

- The accelerating–decelerating method [15]
- Fast primary frequency oscillation method [15]
- PWM dual frequency method. [16]

13.2.5.1 The Accelerating–Decelerating Method

The IM is supplied from an off-the-shelf PWM converter – even one with unidirectional power flow will do. The reference speed is ramped up and down linearly or sinusoidally around the rated frequency synchronous speed so that the IM with free shaft can follow path.

The speed variation range $n_{\min}–n_{\max}$ and its time variation

$$\frac{dn}{dt} \approx \frac{n_{\max} - n_{\min}}{T} \quad (13.27)$$

are used as parameters to try to provide loss equivalence with the direct load test.

Common sense indicates that for equivalent stator copper loss, the same RMS current as in direct load testing is required.

As the speed has to go above rated (base) speed, the inverter voltage ceiling is causing constant voltage to be applied above rated speed with only frequency as a variable.

As $(n_{\max} - n_{\min})/n_{\text{rated}} \approx 0.2-0.4$, the mechanical losses vary. As they are, in general, proportional to speed squared, the process of making it equivalent should not be very difficult.

$$(P_{\text{mec}})_{\text{rated}} = K_m n_n^2 = \frac{1}{(n_{\max} - n_{\min})} \int_{n_{\min}}^{n_{\max}} K_m n^2 dn = \frac{n_{\max}^3 - n_{\min}^3}{3(n_{\max} - n_{\min})} \cdot K_m \quad (13.28)$$

Consequently,

$$n_{\max}^2 + n_{\max} \cdot n_{\min} + n_{\min}^2 = 3n_n^2$$

As the frequency varies, all core losses are considered proportional to voltage squared, and the torque is also proportional with voltage squared (constant slip during the tests), the average core loss is slightly smaller than for direct load tests. In general, $n_{\max}$ and $n_{\min}$ are rather symmetric with respect to rated speed.

Consequently $n_{\max}$ and $n_{\min}$ are found easily.

Now the pace of acceleration is left to be adjusted such that to produce the equivalent RMS stator current over the speed oscillation cycle.

An RMS current estimator based on current acquisition over a few periods can be used as current feedback signal. The reference RMS current will be input to a slow current close loop that outputs the reference speed oscillation frequency between $n_{\min}$ and $n_{\max}$.

The voltages V_a and V_b and the currents I_a and I_b are acquired and transformed to synchronous coordinates:

$$\begin{vmatrix} V_d(I_d) \\ V_q(I_q) \end{vmatrix} = \frac{2}{3} \begin{vmatrix} \cos(-\theta_{\text{es}}) & \cos\left(-\theta_{\text{es}} + \frac{2\pi}{3}\right) & \cos\left(-\theta_{\text{es}} - \frac{2\pi}{3}\right) \\ \sin(-\theta_{\text{es}}) & \left(-\theta_{\text{es}} + \frac{2\pi}{3}\right) & \sin\left(-\theta_{\text{es}} - \frac{2\pi}{3}\right) \end{vmatrix} \cdot \begin{vmatrix} V_a(I_a) \\ V_b(I_b) \\ V_c(I_c) \end{vmatrix}; \quad (13.29)$$

$$V_c = -V_a - V_b;$$

$$I_c = -I_a - I_b;$$

$$\theta_{\text{es}} = \int \omega_1 dt;$$

The frequency f_1 varies with time as the machine accelerates and decelerates.

The input power

$$P_i = \frac{3}{2} (V_d I_d + V_q I_q) \quad (13.30)$$

is in fact the instantaneous active power.

If the reference frame axis d falls along phase a axis at time zero, then $V_q = 0$ and thus only one term appears in (13.30).

The reactive input power Q_i is

$$Q_i = \frac{3}{2} (-V_d I_q + V_q I_d) \quad (13.30')$$

and retains also one term only. Notice that the current has both components.

The main advantage of this variable transformation – to be done offline through a PC – is that it provides “instantaneous” power values without requiring averaging over a few integral electrical periods [15].

Consequently, while the machine travels from $n_{\min}$ to $n_{\max}$ and back, the active and (reactive) instantaneous powers travel a circle when shown as a function of speed (Figure 13.11) [15]. Basically, the machine works as a motor during acceleration from $n_{\min}$ to $n_{\max}$ and as a generator from $n_{\max}$ to $n_{\min}$, and thus the, input instantaneous active power is at times positive or negative.

The average power per cycle represents the average total losses in the IM per cycle P_{loss} . By the same token, the average of positive values yields the average input power P_{inIm} . So the efficiency η is

$$\eta = 1 - P_{\text{loss}}/P_{\text{inIm}} \quad (13.31)$$

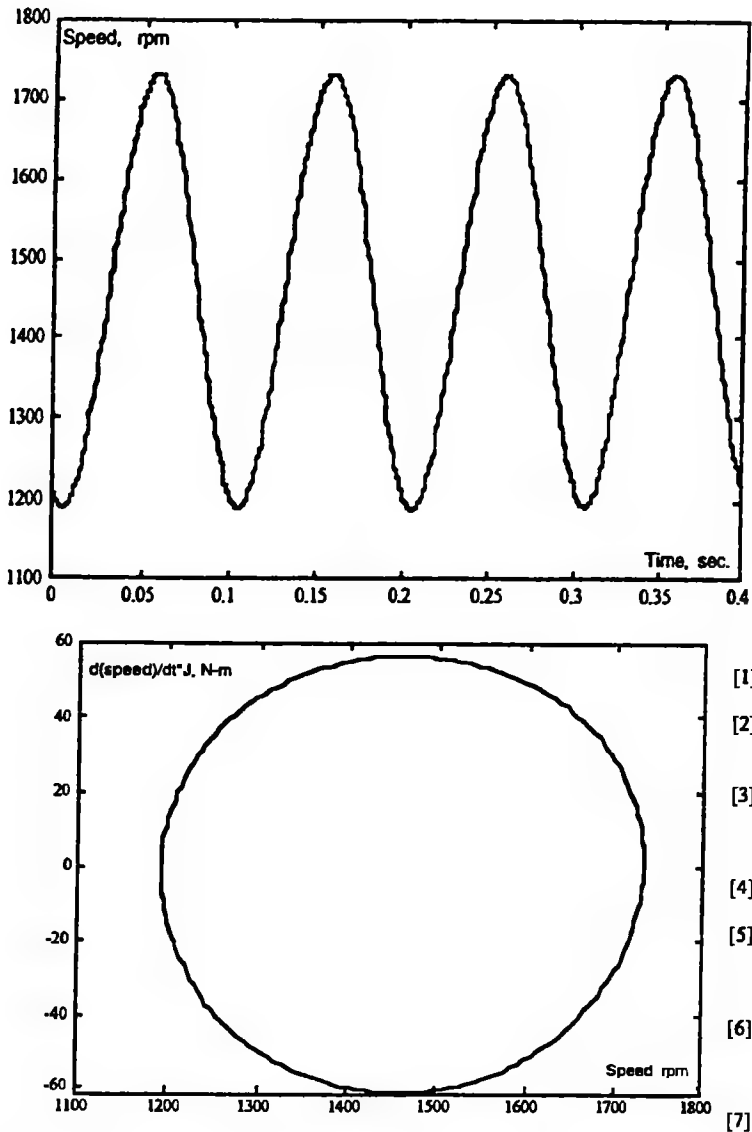


FIGURE 13.11 Speed oscillation and instantaneous active power variation ($P_n = 7.5\text{ kW}$). (After Ref. [15].)

Reference [15] reports <1% difference in efficiency between this method and the direct loading method.

The main demerit of the method is the speed continuous dynamics which gives rise to additional noise and vibration losses in the IM.

If the frequency is changed up and down quickly, the IM speed cannot follow it and thus speed dynamics is avoided. Care must be exercised in this case not to go beyond the D.C. line capacitor threshold voltage during inevitable fast switching from motor to generator mode.

13.2.5.2 The PWM Dual Frequency Test

An inverter with an open control architecture is prepared to produce, through PWM, the $V(t)$ shown in (13.26) which contains two distinct frequencies [16].

This may be chosen around the rated synchronous speed of the IM:

$$\begin{aligned} f_1 &= f_{in} + \Delta f \\ f_2 &= f_{in} - \Delta f \end{aligned} \quad (13.32)$$

Now Δf is the output of a slow RMS current close-loop controller, which will result in the required current load.

Again, Δf is so large that the IM speed oscillates very little somewhere below the average of the two frequencies. The switching from motoring to generating is now carried out, by the electromagnetic field, with a frequency equal to the difference between f_1 and f_2 .

To provide for the same core losses, they are considered proportional to voltage squared and superposition is applied.

Consequently,

$$V_{in}^2 = V_1^2 + V_2^2 \quad (13.33)$$

Still, to determine V_1 and V_2 from (13.33) one of them should be adopted. Alternatively, the amplitude of the magnetic flux may be conserved to provide about the same saturation level:

$$\left(\frac{V_{in}}{f_{in}} \right)^2 = \left(\frac{V_1}{f_1} \right)^2 + \left(\frac{V_2}{f_2} \right)^2 \quad (13.34)$$

with $\Delta f = 5 \cdot 10^{-2} f_{in}$ from (13.33) and (13.34). $V_2 = 0.5383 V_{in}$ and $V_1 = 0.8427 V_{in}$. A rather complete block diagram of the entire system is shown in Figure 13.12. As evident in Figure 13.12, the whole testing process is “mechanized”. This includes starting the IM with a single frequency voltage, which is ramped slowly until the rated frequency is reached. Then, gradually, the dual frequency voltage PWM enters into action and produces the recalculated voltage amplitudes and frequencies. It should be noted that as the load is varied, through the RMS current, not only Δf varies but, also, the voltage amplitudes V_1 and V_2 vary according to (13.33) and (13.34). Typical test results are shown in Figure 13.13 [16].

As expected, the voltage, flux, and current are modulated. Also the capacitor voltage in the D.C. link varies. When it increases, generating mode is encountered.

The instantaneous active power may be calculated as in the previous method, in synchronous coordinates. This time, a few electrical cycles may be averaged, but in essence, the average of instantaneous active power represents the total losses of the machine. The average of the positive values yields the average input power P_{inIm} . So the efficiency may be calculated again as in (13.32) for various reference stator RMS currents. Ultimately, the efficiency can be plotted against output power, which is the difference between input power and losses.

Also temperature measurements may be added either to check the validity of the loss equivalence or, for given temperature, to calibrate the reference RMS current value.

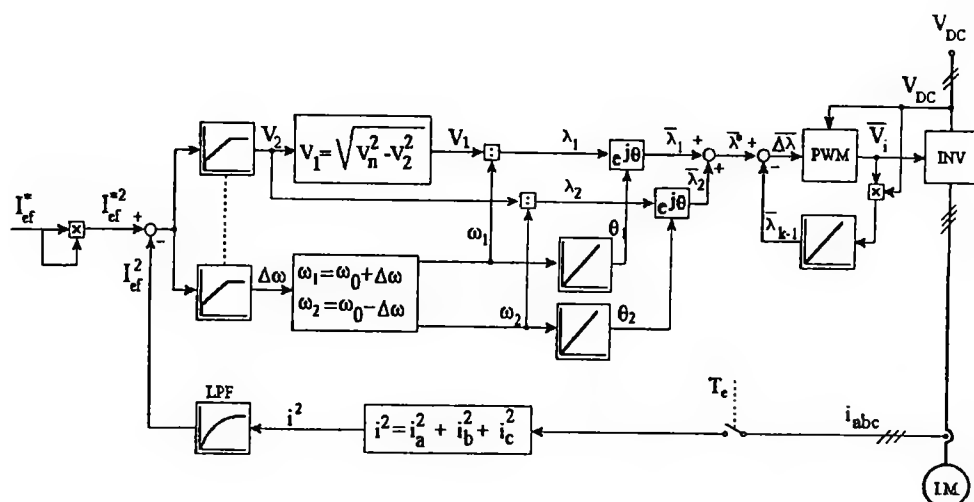


FIGURE 13.12 PWM dual frequency test system: the block diagram. (After Ref. [16].)

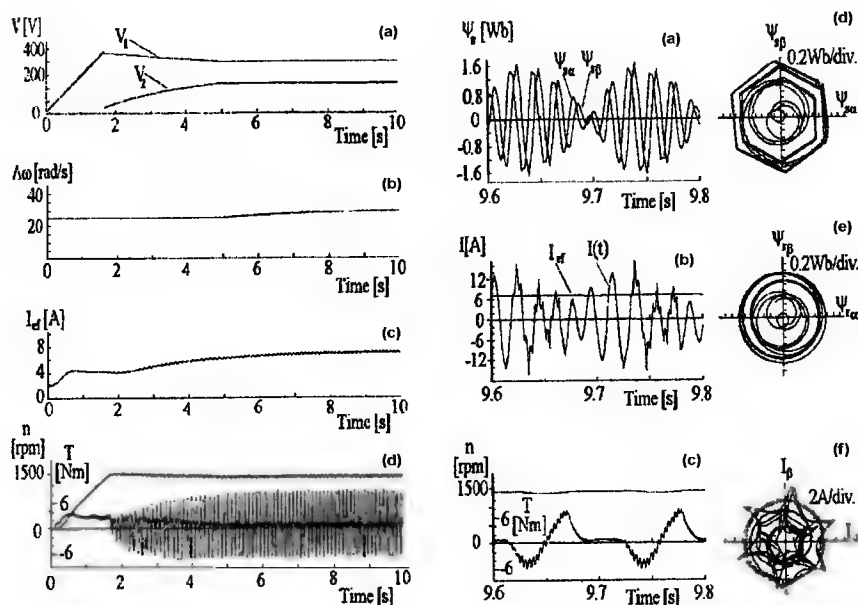


FIGURE 13.13 Sample test results with the PWM dual frequency method. Left side: voltages, speed, current, and torque during acceleration to speed: Right side: steady-state waveforms – flux linkage, stator current, speed, torque, with their hodographs. (After Ref. [16].)

Temperature measurements [16] seem to confirm the method although verifications on many motors are required for full validation.

It should be emphasized that the IM current has been raised up to 150% and thus the complete efficiency test up to 150% load may be replicated by the PWM dual frequency test.

Notice that the slip frequency is around the rated value and thus the skin effect is not different from the case of direct load testing.

The MF methods may be used also for the temperature rise tests.

One more indirect method for temperature rise testing, used extensively by a leading manufacturer for the last 40 years for powers up to 20 MW, is discussed next [17].

13.3 THE TEMPERATURE-RISE TEST VIA FORWARD SHORT-CIRCUIT (FSC) METHOD

For the large-power IMs, direct loading to evaluate efficiency and temperature rise under load involves high costs. To avoid direct (shaft) loading, the superposition and equivalent (artificial) loading methods have been used for large-power, nonstandard frequency or vertical shaft IMs.

The superposition test relies on the fact that temperature rise due to various losses adds up and thus a few special tests could be used to “simulate” the actual temperature rise in the IM. Scaling of results and the rather impossibility to consider correctly the stray load losses render such methods as less practical for IMs [17].

The forward short-circuit test (FSC) [17] performs a replica of load testing from the point of view of losses. It is not standardized yet, but it uses standard hardware. In essence, the tested IM is driven to rated speed slowly by drive motor 1, rated at 10% the rated power of the tested IM (Figure 13.14).

Then, a synchronous generator is rotated by drive motor 2, rated at 10% of generator rated power, and then excited to produce low voltages of frequency f'_1 .

$$f'_1 \approx (0.8 - 0.85)f_{in} \quad (13.35)$$

As the generator excitation current increases, so do the generator voltages supplying the tested motor. The tested motor starts operating as a generator feeding active power to the generator, which becomes now a motor (albeit at low power), and the drive motor 2 works as a generator. The frequency of the currents in the rotor of the tested motor Sf'_1 is

$$Sf'_1 = f'_1 - n_p p_1 = -(0.15 - 0.2)f_{in} = f_{21} \quad (13.36)$$

To secure low active power delivery from the tested motor, the slip frequency f_2 is such that the generator operates beyond peak torque slip frequency:

$$|Sf'_1| > f_{2k} \approx \frac{R_2}{2\pi L_{sc}} \quad (13.37)$$

Consequently, the equivalent impedance of the tested IM is small (large slip values), and, to limit the currents around rated value, the voltage at its terminals has to be low. The synchronous generator excitation current controls the voltage at the tested motor terminals. So low active power is delivered by the IM, albeit at rated current. However, operating beyond the peak torque point may result in instabilities unless the excitation current of the generator is not regulated to “freeze” the operation into a point on the IM descending torque/speed curve.

As low active powers are handled, the drive motors rating may be as low as 10% of tested motor power. So, in fact, the tested motor receives rated current but low power (torque) in the generator regime at rather large slip values (0.15–0.25) and low voltage.

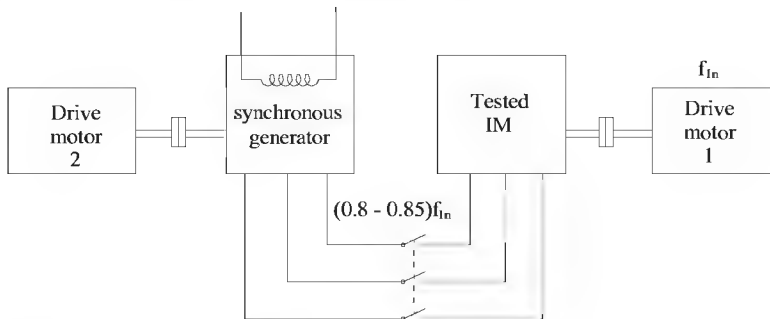


FIGURE 13.14 Test arrangement for FSC test.

The fundamental problem in using this method, intended to replicate temperature rise for rated loading, is, again, equivalence of losses.

But before that, choosing the generator frequency f'_1 is crucial.

When the test f'_1 frequency decreases, the copper losses in the tested motor tend to increase (slip increases) and the required rating of drive motor 1 decreases.

In general, as the slip is rather large, the rotor current, for given stator current, is higher than that for rated load conditions (low slip). The difference is generated by the no-load (magnetization) current, as the rotor current tends to get closer to stator current with increasing slip.

For low no-load current IMs (low number of poles and/or high power), the increase in rotor current in the test with respect to rated power conditions is rather small. Not so for large number of poles and/or low-power IMs.

For $f'_1 = 40$ Hz and $f_{1n} = 50$ Hz, the slip frequency (Sf_1) = 10 Hz, and thus, at least for medium and large-power cage rotor IMs, the skin effect is notable in contrast to rated power conditions.

This is an inherent limitation of the method. Let us now discuss the status of various loss components in the FSC test:

- The stator copper losses

As the current is kept at rated value only, the lower skin effect, due to lower frequency f'_1 ($f'_1 < f_{1n}$), will lead to slightly lower copper losses than in rated load conditions in high-power IMs. If the design data are known, with f'_1 given, the skin effect resistance coefficient may be calculated for f_{1n} and f'_1 (Chapter 9, Vol. 1). Corrections then may be added.

- The fundamental iron losses are definitely lower than those for rated conditions as f'_1 is lower than that for rated conditions and so is the voltage, even the flux.

To the first approximation, the fundamental core losses are proportional to voltage squared (irrespective of frequency). Also, there are some fundamental core losses in the rotor as Sf'_1 is (0.15–0.25) f_1 .

The no-load iron losses at rated voltage and frequency conditions may be compared with the low iron losses estimated for the FSC test with the difference added by increasing stator current to compensate for the difference in the loss balance.

- Rotor cage fundamental losses

As indicated earlier, the rotor cage losses are larger and dependent on the generator frequency f'_1 , but, to calculate their difference, would require the magnetization reactance and the rotor resistance R_2 and leakage reactance X_{l2} at the rather large frequency Sf'_1 (with skin effect considered).

- Stray load losses

A good part of stray load losses occurs in the rotor bars. The frequency $S_v f'_1$ of the rotor bar harmonics currents due to stator mmf space harmonics is

$$S_v f'_1 = [1 - v(1 - S)] f'_1 \quad (13.38)$$

For phase belt harmonics,

$$v = \pm 6K + 1 \quad (13.39)$$

$$\text{with } n = -5, +7, -11, 13, \dots \quad (13.40)$$

If for FST $f'_1 = 40$ Hz and $S = -0.2375$, for rated load conditions $f_{1n} = 50$ Hz and $S_n = 0.01$, the rotor current harmonics frequency $S_v f'_1$ is very close in the two cases (287.5/297.5, 306.5/296.5, 584.5/594.5, 603.5/593.5).

So, from this point of view, the two cases are rather equivalent. That is, the FST method is consistent with the direct load method. However, the rotor current harmonics are also influenced by

slot permeance harmonics, which are voltage dependent. So in general, the stator mmf-caused rotor bar stray losses are smaller for the FST than those for rated conditions.

On the other hand, tooth flux pulsation core losses have to be considered. As the saturation is not present (low voltage; lower main flux level), the flux pulsation in the tooth, for given currents, tends to be larger in the FST, than for rated load conditions.

The rotor slot harmonic pole pairs p_μ is

$$p_\mu = KN_r \pm p_1 \quad (13.41)$$

where N_r – number of rotor slots, p_1 – IM pole pairs.

The frequency f_μ of the eddy currents induced in the stator tooth is

$$f'_\mu = [p_\mu(1-S) + S]f'_1 \quad (13.42)$$

It suffices to consider $K = 1.2$.

For the same example as above ($f'_1 = 40$ Hz, $S = -0.2375$ and $f_1 = 50$ Hz, $S_n = 0.01$) and $K = 1.2$, again frequencies f_μ very close to each other are obtained for the two situations.

So the stator harmonics core losses are equivalent.

Still, we have to remember that the rotor cage fundamental losses tend to be somewhat larger (because of much larger slip and skin effect), whereas the fundamental core losses are much lower.

The degree to which these two effects neutralize each other is an issue to be tackled for every machine separately.

Final results related to temperature rise for three tests – FSC, IEEE-112F and the two frequency methods (MF) – for a 1960 kW four poles, 50 Hz, 6600 V IM [17] are shown in Figure 13.15. The FSC and direct loading tests produce very close results, whereas the MF method overestimates the temperature by 5°C–7°C.

The standard MF method overestimation of temperature rise (Figure 13.15) is due to lower speed, speed oscillations, and due to higher core losses as the supply frequency voltage V_1 comes in with 100% with V_2 added to it. It appears that the PWM dual frequency test is free from such limitations and would produce reliable results in temperature rise as well as in efficiency tests. Returning to

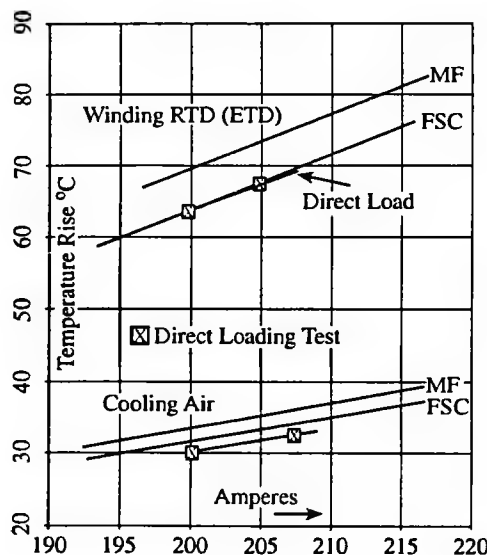


FIGURE 13.15 Steady-state temperature rises with three tests. (After Ref. [17].)

FSC, we notice that if the core losses are neglected (the voltage is reduced), the difference between the generated power P_G and the shaft input power P_{shaft} for the tested motor is

$$P_{\text{shaft}} - P_G = 3I_1^2 R_1 + 3I_2^2 R_2 + p_{\text{stray}} + p_{\text{mec}} + p_{\text{iron}} \left(\frac{V}{V_{\text{In}}} \right)^2 \quad (13.43)$$

The mechanical losses p_{mec} are considered to be known, whereas P_{shaft} , P_G , and I_1 have to be measured. When a D.C. machine is used as drive motor 1, its calibration is easy and thus P_{shaft} may be estimated without a torque-meter. The iron losses at rated voltage are p_{iron} .

Now if we consider the stray losses located in the stator, then the electromagnetic power concept yields

$$\frac{3I_2^2 R_2}{|S|} = P_G + 3R_1 I_1^2 + p_{\text{stray}} + p_{\text{iron}} \left(\frac{V}{V_{\text{In}}} \right)^2 \quad (13.44)$$

Eliminating $I_2^2 R_2$ from (13.43) to (13.44) yields

$$p_{\text{stray}} = \frac{(P_{\text{shaft}} - p_{\text{mec}})}{1 + |S|} - P_G - 3R_1 I_1^2 - p_{\text{iron}} \left(\frac{V}{V_{\text{In}}} \right)^2 \quad (13.45)$$

As all powers involved are rather small, the error of calculating p_{stray} is not expected to be large. With p_{stray} calculated, from (13.45), the rotor fundamental cage losses $3R_2 I_2^2$ are determined. To the first approximation, with $I_2 \approx \sqrt{I_1^2 - I_m^2}$ (I_m – the no-load current at low voltage levels, that is under unsaturated conditions), the rotor resistance R_2 at Sf_1' frequency may be determined as a bonus. Comparing results on parameter calculation from FSC and IEEE–112F shows acceptable correlation for the 1960kW IM investigated in Ref. [17].

The level of voltage at IM terminals during FSC is around 20% rated voltage. A few remarks on FSC are in order

- The loss distribution is changed with more losses in the rotor and less in the stator.
- The skin effect is rather notable in the rotor.
- It is possible, in principle, to replace the drive motor 2 and the A.C. generator by a bi-directional power PWM converter sized around 20% the rated power of the tested IM (Figure 13.16).

The control options, for adequate loss equivalence and stable operation, are improved by the presence and the capabilities of the PWM converter.

13.4 PARAMETER ESTIMATION TESTS

By IM parameters, we mean the resistances and inductances in the adopted circuit model and the inertia.

There are many ways we may develop a circuit model which duplicates better or worse the performance of IM in different operation modes. Magnetic saturation, skin effect, and the space and

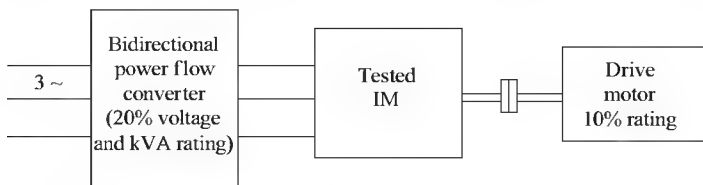


FIGURE 13.16 The FSC test with PWM converter.

time mmf harmonics make the problem of parameter estimation very difficult. Traditionally, the no-load and stalled rotor tests at rated frequency are used to estimate the parameters of the single-cage circuit model, with skin effect neglected.

On the other hand, for IM with deep rotor bars, or double-cage rotors, short-circuit tests at rated frequency produce too a high value for the rotor resistance for the IM running at load and rated frequency.

Standstill frequency response (SSFR) tests are recommended for such cases.

Still, the large values of currents during IM direct starting at the power grid produce saturation in the leakage path of both stator and rotor magnetic fields. Consequently, the leakage inductances are notably reduced. This effect adds to the rotor leakage inductance reduction due to skin effect.

The closed rotor cage slot leakage field, on the other hand, saturates the upper iron bridge for rotor currents above 10% of rated current.

This means that for such a case, the rotor leakage inductance does not vary essentially at currents above 10% rated current, that is during starting or on-load conditions.

For variable-speed IM operation, the level of the airgap flux varies notably. Consequently, the magnetization inductance varies also. The apparent way out of these difficulties may seem to use the traditional single-rotor circuit model with all parameters as variables (Figure 13.17) with primary (f_1), rotor slip frequency Sf_1 , and stator, rotor, and magnetization currents (I_1 , I_2 , and I_m) as measured variables. The trouble is that most parameters depend on more than one variable and thus such an equivalent circuit is hardly practical (Figure 13.17). Also, if space and time harmonics are to be considered, the general equivalent circuit becomes all but manageable.

For the space harmonics, the slip S changes to S_K , whereas for time harmonics, both f_K and S_K vary with the harmonics order K (see Chapter 10, Vol. 1, on time harmonics). We should remember that, while spinning the rotor, slip frequency Sf_1 is different from stator frequency f_1 and thus only tests performed in such conditions match the performance. Also, Finite element modelling (FEM) is now useful enough in the computation of all parameters in any conditions of operation.

Finally, for IM control for drives, a rather simplified equivalent circuit (model) with some parameter variation would be practical. This is how simplified solutions for IM parameter estimation offline and on line evolved.

Among them, in fact, only the no-load and short-circuit single frequency test-derived parameter estimation method is standardized worldwide. The two frequency and the frequency response standstill tests have gained some momentum lately. Modified regression methods to estimate double-cage model parameters based on acquiring data over a direct slow starting period or directly from transients during a fast start have been developed recently. Finally, for variable-speed drives commissioning and their adaptive control, offline and online simplified methods to determine some parameter detuning are currently used. A rough synthesis of these trends is presented next.

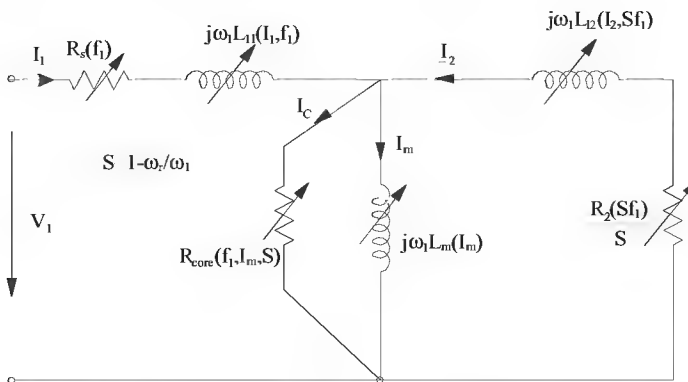


FIGURE 13.17 Fundamental single-cage rotor equivalent circuit with variable parameters.

13.4.1 PARAMETER CALCULATION FROM NO-LOAD AND STANDSTILL TESTS

The no-load motor test – Section 13.1.1 – is hereby used to calculate the electric parameters appearing in the equivalent circuit for zero rotor currents (Figure 13.18).

With voltage V_1 , current I_{10} , and power P_0 measured, we may calculate two parameters:

$$\begin{aligned} R_1 + R_{cs} &= \frac{(P_0 - p_{mec})}{3I_{10}^2} \\ L_{11} + L_m &= \frac{1}{\omega_1} \sqrt{\left(\frac{V_1}{I_{10}}\right)^2 - (R_1 + R_{cs})^2} \end{aligned} \quad (13.46)$$

The stator phase resistances may be D.C. measured or, if possible, in A.C. without the rotor in place. In this case, if a small three-phase voltage is applied, the machine will show a very small R_{cs} (core losses are very small). The calculated phase inductance slightly overestimates the stator leakage inductance.

The overestimation is related to the magnetic energy in the air left by the missing rotor.

The equivalent inductance L_g of the “air” inside stator is, in fact, corresponding to the magnetization inductance of an equivalent airgap of τ/π (τ the pole pitch):

$$L_g = (L_m)_{unsat} \cdot \frac{g\pi}{\tau} \quad (13.47)$$

Suppose we do these measurements (without the rotor) and obtain from power, voltage, and current measurements,

$$\begin{aligned} R_{1-} &= \frac{P_{3-}}{3I_{10-}^2} \\ L_{11} + L_g &= \frac{1}{\omega_1} \sqrt{\left(\frac{V_{1-}}{I_{10-}}\right)^2 - R_{1-}^2} \end{aligned} \quad (13.48)$$

From the no-load test at low voltage, the unsaturated value of L_m is obtained. Then from (13.47), L_g is easily calculated, provided the airgap is known. Finally from (13.48), L_{11} is found with L_g already calculated.

In general, however, the stator leakage inductance is not considered separable from the no-load and short-circuit tests.

The short-circuit (standstill or stall) tests, on the other hand, are performed at low voltage with three-phase or single-phase supply (Section 13.1.4), and R_{sc} and $X_{sc} = \omega_1 L_{sc}$ are calculated from

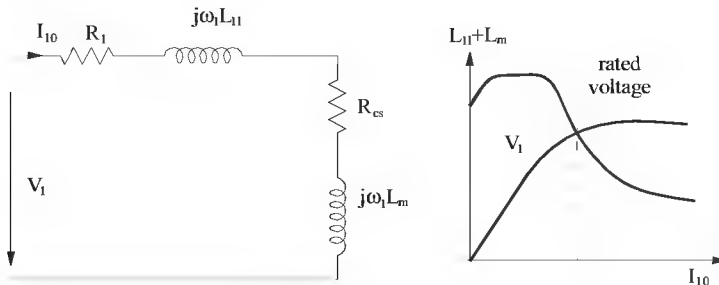


FIGURE 13.18 Fundamental no-load equivalent circuit.

(13.9) to (13.10) and averaged over a few current values up to about rated current. When voltage versus current is plotted, the curve tends to be linear but, for the closed-slot rotor, it does not converge into the origin (Figure 13.19).

The residual voltage E_{bridge} , corresponding to the rapidly saturating iron bridge above the rotor closed slot, may be added to the equivalent circuit in the rotor part. All the other components of the rotor leakage inductance occur in L_{12} . Such an approximation appears to be practical for small IMs.

With R_1 known, R_{2sc} may be calculated from (13.9) to (13.10), that is, the rotor resistance for the rated frequency, with the skin effect accounted for.

The stator and rotor leakage inductances at short-circuit come together in $X_{sc} = (L_{11} + L_{12sc})\omega_1$. Unless L_{11} is measured as indicated above or its design value is known, L_{11} is taken as $L_{11} \approx L_{sc}/2$.

Note that the no-load and short-circuit tests for the wound rotor IM are straightforward as both the stator and rotor currents may be measured directly. Also, the tests may be run with stator or rotor energized. Even both stator and rotor circuits may be energized. Changing the amplitude and phase of rotor voltage until the airgap field is zero leads to a complete separation of the two circuits. The same tests have been proved acceptable to derive torque/current load characteristics for super-high-speed PWM converter-fed IMs [18]. The current is limited in this case by the converter rating. Neglecting the magnetization inductance in the equivalent circuit at standstill ($S = 1$) (Figure 13.20) is valid as long as $\omega_1 L_m \gg R_{2sc}$.

It is known that at full voltage and standstill, the airgap flux is only about (50%–60%) of its no-load value due to the large voltage drop along the stator resistance R_1 and leakage reactance $\omega_1 L_{11}$. So, in any case, the main magnetic circuit is nonsaturated. That is to say that the nonsaturated value of $L_m - (L_m)_{\text{unsat}}$ – has to be introduced in the equivalent circuit at low frequencies, when $\omega_1 L_m$ is not much larger than R_{2sc}' . If the L_m branch circuit is left out in low-frequency standstill tests, the errors become unacceptable.

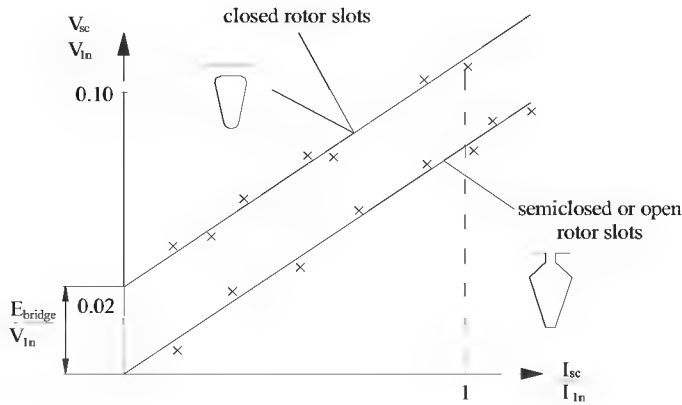


FIGURE 13.19 Typical voltage/current curves at standstill.

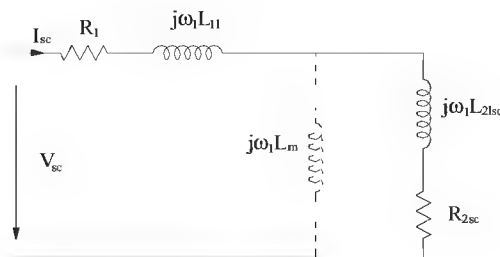


FIGURE 13.20 Equivalent circuit at standstill.

The presence of skin effect suggests that the single-rotor circuit model is not sufficient to accommodate a wide spectrum of operation modes – from standstill to no load.

The first easy step is to perform one more standstill test, but at low frequency: <5% of rated frequency. This is how the so-called two-frequency standstill test was developed.

13.4.2 THE TWO-FREQUENCY STANDSTILL TEST

The second short-circuit test, at low frequency, requires a low-frequency power source. The PWM converter makes a good (easy available) low-frequency power supply.

This time, the equivalent circuit contains basically two loops (Figure 13.21).

To calculate the four rotor parameters R_2 , L_{l2} , R_3 , and L_{l3} , it is assumed that at rated frequency $f_{ln}(\omega_{ln})$, only the second (starting) rotor loop exists. That is, only L_{l3} and R_3 remain as unknowns and the L_m branch may be eliminated. At the second frequency $f'_l \leq 0.05f_{ln}$, only the first (working) cage acts. Thus, only R_2 and L_{l2} are to be calculated with the L_m branch accounted for. L_m is taken as the nonsaturated value of L_m from the no-load test. An iterative procedure may be used with $L_m(I_m)$ from the no-load test as the main circuit may get saturated unless $V_{sc}/V_{ln} < f'_l/f_{ln}$.

The rotor parameters of rotor loops are constant. One loop dominates the performance at large slip frequencies Sf_1 (R_3 , L_{l3}), and the other one prevails at low slip frequency.

The double-cage rotor IM may be assimilated with this case. However, the two cages measured parameters may not overlap with the actual double-cage parameters.

As of now, with the above parameter estimation methods, skin effect is calculated approximately. Also by the $L_m(I_m)$ function, the magnetic saturation of the main circuit is calculated.

Still, the leakage flux path saturation at high currents has not been present in tests. Consequently, calculating the torque, current, and power factor from $S = 1$ to S_n (S_n – rated slip) with the above parameters is bound to produce large errors. And so it does!

The dissatisfaction of IM users with this situation led to quite a few simplified attempts to calculate the parameters, basically R_2 , R_{2sc} , L_{l2} , and L_{l2sc} , based on catalogue data to fit both the starting current, and torque and the rated current and power factor for given efficiency, rated speed, and torque. These approaches may be dubbed as catalogue-based methods.

13.4.3 PARAMETERS FROM CATALOGUE DATA

Let us consider that the starting current and torque I_s , T_{es} , rated speed, frequency, current, power factor, efficiency, and rated power are all catalogue data for the IM. It is well understood that the logic solution to parameter estimation for two extreme situations – start and full load – is to use the equivalent circuit. First, from the no-load current I_0 ,

$$I_0 \approx \frac{V_{ln}}{\omega_l L_m (1 + L_{l1}/L_m)} \quad (13.49)$$

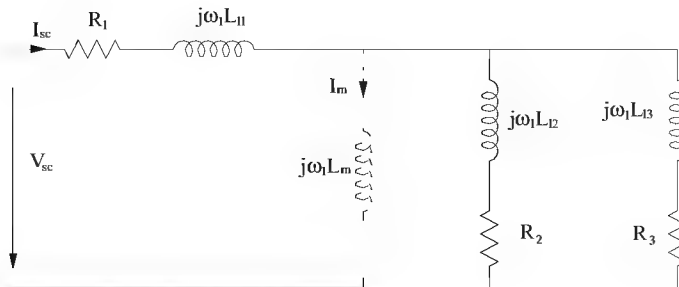


FIGURE 13.21 Dual rotor loop equivalent circuit at standstill.

we obtain an approximate value of L_m . For start $L_{1l}/L_m = 0.02-0.05$. At the end of the estimation process, L_{1l} is obtained.

Then, we may return with a corrected value of L_{1l}/L_m to (13.49) and perform the whole process again until sufficient convergence is reached.

This saturated value is valid for rated load calculations.

For the starting torque, T_{es} at starting stator current I_s

$$T_{es} = \frac{3R_{2sc} \cdot I_{2s}^2 \cdot p_l}{\omega_l} \quad (13.50)$$

$$I_{2s} \approx \sqrt{I_s^2 - I_0^2}$$

From (13.50), R_{2sc} may be determined. The equivalent circuit at start yields

$$(L_{1lsc} + L_{12sc}) = \frac{1}{\omega_l} \sqrt{\left(\frac{V_{ln}}{I_s}\right)^2 - (R_l + R_{2sc})^2} \quad (13.51)$$

The stator resistance R_l may be D.C. measured, and thus, $(L_{1lsc} + L_{12sc})$, as influenced by skin effect and leakage saturation, is determined.

On full load, the mechanical losses are assigned an initial value $p_{mec} = 0.01P_n$.

Thus, the electromagnetic torque is equal to the shaft torque plus the mechanical loss torque.

$$T_{en} = p_l \left(1 + \frac{p_{mec}}{P_n}\right) \frac{P_n}{2\pi f_l (1 - S_n)} = \frac{3R_2 I_{2n}^2}{S_n \omega_l} p_l \quad (13.52)$$

$$I_{2n}^2 \approx \sqrt{I_{ln}^2 - I_0^2}$$

With R_2 determined easily from (13.52), we write the balance of powers at rated slip:

$$Q_l = 3V_{ln} I_{ln} \sin \varphi_{ln} = 3\omega_l L_{1l} I_{ln}^2 + 3\omega_l L_{12} I_{2n}^2 + 3L_m \omega_l \cdot I_0^2 \quad (13.53)$$

$$P_l = 3V_{ln} I_{ln} \cos \varphi_{ln} = 3R_l I_{ln}^2 + p_{iron} + p_{stray} + 3 \frac{R_2 I_{2n}^2}{S_n} \quad (13.54)$$

$$P_n \left(1 - \frac{1}{\eta}\right) = 3R_l I_l^2 + p_{iron} + p_{stray} + 3R_2 I_{2n}^2 + p_{mec} \quad (13.55)$$

There are five unknowns – L_{1l} , L_{12} , p_{iron} , p_{stray} , and p_{mec} – in Equations (13.53)–(13.55).

However from (13.54), $p_{iron} + p_{stray}$ can be calculated directly. They are introduced in (13.55) to calculate mechanical loss p_{mec} . If they differ from the initial value of 1% of rated power P_n , their value is corrected in (13.52) and $p_{core} + p_{stray}$ and p_{mec} are recalculated again until sufficient convergence in p_{mec} is obtained.

In Equation (13.53), we do have two unknowns remaining: the stator and rotor leakage inductances. They may not be separated, and thus, we consider them equal to each other: $L_{1l} = L_{12}$.

This way

$$L_{1l} = L_{12} = \frac{(3V_{ln} I_{ln} \sin \varphi_{ln} - 3\omega_l L_m I_0^2)}{3\omega_l (2I_{ln}^2 - I_0^2)} \quad (13.56)$$

Now going back to (13.51), if the IM has open stator slots, there will be no leakage flux path saturation in the stator, and thus, $L_{l1sc} = L_{l1} = L_{l2}$. Consequently, from (13.51), L_{l2sc} is found. With semi-closed stator slots, both stator and rotor leakage flux paths may saturate. So, we might consider $L_{l1sc} = L_{l2sc}$ as well. Again, $L_{l2sc} = L_{l1sc}$ is found from (13.51).

Attention has to be paid to (13.56) as computation precision is important because most of the reactive power is “spent” in the airgap, in L_m . In general, $2L_{l1} > L_{l1sc} + L_{l2sc}$. If $2L_{l1} \leq L_{l1sc} + L_{l2sc}$ is obtained, the skin and saturation effects are mild and the short-circuit values hold as good for all slip values.

An approximate dependence of rotor resistance and leakage inductances with slip, between $S = 1$ and $S = S_n$, is

$$R_2(S) = R_2 + (R_{2sc} - R_2) \sqrt{\frac{(S - S_n)}{1 - S_n}}; \quad S \geq S_n \quad (13.57)$$

$$L_{l2}(S) = L_{l2} - (L_{l2} - L_{l2sc}) \frac{S - S_n}{1 - S_n}; \quad S \geq S_n \quad (13.58)$$

$$L_{l1}(S) = L_{l1} - (L_{l1} - L_{l1sc}) \frac{S - S_n}{1 - S_n}; \quad S \geq S_n \quad (13.59)$$

This way, the parameter dependence on slip is known, so torque, current, power factor versus slip are calculated. Alternatively, we may consider that the rotor parameters at standstill R_{2sc} and L_{l2sc} represent the starting cage and R_{2l} , L_{l2} , the working cage. Such an attitude may prove practical for the investigation of IM transients and control.

It should be emphasized that catalogue data methods provide for complete agreement between the circuit model and the actual machine performance only at standstill and at full load. There is no guarantee that the rotor parameters vary linearly or as in (13.57) with slip for all rotor (stator) slot geometries.

In other words, the agreement of, say, the torque and current versus slip curves for all slips is not guaranteed to any definable error.

From the need to secure good agreement between the circuit model and the real machine, for ever wider operation modes, more sophisticated methods have been proposed.

We introduce here two of the most representative ones: the SSFR method and the step-wise regression general method.

13.4.4 STANDSTILL FREQUENCY RESPONSE METHOD

Standstill tests, especially when a single-phase A.C. supply is used (zero torque), require less man power, equipment, and electrical energy to perform than running tests. To replicate the machine performance with different rotor slip frequency Sf_1 , tests at standstill with a variable voltage and frequency supply have been proposed [19,20]. Dubbed as SSFR, such tests have been first used for synchronous generator parameter estimation.

In essence, the IM is fed with variable voltage and frequency from a PWM converter supply.

Single-phase supply is preferred as the torque is zero, and thus, no mechanical fixture to stall the rotor is required. However, as three-phase PWM converters are available up to high powers and the tests are performed at low currents (less than 10% of rated current), three-phase testing seems more practical (Figure 13.22).

The test provides for the consideration of skin effect, which is frequency dependent. However, as the current is low, to avoid IM overheating, the SSFR test methods should not be expected to produce good agreement with the real machine in the torque/slip or current/slip curves at rated voltage.

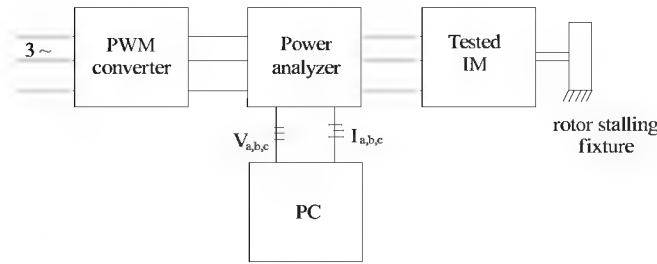


FIGURE 13.22 Standstill frequency test arrangement.

This is so because the leakage saturation effect of reducing the leakage inductances is not considered. A way out of this difficulty is to adopt a certain functional for stator leakage inductance variation with stator current.

Also at low frequencies, 0.01 Hz, two to three periods acquisition time means 200–300 seconds. So, in general, acquiring data up to 50–100 Hz frequencies is time prohibitive. The voltages and currents are acquired, and their fundamental wave amplitudes and ratio of amplitudes and phase lag are calculated.

A two (even three)-rotor loop circuit model is used (Figure 13.23).

The operational inductance $L(p)$ for a double-cage model (see Chapter 1) in rotor coordinates is

$$L(p) = (L_{11} + L_m) \frac{(1 + pT')(1 + pT'')}{(1 + pT'_0)(1 + pT''_0)} \quad (13.60)$$

The equivalent circuit does not show up the mutual leakage inductance between the two virtual cages (L_{123}) because the latter cannot be measured directly.

The time constants in (13.60) are related to the equivalent circuit parameters as follows:

$$T' = \frac{1}{R_3} \left[L_{31} + \frac{L_m \cdot L_{11}}{L_m + L_{11}} \right] \quad (13.61)$$

$$T'' = \frac{1}{R_2} \left[L_{12} + \frac{L_m L_{11} L_{13}}{L_{13} L_m + L_{13} L_{11} + L_m L_{11}} \right] \quad (13.62)$$

$$T'_0 = \frac{1}{R_3} (L_m + L_{11} + L_{13}) \quad (13.63)$$

$$T''_0 = \frac{1}{R_2} \left[L_{12} + \frac{L_m L_{13}}{L_m + L_{13}} \right] \quad (13.64)$$

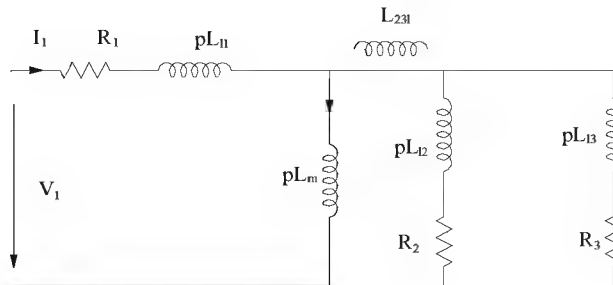


FIGURE 13.23 Double-cage operational equivalent circuit at standstill.

The unsaturated value of the no-load inductance $L_s = L_{ll} + L_m$ has to be determined separately from the no-load test at lower than rated voltage.

Also the stator leakage inductance value has to be known from design or is being assigned a “reasonable” value of 2%–5% of no-load inductance L_s .

The measurements will result in impedance estimation for every frequency the test is performed.

$$\underline{Z}_{sc}(\omega) = \frac{V_{sc}}{I_{sc}} = R_{sc}(\omega) + jX_{sc}(\omega) \quad (13.65)$$

When the IM is single-phase supply fed between two phases, a factor of two occurs in the denominator of (13.65).

We now calculate $L(j\omega)$ from (13.65):

$$L(j\omega) = \frac{(\underline{Z}_{sc} - R_l)}{j\omega_l} = \text{Re}(\omega) + j\text{Im}(\omega) \quad (13.66)$$

Equating the real and imaginary parts in (13.60) and (13.66), we obtain

$$\text{Re}(\omega) = \omega L_s + \omega \text{Im}(\omega)(T'_0 + T''_0) - \omega^3 L_s T' T'' + \omega^2 \text{Re}(\omega) \cdot T'_0 \cdot T''_0 \quad (13.67)$$

$$\text{Im}(\omega) = \omega^2 L_s (T' + T'') - \omega \text{Re}(\omega)(T'_0 + T''_0) - \omega^2 \text{Im}(\omega) T'_0 \cdot T''_0 \quad (13.68)$$

A new set of unknowns, instead of T' , T'' , T'_0 , T''_0 , is now defined [21].

$$\begin{aligned} X_1 &= L_s \\ X_2 &= T'_0 + T''_0 \\ X_3 &= L_s T' T'' \\ X_4 &= T'_0 T''_0 \\ X_5 &= L_s (T' + T'') \end{aligned} \quad (13.69)$$

L_s may be discarded by introducing the known value of the nonsaturated no-load inductance.

A linear system is obtained.

In matrix form,

$$|Z_i| = |H_i| |X|; i = 1, 2, \dots, m \quad (13.70)$$

with

$$|Z_i| = |\text{Re}(\omega), \text{Im}(\omega)|_i^T \quad (13.71)$$

$$|X| = |X_1, X_2, X_3, X_4, X_5| \quad (13.72)$$

$$|H_i| = \begin{vmatrix} \omega & \omega \text{Im}(\omega) & -\omega^3 & \omega^2 \text{Re}(\omega) & 0 \\ 0 & -\omega \text{Re}(\omega) & 0 & \omega^2 \text{Im}(\omega) & \omega^2 \end{vmatrix} \quad (13.73)$$

The measurements are taken for $m \geq 5$ distinct frequencies.

As the number of equations is larger than the number of unknowns, various approximation methods may be used to solve the problem.

The solution of (13.70) in the least squared error method sense is

$$|X| = [([H]^T \cdot [H])]^{-1} \cdot [H]^T \cdot [Z] \quad (13.74)$$

Once the variable vector $|X|$ is known, the actual unknowns $L_s, T', T'', T'_0, T''_0$ are

$$L_s = X_1$$

$$T', T'' = \frac{1}{2X_1} \left(X_5 \pm \sqrt{X_5^2 - 4X_3X_1} \right) \quad (13.75)$$

$$T'_0, T''_0 = \frac{1}{2} \left(X_2 \pm \sqrt{X_2^2 - 4X_4} \right) \quad (13.76)$$

The least squared error method has been successfully applied to a 1288kW IM in [21].

Typical direct impedance results are shown in Figure 13.24 [21].

Comparative results [20,21] of parameter values estimated with Bode diagram and, respectively, by the least squared method, for same machine, are given in Table 13.3 [21].

Good agreement between the two methods of solution is evident.

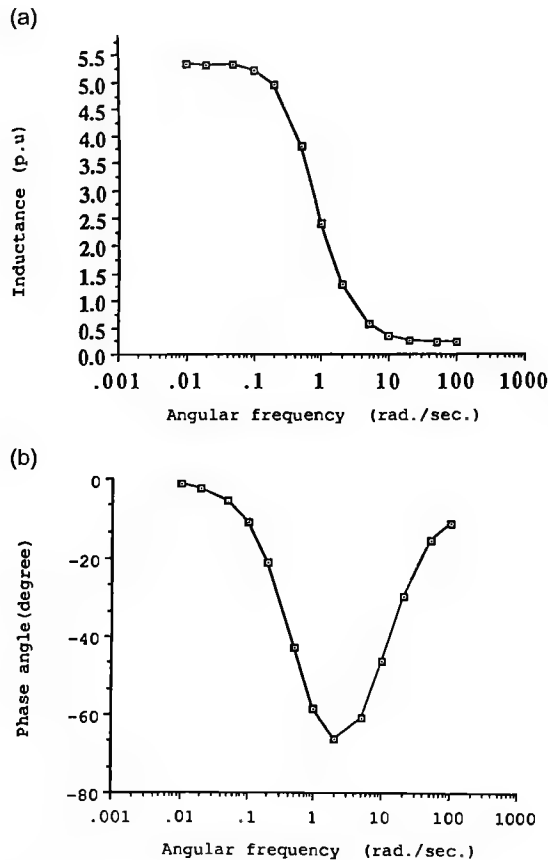


FIGURE 13.24 SSFR of a 1288kW IM. (After Ref. [21].)

TABLE 13.3
Comparative Parameter Estimates for an 1288kW IM

Parameter	Bode Diagram [20]	Least Squared Method [21]
I_s p.u.	5.3	5.299
T'_s	$88 \cdot 10^{-3}$	$87.99 \cdot 10^{-3}$
T''_s	$3.6 \cdot 10^{-3}$	$3.6 \cdot 10^{-3}$
T'_0 s	2.0	2.00
T''_0 s	$4.55 \cdot 10^{-3}$	4.55

However, as [20] shows, when applying these parameters to the torque/slip or current/slip curves at rated load, the discrepancies in torque and current at high values of slip are inadmissible. This may be explained by the neglect of leakage saturation, which in the real machine with high starting torque and current is notable.

As always in such cases, an empirical expression of the stator leakage inductance, which decreases with stator current, may be introduced to improve the model performance at high currents.

Still, there seem to be some notable discrepancies in the breaking torque region, which hold in there even after correcting the stator leakage inductance for leakage saturation. A three-circuit cage seems necessary in this particular case.

This raises the question of when the SSFR parameter estimation may be used with guaranteed success. It appears only for small periodic load perturbations around rated speed or for estimating the machine behaviour with respect to high-frequency time harmonics so typical in PWM converter-fed IMs.

Step response methods to different levels of voltage and current peak values may be the response to the leakage saturation at high currents inclusion into the IM models.

Short voltage pulses may be injected in the IM at standstill between two phases until the current peaks to a threshold, predetermined value. A pair of positive and negative voltage pulses may be applied to eliminate the influence of hysteresis.

The current and voltage are acquired and processed using the same operational inductance (13.60).

$$v(p) = 2 \cdot i(p) [R_1 + pL(p)] \quad (13.77)$$

The two cage model may be used, with its parameters determined to fit the current/time transient experimental curve via an approximation method (least squared method, etc.)

The voltage signals will stop, via a fast static power switch, at values of current up to the peak starting current to explore fully the leakage saturation phenomenon. The pulses are so short that the motor does not get warm after 10–15 threshold current level tests. Also, the testing will last only 1–2 minutes in all and requires simplified power electronics equipment. It may be argued that the step voltage is too a fast signal and thus the rotor leakage inductances and resistances are too much influenced by the skin effect. Only extensive tests hold the answer to this serious question.

The method to estimate the parameters from this test is somehow included in the general regression method that follows.

13.4.5 THE GENERAL REGRESSION METHOD FOR PARAMETERS ESTIMATION

As variable parameters, with current and/or rotor slip frequency, are difficult to handle, constant parameters are preferred. By using a two-rotor cage circuit model, the variable parameters of a single cage with slip frequency are calculated. Not so for magnetic saturation effects.

The deterministic way of defining variable parameters for the complete spectrum of frequency and current has to be apparently abandoned for constant parameter models which, for a given

spectrum of operation modes, give general satisfaction according to an optimization criterion, such as the least squared error over the investigated operation mode spectrum.

For transient modes, the transient model has to be used, whereas for steady-state modes, the steady-state equations are applied.

The d.q. model of double-cage IM in synchronous coordinates in space phasor quantities is (Chapter 1)

$$V_1 = R_1 \bar{I}_1 + \frac{d\bar{\Psi}_1}{dt} + j\omega_1 \bar{\Psi}_1; \quad \bar{\Psi}_1 = \bar{\Psi}_m + L_{11} \bar{I}_1 \quad (13.78)$$

$$0 = R_2 \bar{I}_2 + \frac{d\bar{\Psi}_2}{dt} + jS\omega_1 \bar{\Psi}_2; \quad \bar{\Psi}_2 = \bar{\Psi}_m + L_{12} \bar{I}_2 \quad (13.79)$$

$$0 = R_3 \bar{I}_3 + \frac{d\bar{\Psi}_3}{dt} + jS\omega_1 \bar{\Psi}_3; \quad \bar{\Psi}_3 = \bar{\Psi}_m + L_{13} \bar{I}_3 \quad (13.80)$$

$$\bar{\Psi}_m = L_m (\bar{I}_1 + \bar{I}_2 + \bar{I}_3); \quad \bar{I}_2 = I_{d2} + jI_{q2}; \quad \bar{I}_3 = I_{d3} + jI_{q3} \quad (13.81)$$

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{load}; \quad T_e = \frac{3}{2} p_1 L_m \cdot \text{Imag} \left[\bar{I}_1 \cdot (\bar{I}_2^* + \bar{I}_3^*) \right] \quad (13.82)$$

For sinusoidal voltages,

$$\bar{V}_1 = V\sqrt{2} \cos \gamma - jV\sqrt{2} \sin \gamma; \quad \bar{I}_1 = I_{d1} + jI_{q1} \quad (13.83)$$

Equations (13.78)–(13.82) represent a seven-order system with seven variables (I_{d1} , I_{q1} , I_{d2} , I_{q2} , I_{d3} , I_{q3} , and ω_r).

For steady-state, $d/dt = 0$ in Equations (13.78)–(13.80) – synchronous coordinates.

When the IM is fed from a nonsinusoidal voltage source, $\bar{V}_s$ has to be calculated according to Park transformation:

$$\bar{V}_s = \frac{2}{3} \left[V_a(t) + V_b(t)e^{j\frac{2\pi}{3}} + V_c(t)e^{-j\frac{2\pi}{3}} \right] e^{-j\theta_1} \quad (13.84)$$

$$\frac{d\theta_1}{dt} = \omega_1 \quad (13.85)$$

Equation (13.85) represents the case when the fundamental frequency varies during the parameter estimation testing process.

For steady state, (13.79)–(13.81) are solved for torque T_e , stator current, I_1 , and input power P_1 [22].

$$T_e(S) = 3V^2 \cdot \frac{p_1}{\omega_1} \cdot \frac{R_r}{S(A_2 + B_2)} \quad (13.86)$$

$$I_1(S) = V \sqrt{\frac{C^2 + D^2}{A^2 + B^2}} \quad (13.87)$$

$$P(S) = 3V^2 \frac{AC - BD}{A^2 + B^2} \quad (13.88)$$

$$R_r = \frac{R_2 R_3 (R_2 + R_3) + (R_2 X_{l3}^2 + R_3 X_{l2}^2) S^2}{(R_2 + R_3)^2 + (X_{l2} + X_{l3})^2 S^2} \quad (13.89)$$

$$X_{dl} = \frac{(X_{l2} X_{l3} (X_{l2} + X_{l3}) S^2 + R_2^2 X_{l3} + R_3^2 X_{l2})}{(R_2 + R_3)^2 + (X_{l2} + X_{l3})^2 S^2} \quad (13.90)$$

$$C = 1 + X_n/X_m; \quad D = R_r/(S X_m); \quad E = 1 + X_{l1}/X_m \quad (13.91)$$

$$A = R_l C + \frac{R_r}{S} E; \quad B = X_{l1} + X_n \cdot E - R_l D \quad (13.92)$$

$$X_i = \omega_l L_i \quad (13.93)$$

The steady-state and transient models have the phase voltage RMS value V , its phase angle γ , p_l pole pairs, and frequency ω_l as inputs.

For steady state, the slip value is also a given. The stator resistance R_l , stator leakage L_{l1} , and magnetization inductances L_m are to be known a priori.

The stator current or torque or power under steady state is measured.

Using torque, or current or input power measured functions of slip (from S_1 to S_2), the rotor double-cage parameters that match the measured curve have to be found. This is only an example for steady state. In a similar way, the observation vector may be changed for transients into stator currents d–q components versus time: $I_{dl}(t)$, $I_{ql}(t)$. For a number of time steps $t_1, \dots, t_n$, the two currents are calculated from the measured stator current after transformation through Park transformation (13.84).

To summarize, the parameters vector $|X|$ can be estimated from a row $|Y|$ of test data at steady state or transients:

$$|X| = [R_2, R_3, L_{l2}, L_{l3}]^T \quad (13.94)$$

$$|Y| = [Y(S_1), Y(S_2), \dots, Y(S_n)] \quad (13.95)$$

$$Y(S_k) = T_e(S_k) \quad \text{or} \quad I_l(S_k) \quad \text{or} \quad P_l(S_k) \quad (13.96)$$

where $S_1, \dots, S_n$ are the n values of slip considered for steady state. For transients, the test data $I_{dl}(t)$ and $I_{ql}(t)$ are acquired at n time instants $t_1, \dots, t_n$. The stator current d–q components in synchronous coordinates constitute now the observation vector:

$$|Y| = [Y(t_1), Y(t_2), \dots, Y(t_n)] \quad (13.97)$$

$$Y(t_k) = I_{dl}(t_k) \quad \text{or} \quad I_{ql}(t_k) \quad (13.98)$$

In general, the problem to solve may be written as

$$Y = f(\zeta_k, X); \quad \xi_k = 1, \dots, n \quad (13.99)$$

with $\zeta = S_k$ for steady-state estimation and $\zeta = t_k$ for transients estimation. The parameter ξ_k is the error for the K th value of S_k (or t_k).

There are many ways to solve such a problem iteratively. Gauss–Newton method, when used after linearization, tends to run into almost singularity matrix situations. On the contrary, the step-wise

TABLE 13.4
Estimated Parameters for Various Observation Vectors

	Estimated Parameters [Ω]				
	R_2	R_3	X_{l2}	X_{l3}	X_{ll}
Values used to calculate the input (theoretical) characteristics and runs	3.1144	0.2717	3.0187	4.6098	3.1316
Values obtained after estimation from $T_e = f(S)$	3.1225	0.2717	3.032	4.6145	3.1261
Values obtained after estimation from $I_s = f(S)$	3.0221	0.2722	2.868	4.5552	3.1946
Values obtained after estimation from $i_a = f(S)$	3.0842	0.2718	2.9706	4.5931	3.1513
Values obtained after estimation from $li_s l = f(t)$	3.0525	0.2721	2.9099	4.5671	3.1762
Values obtained after estimation from $T_e = f(t)$	3.1114	0.2664	3.0693	4.5831	3.093

Source: After Ref. [22].

regression method, based on Gauss–Jordan pivot concept [21], seems to produce safely good results across the board. For details on the computation algorithm, see Ref. [22]. The method’s convergence should not depend on the width of change limits of parameters or on their assigned start values.

Typical results obtained in [22] for steady-state and dynamic (direct starting) operation modes are given in Table 13.4.

Sample results on convergence progress for $T_e(S)$ observation vector show fast convergence rates in Figure 13.25 [22].

The corresponding torque/slip curve fitting is, as expected, very good (Figure 13.26).

Similarly good agreement for the torque–time curve during starting transients is shown in Figure 13.27 when the $T_e(t)$ function is observed.

A few remarks are

- The curve fitting of the observed vectors over the entire explored zone is very good, and the rate of convergence is safe and rather independent of the extension of parameter existence range.
- The estimated parameter vectors (Table 13.4) have only up to 3% variations depending on the observation vector: torque or current. This is a strong indication that if the torque/slip is observed, the current/slip curve will also fit rather well.

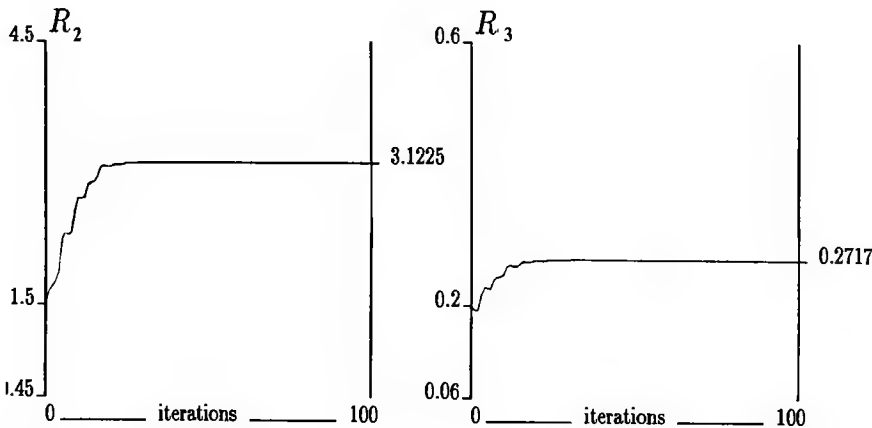


FIGURE 13.25 Convergence progress in rotor resistance estimation from $T_e(S)$. (After Ref. [22].)

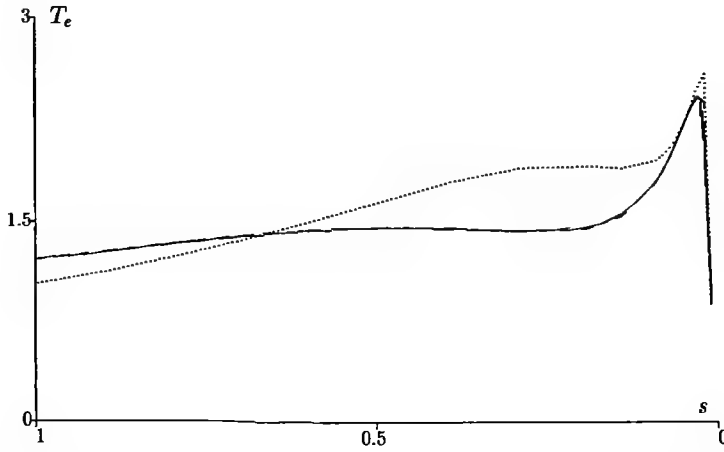


FIGURE 13.26 The static characteristic of electromagnetic torque $T_e = f(s)$: (after Ref. [22]) solid line (—), the theoretical characteristic of $T_e = f(s)$; dotted line (.....), the characteristic calculated for starting values of parameters; dashed line (-----), the characteristic calculated for final values of estimated parameters.

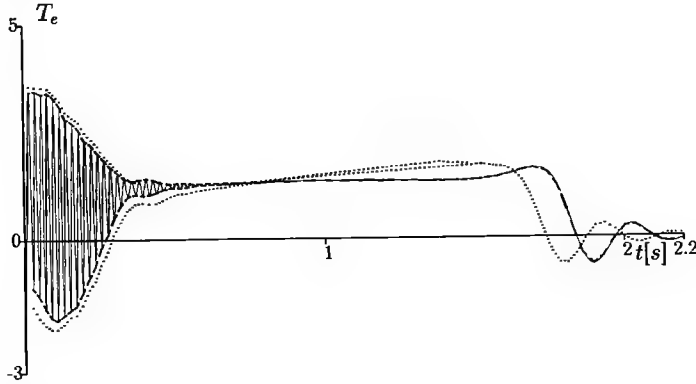


FIGURE 13.27 Runs of the electromagnetic torque T_e during the start-up: (after Ref. [22]) solid line (—), the theoretical run of $T_e = f(t)$; dotted line (.....), the run calculated for starting values of parameters; dashed line (-----), the run calculated for final values of estimated parameters.

- The phase current measurement is straightforward, but the torque measurement is not so. We may however calculate the torque offline from measured voltages and currents V_a , V_b , I_a , and I_b :

$$T_e = \frac{3}{2} p_l (\Psi_\alpha I_\beta - \Psi_\beta I_\alpha) \quad (13.100)$$

$$V_\alpha = V_a$$

$$V_\beta = \frac{1}{\sqrt{3}} (V_a + 2V_b) \quad (13.101)$$

$$I_\alpha = I_a$$

$$I_\beta = \frac{1}{\sqrt{3}} (I_a + 2I_b)$$

$$V_{\alpha} = \int (V_{\alpha} - R_1 I_{\alpha}) dt$$

$$V_{\beta} = \int (V_{\beta} - R_1 I_{\beta}) dt$$

The precision of torque estimates by (13.100) is good at rated frequency but smaller at low frequencies f_1 . The integrator should be implemented as a filter to cancel the offset. In (13.100), the reduction of the electromagnetic torque due to core loss, which is smaller than 1% of rated torque in general, is neglected. If core loss p_{core} is known from the no-load test, the torque of (13.100) can be reduced by ΔT_{ecore} :

$$\Delta T_{\text{ecore}} = p_{\text{core}} \frac{p_1}{\omega_1} \quad (13.102)$$

- The direct start is easy to perform, as only two voltages and two currents are to be acquired and the data are processed offline. It shows that this test may be the best way to estimate the IM parameters for a large spectrum of operation modes. Such a way is especially practical for medium–large-power IMs.
- Very large-power machines direct starting is rather slow as the relative inertia is large and the power grid voltage drops notably during the process. In such a case, the method of parameter estimation data may be simplified as shown in the following section where also the inertia is estimated.

13.4.6 LARGE IM INERTIA AND PARAMETERS FROM DIRECT STARTING ACCELERATION AND DECELERATION DATA

Testing large-power IMs takes time and requires notably human effort. Any attempt to reduce it while preserving the quality of the results should be considered.

Direct connection to the power grid, even if through a transformer at lower than rated voltage in a limited power manufacturer's laboratory, seems a practical way to do it. The essence of such a test consists of the fact that the acceleration would be slow enough to validate steady-state operation mode approximation all throughout the process. In large-power IMs, reducing the supply voltage to 60% would do it, in general. We assume that the core loss is proportional to voltage squared. By performing no-load tests at decreasing voltages, we can separate first the mechanical from iron losses by the standard loss segregation method. The mechanical losses vary with speed. To begin, we may suppose a linear dependence:

$$p_{\text{mec}} \approx p_{\text{mec0}} \cdot \left(\frac{n}{f_1/p} \right)^2$$

$$p_{\text{iron}} = p_{\text{iron0}} \cdot \left(\frac{V}{V_0} \right)^2 \quad (13.103)$$

During the no-load acceleration test, the voltage and current of two phases are acquired together with the speed measurement. Based on this, the average power is calculated. To simplify the process of active power calculation, synchronous coordinates can be used:

$$\begin{vmatrix} V_d \\ V_q \end{vmatrix} = \frac{2}{3} \begin{vmatrix} \cos(-\omega_1 t) & \cos\left(-\omega_1 t + \frac{2\pi}{3}\right) & \cos\left(-\omega_1 t - \frac{2\pi}{3}\right) \\ \sin(-\omega_1 t) & \sin\left(-\omega_1 t + \frac{2\pi}{3}\right) & \sin\left(-\omega_1 t - \frac{2\pi}{3}\right) \end{vmatrix} \cdot \begin{vmatrix} V_a \\ V_b \\ -(V_a + V_b) \end{vmatrix} \quad (13.104)$$

The same formula is valid for the currents I_d and I_q . Finally, the instantaneous active power P_i is

$$P_i = \frac{3}{2} (V_d I_d + V_q I_q) \quad (13.105)$$

As V_d , V_q , I_d , and I_q , for constant frequency ω_1 , are essentially D.C. variables, the power signal P_i is rather clean and no averaging is required.

We might define now the kinetic useful power P_K as

$$\begin{aligned} P_K(n) &= P_{elm}(1-S) + p_{mec} \\ P_{elm} &= P_i - 3R_s I_1^2 - p_{iron} \end{aligned} \quad (13.106)$$

As all terms in (13.106) are known for each value of speed, the kinetic useful power $P_K(n)$, dependent on speed, may be found. The time integral of $P_K(n)$ gives the kinetic energy of the rotor E_c :

$$E_c = \int_0^t P_K(n) dt = \frac{J}{2} 4\pi^2 n_f^2 \quad (13.107)$$

where n_f is the final speed considered.

We may terminate the integration at any time (or speed) during acceleration.

Alternatively,

$$P_K = J 4\pi^2 n \frac{dn}{dt} \quad (13.108)$$

At every moment during the acceleration process, we may calculate P_K , measure speed n , and build a filter to determine dn/dt .

Consequently, the inertia J may be determined from either (13.107) or (13.108) for many values of speed (time) and take an average.

However, in (13.107) the derivative of n is avoided and an integral is performed. and thus, the results are much smoother.

Sample measurements results on a 7500kW, 6000 V, 1490rpm, 50Hz, IM with $R_1 = 0.0173 \Omega$, $P_{mec0} = 44.643 \text{ kW}$ and $P_{iron} = 78.628 \text{ kW}$ are shown in Figure 13.28 [23].

The moment of inertia, calculated during the acceleration process, by (13.108), is shown in Figure 13.29.

Even with dn/dt calculated, J is rather smooth until we get close to the settling speed zone, which has to be eliminated when the average J is calculated.

The torque can also be calculated from two different expressions (Figure 13.30):

$$T = 2\pi J \frac{dn}{dt} \quad (13.109)$$

$$T_e = \frac{P_K}{2\pi n} \quad (13.110)$$

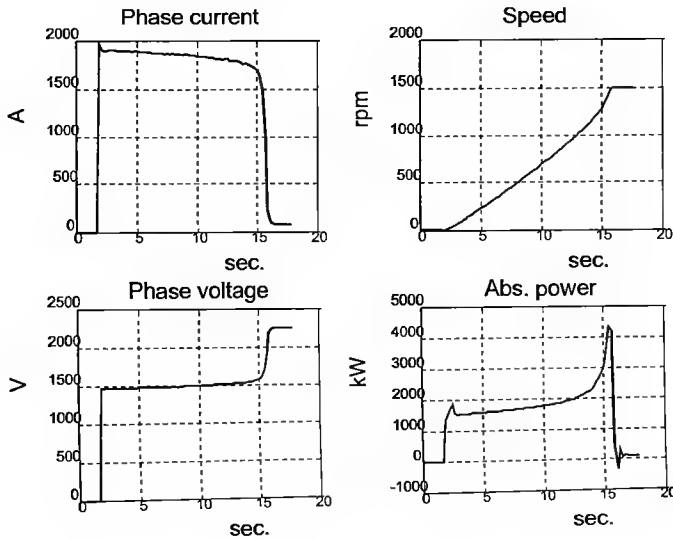


FIGURE 13.28 Current, phase, voltage, power and speed during direct starting of a 7500kW IM. (After Ref. [23].)

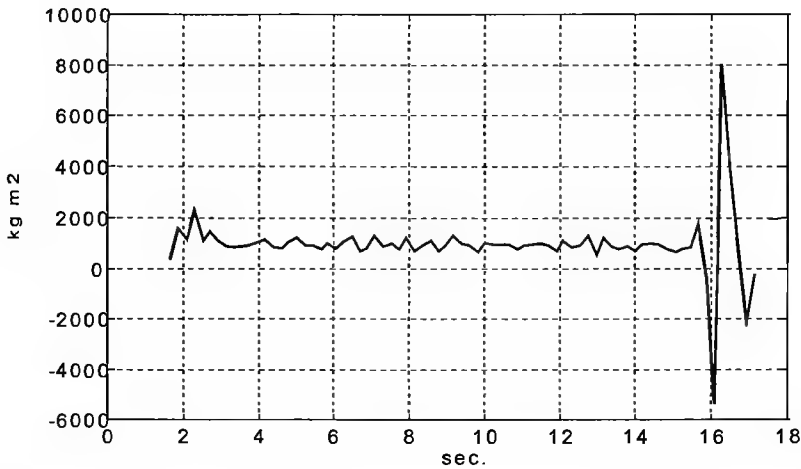


FIGURE 13.29 Inertia calculated during the acceleration process using the speed derivative (13.108). (After Ref. [23].)

Again, Equation (13.110), with $P_k(n)$ from (13.106) which avoids dn/dt , produces a smoother torque/time curve.

The torque may be represented as a function of speed also, as speed has been acquired.

The torque shown in Figure 13.30 is scaled up to 6kV line voltage (star connection), assuming that the torque is proportional to voltage squared.

The testing may add a free deceleration test from which, with inertia known from the above procedure, the more exact mechanical loss dependence on speed $p_{mec}(n)$ can be determined:

$$p_{mec}(n) = -J4\pi^2 \frac{dn}{dt} n \quad (13.111)$$

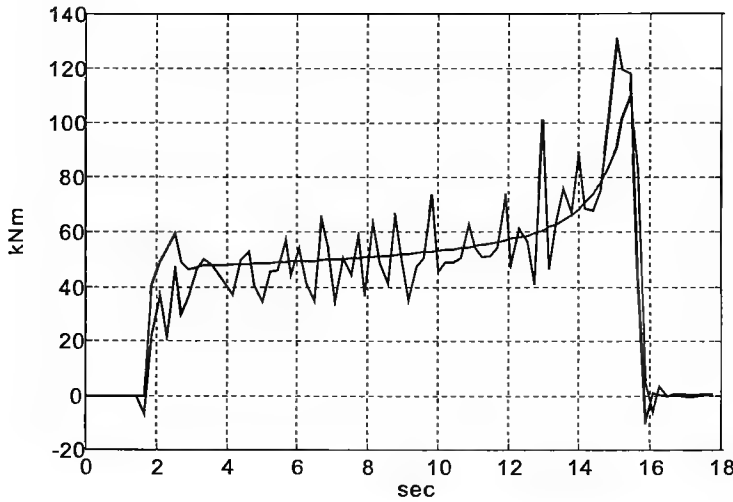


FIGURE 13.30 Torque versus time during acceleration from (13.109) and (13.110). (After Ref. [23].)

Then, $p_{mec}(n)$ from (13.111) may be used in the acceleration tests processing to get a better inertia value. The process proved to converge quickly. Finally, an exponential regression of mechanical losses is calculated:

$$p_{mec} = \alpha n^{2.4}; \quad \alpha = 0.001533 \quad (13.112)$$

The stator voltage attenuation during free deceleration has also been acquired.

Sample deceleration test results are shown in Figure 13.31a–c.

The mechanical losses, determined from the deceleration tests, have been consistently smaller (i.e. for quite a few tested machines: 800kW, 2200kW, 2800kW, 7500kW) than those obtained from loss segregation method.

In our case at no-load speed $p_{mec} = 64.631$ kW from the free deceleration tests, that is 14kW less than from the loss segregation method.

This difference may be attributed to additional (stray losses).

To calculate the stray load losses, we can use the following formula:

$$P_{stray}(I) = \left[(P_{mec})_{l.s.m.} - (P_{mec})_{f.d.m.} \right] \cdot \left(\frac{I}{I_0} \right)^2 \quad (13.113)$$

l.s.m. – loss segregation method

f.d.m. – free deceleration method

where I is the stator current and I_0 the no-load current.

To complete loss segregation and secure performance assessment, all IM parameters have to be determined. We may accomplish this goal here avoiding the short-circuit test due to both time (work) needed and also the fact that large IMs tend to have strong rotor skin effect at standstill and thus a double-cage rotor circuit has to be adopted.

The IM parameters may also be identified from the acceleration and free deceleration test data alone through a regression method, while even the rotor speed $n(t)$ may be estimated (no speed sensor) [23].

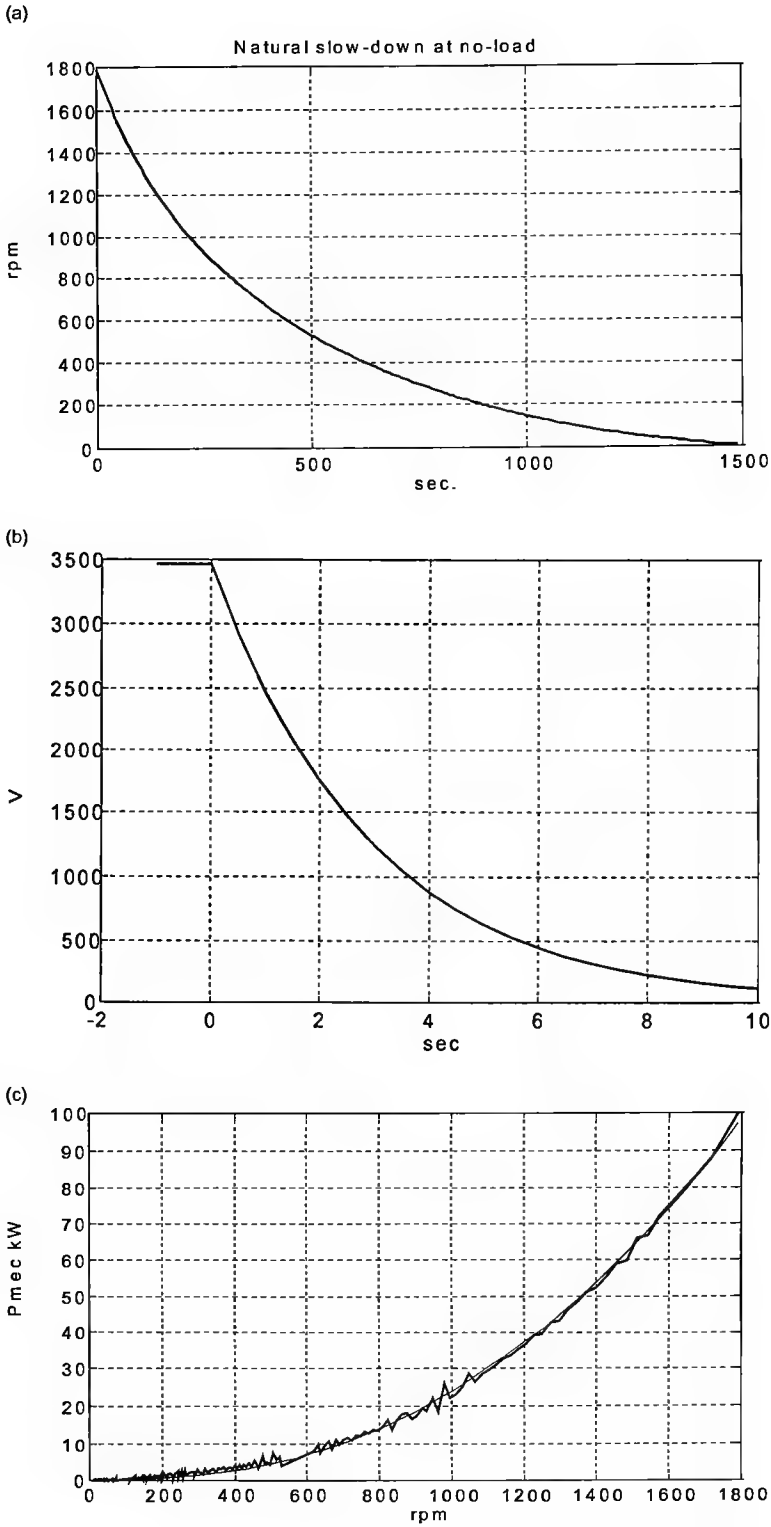


FIGURE 13.31 Free deceleration tests [23]: (a) speed versus time, (b) stator voltage amplitude versus time, and (c) mechanical losses versus speed.

13.5 NOISE AND VIBRATION MEASUREMENTS: FROM NO LOAD TO LOAD

Noise in IM has electromagnetic and mechanical origins. The noise level accepted depends on the environment in which the IM works. At the present time, standards (ISO1680) prescribe noise limits for IMs, based on sound power levels measured at no-load conditions, if the noise level does not vary with load. It is not yet clear how to set the sufficient conditions for no-load noise tests.

The load machine produces additional noise, and thus, it is simpler to retort to IM no-load noise tests only.

If the electromagnetic noise is relatively large, it is very likely that under load, it will increase further [22]. Also, when PWM converter fed, the IM noise will change from no-load to load conditions. Skipping the basics of sound theory [24] in electrical machines, we will synthesize here ways to measure the noise under load in IMs.

13.5.1 WHEN ON-LOAD NOISE TESTS ARE NECESSARY?

Noise tests with the machine on no load (free at shaft) are much easier to perform, eventually in an acoustic chamber (anechoic room for free field or reverberant room for diffuse field). So it is natural to take the pains and perform noise measurements on-load only when the difference in noise level between the two situations is likely to be notable.

An answer to this question may be that the on-load noise measurements are required when the electromagnetic noise at no load is relatively important. Under load, space harmonics fields accentuate and are likely to produce increased noise, aside from the noise related to torque pulsations (parasitic torques) and uncompensated radial forces.

To separate the no-load electromagnetic noise sound power level $L_{W,em}$, three tests on no load are required [25,26]:

- The no-load sound power L_{W_0} is measured according to standards.
- After taking off the ventilator, the sound power level L_{Wf} is measured. Subtracting L_{Wf} from L_{W_0} , we determine the ventilator system sound power level $L_{W,vent}$:

$$L_{W,vent} = 10 \log(10^{0.1L_{W_0}} - 10^{0.1L_{Wf}}) \quad (13.114)$$

- Without the ventilator, the no-load noise measurements are carried out at ever smaller voltage level before the speed decreases. By extrapolating for zero voltage, the mechanical sound power level $L_{W,mec}$ is obtained. Consequently, the electromagnetic sound level at no load $L_{W,em}$ is

$$L_{W,em} = 10 \log(10^{0.1L_{W_0}} - 10^{0.1L_{W,mec}}) \quad (13.115)$$

Now if the electromagnetic sound levels $L_{W,em}$ are more than 8 dB smaller than the total mechanical sound power, the noise measurements on-load are not necessary.

$$L_{W,em} < -8\text{dB} + 10 \log(10^{0.1L_{W,mec}} + 10^{0.1L_{W,vent}}) \quad (13.116)$$

13.5.2 HOW TO MEASURE THE NOISE ON-LOAD

There are a few conditions to meet for noise tests:

- The sound pressure has to be measured by microphones placed in the acoustic far field (about 1m from the motor).

- The background sound pressure level radiated by any sound source has to be 10dB lower than the sound pressure generated by the motor itself.
- Every structure-based vibration has to be eliminated.

Acoustical chambers fulfil these conditions, but large machine shops, especially in the after hours, are also suitable for noise measurements.

Under on-load conditions, the loading machine and the mechanical transmission (clutch) produce additional noise. As the latter is located close to the IM, its contribution to the resultant noise power level is not easy to segregate.

Separation of the loading machine noise may be performed through putting it outside the acoustic chamber which contains the tested motor.

Also, an acoustic capsule may be built around the coupling plus the load machine with the tested motor placed in the machine shop.

Both solutions are costly and hardly practical unless a special “silent” motor is to be tested thoroughly.

Thus, in situ noise measurements, in large machine shops, after hours, appear as the practical solution for noise measurements on-load.

The sound pressure or power is measured in the far field 1m from the motor, in general. To reduce the influence of the loading machine noise, the measurements are taken at near field 0.2m from the motor.

However, in this case, near-field errors occur mainly due to motor vibration modes.

To eliminate the near-field errors, comparative measurements in the same points are made.

For example, in a near-field point (0.2m from the motor), the sound pressure is measured on no-load and with the load coupled to obtain $L_{W,on}$ and $L_{W,n}$. The difference between the two represents the load contribution to noise and is not affected by the near field error:

$$\Delta L_n = L_{W,n} - L_{W,on} \quad (13.117)$$

This method of near-field measurements is most adequate for tests at the working place of the IMs.

Noise, in the far or near field, may be measured directly by sound pressure microphones (Figure 13.32) or calculated based on vibration speed measurements.

The microphone method measures sound pressure and produces very good results if the microphone placement avoids the aggregation of load machine noise contributions. Its placement in zone 2 (Figure 13.32) at $d = 1$ m from motor, though the best available solution, may still lead to about 3 dB increase in noise as loading machine contribution. This may be considered a systematic error at worst.

The artificial loading by PWM dual frequency methods avoids the presence of the load machine, and it may constitute a viable method to verify IM noise measurements under load.

The noise level may be also calculated from vibration mode speeds measurement. Measurements of IM frame vibration mode speeds are made on its frame. This is the traditional method and is described in detail in the literature [24.25,27–29].

Experimental results reported in [25] lead to conclusions such as

- The sound power level may be determined directly by sound intensity (pressure) measurement in the far field (1 m away from IM) through microphones (Figure 13.32).
- The sound power level of the IM under load may be determined indirectly via measurements by microphones, on no-load and on-load, made in the near field (0.2m) of the IM (Equations (13.115)–(13.116)).
- Finally, the sound power may be calculated from noise or vibration measurements depending on which the mechanical or electromagnetic noise dominates.

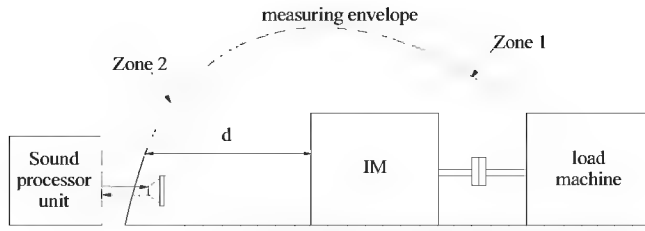


FIGURE 13.32 Sound intensity measurement arrangements.

- All these procedures seem worth further consideration, as their results are close. In [25], the far-field microphone method departed from the other two by 0.80 dB (from 78.32 to 79.12 dB on-load).
- A ΔL_n of 6.84 dB increase due to load was measured for a 5 kW IM in [25].

For details on noise and vibration essentials, see Ref. [29].

In the power electronics era, IM noise theory, testing and reduction are expected to prompt new R&D efforts.

13.6 RECENT TRENDS IN IM TESTING

In view of recent developments in IM optimal design, their use fed by PWM converter for variable speed drives, supply voltage asymmetries or time harmonics in line-start applications, new modelling methods, a few trends in IM testing may be deciphered:

- Direct loading tests of PWM converter-fed IM to determine motor losses and their comparison with laborious FEA [30].
- In situ efficiency estimation of IM under unbalanced voltages [31].
- Space vector versus FEM models to identify torsional vibration of cage IMs [32] and IM acoustic spectra.
- Parameter estimation of multiphase IMs [33].
- Offline identification of IM parameters (with core loss estimation) from stator current locus [34].
- Self-commissioning cycle to estimate IM parameters in PWM inverter-fed variable speed sensorless drives [35].
- Parameter and performance, estimation in doubly fed induction motor/generators (DFIM/G) for limited speed variable/speed applications.
- Dual stator winding cage (or loop cage) brushless DFIM(G) testing for parameters and performance in limited variable speed applications [36].
- Internal and external fault-detection measurement methods for IMs beyond stator current signature analysis [36–38].
- The list may go on, and we end up with a section on the testing of cage-PM rotor line-start induction motors as they tend to be used on superpremium efficiency applications and on linear induction motor (LIM) testing.

13.7 CAGE-PM ROTOR LINE-START IM TESTING

The cage-PM rotor IMs are now considered to be used in superpremium efficiency line-start applications for energy savings.

In addition, the PM-assisted cage rotor IM starts as a regular induction motor, but then, at a certain speed (slip S_s), it self-synchronizes and then operates as a PM synchronous motor, with notably increased efficiency (by as much as up to 4%–6%) in comparison with an IM in the same frame.

Also the power factor is improved (especially for $2p \geq 4$ poles). However, these merits come accompanied by a few challenges:

- The starting/rated current ratio $I_{\text{start}}/I_{\text{rated}} \approx 6.3\text{--}6.8$ (p.u.).
- High enough starting torque to overcome the additional braking torque of PM motion induced currents in the stator during asynchronous operation.
- Enough torque at slip S_s to secure successful self-synchronization which implies strong magnets unless a large magnetic anisotropy in the rotor (multiple flux barriers) produces a large enough reluctance synchronous torque.
- PM demagnetization during starting due to not enough shielding from the rotor cage currents and during synchronous operation at overloads.
- Additional notable initial costs when high energy PMs are used; the payback time for these extra costs has to be $< 2.5\text{--}3$ years to justify the effort; applications with more than 8 hours/day full load operation may qualify. Also, lower speed ($2p > 4$) applications seem adequate as IMs perform poorly in such topologies.

In view of the above, the testing of such motors becomes a critical issue. Basically, a combination of IM tests with some tests from inverter-fed PMSM tests is used for the implementation.

So far a coherent set of tests to characterize such machines fully is still yet due, not to mention a dedicated standard.

Consequently, we end up here by a few mere educated guesses on tests to fully characterize cage-PM rotor IMs:

- Slowly free accelerating test followed by free deceleration test.
- Artificial loading (as presented for IMs) with special PWM converter to oscillate the phase of the output voltage vector in order to swap quickly from motoring to generating [39]; to determine efficiency and perform the heating test.
- Back-to-back loading of twin machines: one connected to the grid and motoring and the other operating as a generator supplying a resistive load, to validate efficiency and the heating-cooling; the no-load voltage as generator will illustrate the PM flux linkage waveform and the state of the magnets before and after critical conditions (during starting and under heavy overload (if needed by the application)).

13.8 LINEAR INDUCTION MOTOR (LIM) TESTING

LIM testing is constraint by the fact that no mechanical transmission is placed between it and the linear motion load. Also the primary (inductor) and secondary have different lengths; in general, the secondary (a conductor sheet on iron or cage type) is longer than the primary. Finally, in most applications, PWM converter variable speed (position) control is required. Attempts to develop recommendations for a standard for testing of LIMs have been made starting in 1990, but it seems that no widely accepted version is available yet.

In view of this situation here, a few suggestions on coherent LIM testing and performance are made:

- Stall testing at quite a few frequencies ($f_1 = f_2$ slip frequency) with one phase in series with the other two phases in parallel up to rated current to calculate the LIM circuit parameters R_{sc} and L_{sc} :

$$R_{\text{sc}} = R_1 + R'_2(f_2); \quad L_{\text{sc}} \approx L_{11} + L'_{21}(f_2) \quad (13.118)$$

- To estimate R_1 and L_{11} approximately, the phases are connected in series and fed in A.C. at a few frequencies with the homopolar impedance measured: $R_0(f_1)$, $L_{10}(f_1)$; approximately $L_{10} \approx L_{11}$.

- The main problem is the estimation of magnetization inductance L_m . Same stall tests at low frequencies provide enough information to calculate $L_m(i_m)$ with $R'_2(f_2)$, $L_{11}(f_2)$ already determined above.
- Finally, the dynamic longitudinal effects influence on R'_2 and L_m can be estimated only during running tests (Chapter 12).
- Running tests may be performed with the actual LIM in a back-to-back configuration with two twin LIMs; one will operate as a motor and the other – with parallel terminal capacitor – will work in the generator mode on a resistive loading. From such a test, the dynamic longitudinal end effects on the circuit model (on R'_2 and L_m) as a function of speed (slip) and current level as well as efficiency may be obtained.
- However, to reduce the costs, when a new LIM prototype is design, a dual arch stator (primary) rotary rotor may be used as a surrogate back-to-back test rig. The two arch stator may be placed diametrically to cancel radial forces on the rotor, and there should be large free air between the two tangentially such that the dynamic longitudinal end effect rotor (secondary) currents die out from the exit of one stator arch to the entrance under the next one.

Again, one stator winding will be fed as motor and the other as generator (with capacitor plus resistive load or through an inverter drive).

With the second stator winding having various capacitors at terminals and no resistive load as generator, the variation of magnetization inductance can be calculated as the no-load voltage V_0 :

$$V_0 \approx \omega_1 (L_{11} + L_m(i_0)) \approx \frac{i_0}{\omega_1 C} \quad (13.119)$$

- Artificial loading may be exercised on the arch–stator–rotor counterpart of LIM as for the rotary IM. In this case, only one stator arch is required, but there is an uncompensated radial force on the stator (rotor). However, this uncompensated radial force if measured (on the arch stator) will give an approximate value of the normal force of LIM for various values of stator current, slip frequency (Sf), and speed U .

13.9 SUMMARY

- IM testing refers essentially to loss segregation, direct and indirect load testing, parameter estimation, and noise and vibration measurements.
- Loss segregation is performed to verify design calculations and avoid full load testing for efficiency calculation.
- The no-load and standstill tests are standard in segregating the mechanical losses, fundamental core losses, and winding losses.
- The key absence in loss segregation is however the so-called stray load losses.
- Stray load losses may be defined by identifying their origins: rotor and stator surface and teeth flux pulsation space harmonics core losses, the rotor cage non-fundamental losses. The latter are caused by stator mmf space harmonics, augmented by airgap permeance harmonics, and/or inter-bar rotor core losses. Separating these components in measurements is hardly practical.
- In face of such a complex problem, the stray load losses have been defined as the difference between the total losses calculated from load tests and the ones obtained from no-load and standstill (loss segregation) tests.
- Although many tests to measure directly the stray load losses have been proposed, so far only two seem to hold.
- In reverse motion test, the IM fed at low voltage is driven at rated speed but in reverse direction to the stator voltage system. The slip $S \approx 2$. Although the loss equivalence with

the rated load operation is far from complete, the elimination of rotor cage fundamental loss from calculations “makes” the method acceptable in industry.

- In the no-load test extension over rated voltage – performed so far only on small IMs – the power input minus stator winding losses deviate from its proportionality to voltage squared. This difference is attributed to additional “stray load” losses considered then proportional to current squared.
- The stator resistance is traditionally D.C. measured. However, for large IMs, A.C. measurements would be needed. The A.C. test at low voltage with the rotor outside the machine is considered to be pertinent for determining both the stator resistance and leakage inductance.
- The no-load and standstill tests may be performed by using PWM converter supplies, especially for IM destined for variable speed drives. Additional losses, in comparison with sinusoidal voltage supply, occur at no-load test, not at standstill test, as the switching frequency in the converter is a few kilohertz.
- If the distribution of the losses is known, in the moment their source is eliminated by turning off the stator winding, the temperature gradient is proportional to those losses. Consequently, measuring temperature versus time leads to loss value. Unfortunately, loss distribution is not easy to find unless FEM is used, while temperature sensors, placed at sensitive points, which represent an intrusion, may be accepted only for prototyping.
- On the contrary, the calorimetric method – in the dual chamber version – leads to the direct determination of overall losses, with an error of <5% generally.
- Load tests are performed to determine efficiency and temperature rise for various loads.
- Efficiency may be calculated based on loss segregation methods or by direct (or indirect: artificial) loading tests. In the latter case, both the input and output powers are measured.
- Standard methods to determine efficiency are amply presented in the international standards IEEE–112 (USA), IEC–34–2 (Europe), and JEC (Japan). The essential difference between these three standards lies in the treatment of non-fundamental (or stray load) losses. JEC used to consider the stray load losses as zero, whereas IEC–34–2 took them as a constant until very recently: 0.5% of rated power (a new European standard, closer to IEEE–112, is pending or already released). IEEE–112, through its many alternative methods, determines them as the difference between overall losses in direct load tests and no-load losses at rated voltage plus standstill losses at rated current. Alternatively, the latter sets constant values for stray load losses, all higher than 0.5% – from 1.8% below 90kW to 0.9% above 1800kW. So, in fact, the same IM tested according to these representative standards would have different labelled efficiency. Or, conversely, an IM with same label efficiency and power will produce more power if the efficiency was measured according to IEEE–112 and least power for JEC-like efficiency definition. The differences may run as high as 2%.
- IEEE–112B by its provisions seems much closer to reality.
- The direct torque measurement using a brushless torque-meter in direct load tests may be avoided either by a calibrated D.C. generator loading machine or by estimating the torque of an inverter-fed IM machine load. Alternatively, the tested IM may be run both as a motor and generator at equal slip values. Rather simple calculations allow for stray load loss and efficiency computation, based on input power, current, voltage, and slip precise measurements.
- Avoiding altogether the burden of coupling a second machine at the tested IM shaft led to artificial (indirect) loading. The two-frequency method introduced in 1921 has been used only for temperature rise calculations as loss equivalence with direct load tests was not done equitably.
- The use of PWM converter to mix frequencies and provide indirect (artificial) loading apparently changes the picture. Equivalence of losses may be now pursued elegantly (though not completely), and the whole process may be mechanized through PC control.

Good agreement in efficiency with the direct methods has been reported. Is this the way of the future? More experience with this method will only tell.

- A rather conventional indirect method, applied by a leading manufacturer for 40 years, is the forward short-circuit approach. In essence, the unsupplied IM is first driven at rated speed by a 10% rating IM of the same number of poles. Then, another 10% rating motor drives a synchronous generator to produce about 80%–85% rated frequency of the tested IM. Finally, the A.C. generator is excited, and its terminals are connected to the tested motor. The voltage is raised to reach the desired current. The tested IM works at high slip as a generator and at rated speed, and thus, the active power output is low: 10% of rated power. Despite the fact that the distribution of losses differs from that in direct load tests, with more losses on rotor and less on stator and skin rotor effect presence, stator temperature rise in good agreement with direct load tests has been reported for large IMs. The introduction of bi-directional power electronics to replace the second drive motor plus A.C. generator might prove a valuable way to improve on this genuine and rather practical method, for large-power IMs.
- Parameter estimation is essential for IM precise modelling over a wide spectrum of operation modes.
- Single and multiple rotor loop circuit models are used for modelling.
- Basically, only no-load and short-circuit tests are standardized for parameter estimation.
- No-load tests provide reliable results on the core loss resistance and magnetization inductance dependence on magnetization current.
- Short-circuit tests performed at standstill and at rated frequency contain too much skin effect influence on rotor resistance and leakage inductance, which precludes their usage for calculating on-load IM performance.
- Even for starting torque and current estimation at rated voltage, they are not proper as they are performed up to rated current and thus the leakage flux path saturation is not considered. Large errors at full voltage – up to 60%–70% – may be expected in starting torque in extreme cases.
- Two-frequency short-circuit tests at rated current or so – one at rated frequency and the other at 5% rated frequency – could lead to a dual cage rotor model good for both large and low slip values, provided the current is not larger than 150%–200% of rated current; most PWM converter-fed general variable-speed IM drives qualify for these conditions.
- The SSFR method is an improvement on the two-frequency method but, being performed at low current and many frequencies, excludes leakage saturation effect as well and is time prohibitive. Its validity is restricted to small deviation transients and stability analysis in large IMs.
- Even catalogue data-based methods have been proposed to match (by a single or dual virtual cage rotor model) both the starting and rated torque and currents. However, in between, around breakdown torque, large errors persist [40].
- The general regression methods, based on steady-state or transients measurements, for wide speed range, from standstill to no-load speed, have been successfully used to estimate the parameters of an equivalent dual cage rotor model according to the criterion of least squared error. Thus, a wide spectrum of operation modes can be handled by a single but adequate model. A slow acceleration start and a direct (fast) one are required to cover large ranges of current and rotor slip frequency. The slow start may be obtained by mounting an inertia disk on the shaft. However, direct fast start transients tend to produce, by the general regression method, about the same parameters as the steady-state test and thus may be preferred.
- A simplified version of the naturally slow acceleration/deceleration (quasi-steady-state electromagnetic-wise) method for very large IMs has been successfully used to separate the losses, calculate the inertia and dual cage rotor model parameters that fit the whole slip range, even without a speed feedback.

- Noise and vibration tests are environmental constraints. Only no-load noise tests are, in general, standardized, but sometimes, on-load noise is larger by 6–8 dB or more.
- When and how on-load in situ noise tests are required (done) is discussed in some detail. For PWM converter-fed IMs, variable speed noise tests are necessary [26].
- We did not exhaust, rather prioritize, the subject of IM testing. Many other methods have been proposed and will continue to be introduced [41,42] as the IM experimental investigation may see a new boost by making full use of PWM converter variable voltage and frequency supplies [43,44].
- An interesting example is the revival of the method of driving the IM on no load through the synchronous speed and detecting the power input jump at that speed. This power jump corresponds basically to hysteresis losses. The test is made easier to do with a PWM converter supply with a precise speed control loop.
- New identification methods of IM nonlinear model keep surfacing [45].
- New methods, based on single-phase operation, to assess stray load losses [46] and to identify three-phase IM parameters from catalogue data, based on regressive algorithms and thorough experiments, are also proposed.
- Special IMs, such as high-power high-speed IMs (6MW, 15000rpm) with active magnetic bearings (AMB) require special testing sequences [47].
- Locked rotor tests in line-start IMs (or PM-IMs) are still in debate in terms of maximum allowable test time at full voltage [48] especially with higher efficiency motors (IE3–5) which are known to have in general higher starting currents [49].
- Direct start-up test refinements in impedance-based estimation of deep-bar parameters have been recently proposed [50].

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14 Single-Phase IM Testing

14.1 INTRODUCTION

The elliptic magnetic field in the airgap of single-phase IMs in the presence of space mmf harmonics, magnetic saturation, rotor skin effect, and interbar rotor currents makes a complete theoretical modelling a formidable task.

In previous chapters, we did touch all these subjects through basically refined analytical approaches. Ideally, a 3D finite element modelling (3D-FEM), with eddy currents computation and circuit model coupling, should be used to tackle simultaneously all the above phenomena.

However, such a task still requires a prohibitive amount of programming and computation effort.

As engineering implies intelligent compromises between results and costs, especially for single-phase IMs, characterized by low powers, experimental investigation is highly recommended. But, again, it is our tendency to make tests under particular operation modes such as locked rotor (short circuit) and no-load tests to segregate different kinds of losses and then use them to calculate on-load performance.

Finally, on-load tests are used to check the loss segregation approach.

For three-phase IMs, losses from segregation methods and direct on-load tests are averaged to produce safe practical values of stray load losses and efficiency (IEEE Standard 112B).

The presence of rotor currents even at zero slip ($S = 0$) – due to the backward field component – in single-phase induction machine (IM) makes the segregation of losses and equivalent circuit parameter computation rather difficult. Among many potential tests to determine single-phase IM parameters and loss segregation, two of them have gained rather large acceptance.

One is based on single-phase supplying of either main or auxiliary winding of the single-phase IM at zero speed ($S = 1$) and on no load. The motor may be started as capacitor or split-phase motor, and then, the auxiliary phase is turned off with the motor free at shaft [1,2].

The second method is based on the principle of supplying the single-phase IM from a symmetrical voltage supply. The auxiliary winding voltage V_a is 90° ahead of the main winding voltage and $V_a = V_m/a$, such that the current I_a is $I_a = I_m/a$, where a is the ratio between main and auxiliary winding effective turns [3].

This means in fact that pure forward travelling field conditions are provided. The 90° shifted voltage source is obtained with two single-phase transformers with modified Scott connection and a Variac, or by power electronics.

Again, short-circuit (zero speed) – at low voltage – and no-load testing is exercised. Moreover, ideal no-load operation ($S = 0$) is performed by using a drive at synchronous speed $n_1 = f_1/p_1$.

Then, both segregation methods are compared with full (direct) load testing [3]. For the case studies considered both methods claim superior results.

The two methods have a few common attributes:

- They ignore the stray losses (or take them as additional core losses already present under no-load tests).
- They consider the magnetization inductance as constant from short-circuit ($S = 1$) to no-load and load conditions.
- They ignore the space mmf harmonics and, in general, consider the current in the machine as sinusoidal in time.
- They neglect the skin effect in the rotor cage. Though the value of the rotor slot depth is not likely to go over $15 \cdot 10^{-3}$ m (the power per unit is limited to a few kW), the backward

field produces a rotor current component whose frequency $f_{2b} = f_1(2-S)$ varies from f_1 to $2f_1$ when the motor accelerates from zero to rated speed. The penetration depth of electromagnetic field in aluminium is about $12 \cdot 10^{-3}$ m at 50 Hz and $(12/\sqrt{2}) \times 10^{-3}$ m at 100 Hz. The symmetrical voltage method [3] does not have to deal with the backward field, and thus, the rotor current has a single frequency ($f_{2f} = Sf_1$). Consequently, the skin effect may be neglected. The trouble is that during normal variable load operation, the backward field exists and thus the rotor skin effect is present. In the single-phase voltage method [1,2], the backward field is present even under no load, and thus, it may be claimed that somehow the skin effect is accounted for. It is true that, as for both methods, the short-circuit tests ($S = 1$) are made at rated frequency, the rotor resistance thus determined already contains a substantial skin effect. This may explain why both methods give results which are not far away from full-load tests.

- Avoiding full-load tests, both methods measure, under no-load tests, smaller interbar current losses in the rotor than they would be under load. Although there are methods to reduce the interbar currents, there are cases when they are reported to be important, especially with skewed rotors. However, in this case, the rotor surface core additional losses are reduced, and thus, some compensation of errors may occur to yield good overall loss values.
- Due to the applied simplifications, neither of the methods is to be used to calculate the torque/speed curve of the single-phase IM beyond the rated slip ($S > S_n$), especially for tapped winding or split-phase IMs which tend to have a marked third-order mmf space harmonic that causes a visible deep in the torque/speed curve around 33% of ideal no-load speed ($S = 0$).

As the symmetrical voltage method of loss segregation and parameter computation are quite similar to that used for three-phase IM testing (Chapter 13), we will concentrate here on Veinott's method [1] as it sheds more light on single-phase IM peculiarities, given by the presence of backward travelling field. The presentation here will try to retain the essentials of Veinott's method while make it show a simple form, for the potential user.

14.2 LOSS SEGREGATION IN SPLIT-PHASE AND CAPACITOR-START IMS

The split-phase and capacitor-start IMs start with the auxiliary winding on but end up operating only with the main stator winding connected to the power grid.

It seems practical to start with this case by exploring the short circuit (zero speed) and no-load operation modes with the main winding only on, for parameter computation and loss segregation.

We will make use of the cross field model (see Chapter 14, Volume 1, Figure 14.21), although the travelling field (+, -) model would give similar results, since constant motor parameters are considered.

For the zero speed test ($S = 1$), $Z_f = Z_b$ and thus

$$R_{sm} + R_{rm} \approx \frac{P_{sc}}{I_{msc}^2}; \quad I_a = 0 \quad (14.1)$$

$$|Z_{sc}| = \frac{V_s}{I_{msc}} \quad (14.2)$$

$$X_{sm} + X_{rm} \approx \sqrt{\left(\frac{V_s}{I_{msc}}\right)^2 - (R_{sm} + R_{rm})^2} \quad (14.3)$$

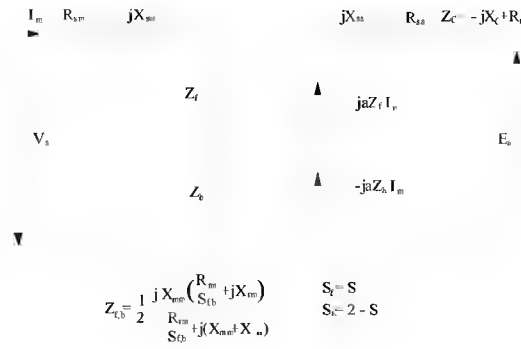


FIGURE 14.1 The cross-field single-phase IM with auxiliary winding open ($I_a = 0$).

With R_{sm} D.C.-measured and temperature-corrected, and $X_{sm} \approx X_{rm}$ for first iteration, the values of R_{sm} , $X_{sm} = X_{rm}$, and R_{rm} are determined. For the no-load test (still $I_a = 0$), we may measure the slip value S_0 or we may not. If we do, we make use of it. If not, $S_0 \approx 0$.

Making use of the equivalent circuit of Figure 14.1, for $S = S_0 = 0$ and with $I_{m0}(A)$, $V_s(V)$, $P_m(W)$ and E_a measured, we have the following mathematical relations:

$$Z_f \approx \frac{1}{2} jX_{mm} \quad (14.4)$$

$$Z_b \approx \frac{1}{2} \left[\frac{R_{rm}}{2} + jX_{rm} \right] \quad (14.5)$$

$$\text{From } \frac{E_a}{a} \approx \left| (Z_f - Z_b) \right| I_{m0} \quad (14.6)$$

with R_{rm} and X_{sm} already determined from the short-circuit test and I_{m0} and E_a measured, we need the value of a in Equation 14.6, to determine the magnetization reactance X_{mm} :

$$X_{mm} = 2 \sqrt{\frac{E_a^2}{a^2 I_{m0}^2} - \frac{R_{rm}^2}{16}} - X_{rm} \quad (14.7)$$

A rather good value of a may be determined by running on no load the machine additionally, with the main winding open and the auxiliary winding fed from the voltage $V_{a0} \approx 1.2E_a$. With E_m measured, [1]

$$a = \sqrt{\frac{E_a V_{a0}}{V_s E_m}} \quad (14.8)$$

Once X_{mm} is known, from (14.7) with (14.8), we may make use of the measured P_{m0} , I_{m0} , and V_s (see Figure 14.1) to determine the sum of iron and mechanical losses:

$$p_{mec} + p_{iron} = P_{m0} - \left(R_{sm} + \frac{R_{rm}}{4} \right) I_{m0}^2 \quad (14.9)$$

The no-load test may be performed at different values of V_s , below rated value, until the current I_{m0} starts increasing; a sign that the slip is likely to increase too much.

As for the three-phase IM, the separation of mechanical and core losses may be done by taking the ordinate at zero speed of the rather straight-line dependence of $(p_{mec} + p_{iron})$ of V_s^2 (Figure 14.2). Alternatively, we may use only the results for two voltages to segregate p_{mec} from p_{iron} .

A standard straight-line curve fitting method may be used for better precision.

The core loss is, in fact, dependent on the emf E_m (not on input voltage V_m) and on the magnetic field ellipticity.

The field ellipticity decreases with load, and this is why, in general, the core losses are attributed to the forward component.

Consequently, the core loss resistance R_{miron} may be placed in series with the magnetization reactance X_{mm} and thus (Figure 14.3)

$$R_{miron} \approx \frac{p_{iron}}{2I_{m0}^2} \quad (14.10)$$

The impedance at no load Z_{om} is

$$|Z_{om}| = \frac{V_s}{I_{m0}} = \sqrt{\left(R_{sm} + R_{miron} + \frac{R_{rm}}{4}\right)^2 + \left(\frac{X_{mm} + X_{rm}}{2} + X_{sm}\right)^2} \quad (14.11)$$

Equation (14.11) allows us to calculate again X_{mm} . An average of the value obtained from (14.7) and (14.11) may be used for more confidence.

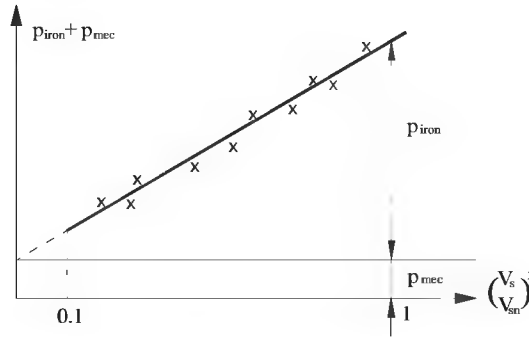


FIGURE 14.2 Mechanical plus core losses at no-load (open auxiliary winding).

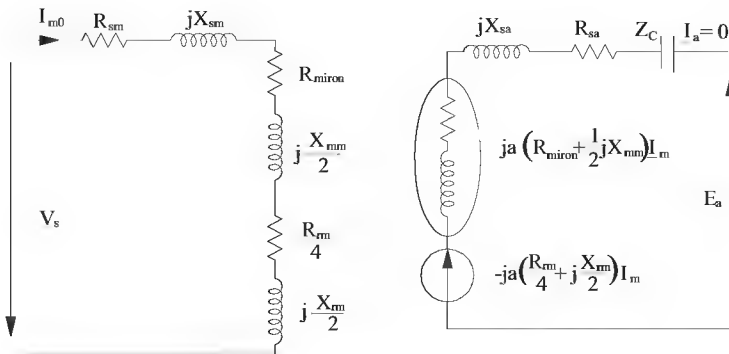


FIGURE 14.3 Simplified no-load ($S = 0$) equivalent circuit with series core loss resistance R_{miron} in both circuits (the auxiliary winding is open).

Now that all parameters are known, it is possible to refine the results by introducing the magnetization reactance X_{mm} in the short-circuit impedance, while still $X_{sm} = X_{rm}$, to improve the values of R_{rm} and X_{sm} , until sufficient convergence is obtained.

Today numerical methods, available through many general softwares on PCs, allow for such iterative procedures to be applied rather easily.

Example 14.1

An 123 W (1/6 hp), 6 poles, 60 Hz, 110 V, split-phase motor was tested as follows [1]

$R_{sm} = 2.54 \Omega$ after locked rotor reading, $R_{sm} = 2.65 \Omega$ after no-load single-phase running.

Locked rotor watts at 110 V is $P_{sc} = 851$ W.

No-load current I_{m0} at $V_{s0} = 105$ V is $I_{m0} = 2.68$ A. Locked rotor current is $I_{sc} = 11.65$ A.

$$V_{s0} = V_{sn} - I_{mn} R_{sm}^h \cos \phi_n = 110 - 3.17 \times 2.85 \times 0.588 \approx 105 \text{ V} \quad (14.12)$$

where $I_{mn} = 3.17$ A, $R_{sm}^h = 2.85 \Omega$, $\cos \phi_n = 0.588$, full-load slip $S_n = 0.033$. Full-load input power $P_{in} = 205$ W and rated efficiency $\eta_n = 60.5\%$ have been obtained from a direct load (brake) test.

The no-load power versus applied volts V_s , produces $(p_{iron})_{V_{s0}} = 24.7$ W and mechanical losses $P_{mec} = 1.5$ W.

The no-load auxiliary winding voltage $E_a = 140.0$ V.

The value of turns ratio a is found from an additional no-load test with the auxiliary winding fed at $1.2 E_a$ (V). With the measured no-load main winding voltage $E_{m0} = 105$ V, a is (14.8)

$$a = \sqrt{\frac{140 \times 140 \times 1.2}{105 \times 105}} = 1.46$$

Let us now find the motor parameters and check directly the efficiency measured by the loss segregation method.

The rotor resistance (referred to the main winding) R_{rm} is (14.1)

$$R_{rm} = \frac{851}{11.65^2} - 2.54 = 3.73 \Omega$$

The stator and rotor leakage reactances $X_{rm} = X_{sm}$ from (14.3) are

$$X_{sm} = X_{rm} = \frac{1}{2} \sqrt{\left(\frac{105}{11.65} \right)^2 - (3.73 + 2.54)^2} = 3.237 \Omega$$

R_{rm} is getting larger during the no-load test, due to heating, to the same extent that R_{sm} does

$$R_{rm}^o = R_{rm} \times \frac{R_{sm}^o}{R_{sm}} = 3.73 \times \frac{2.65}{2.54} = 3.89 \Omega$$

Now, the magnetization reactance X_{mm} , from (14.7), is

$$X_{mm} = 2 \sqrt{\left(\frac{140.0}{1.46 \times 2.58} \right)^2 - \left(\frac{3.89}{4} \right)^2} - 3.237 = 77.57 \Omega$$

with the core resistance R_{miron} (14.10)

$$R_{\text{miron}} = \frac{P_{\text{iron}}}{2I_{m0}^2} = \frac{24.7}{2 \cdot 2.58^2} = 1.87 \Omega$$

From (14.11), we may recalculate X_{rm}

$$X_{\text{rm}} = 2 \sqrt{\left(\frac{105}{2.58}\right)^2 - \left(2.65 + 1.87 + \frac{3.89}{4}\right)^2} - 3.237 = 74.71 \Omega$$

An average of the two X_{rm} values would be 76.1235 Ω .

By now the parameter problem has been solved. A few iterations may be used with the complete circuit at $S = 1$ to get better values for R_{rm} and $X_{\text{sm}} = X_{\text{rm}}$. However, we should note that $X_{\text{rm}}/X_{\text{rm}} \approx 20$, and thus, not much is to gain from these refinements.

For efficiency checking, we need to calculate the winding losses at rated slip $S_n = 0.033$ with the following parameters:

$$R_{\text{sm}}^h = 2.85 \Omega, R_{\text{rm}}^h = R_{\text{rm}}^o \times \frac{R_{\text{sm}}^h}{R_{\text{sm}}^o} = 3.89 \times \frac{2.85}{2.65} = 4.1835 \Omega$$

$$X_{\text{sm}} = X_{\text{rm}} = 3.237 \Omega, X_{\text{mm}} = 76.12 \Omega$$

The core resistance R_{miron} may be neglected when the rotor currents are calculated:

$$I_{\text{rmf}} = I_m \left| \frac{jX_{\text{mm}}}{\frac{R_{\text{rm}}^h}{S_n} + j(X_{\text{mm}} + X_{\text{rm}})} \right| = 3.117 \cdot \left| \frac{j76.12}{\frac{4.1835}{0.033} + j(76.12 + 3.237)} \right| = 1.586 \text{ A}$$

$$I_{\text{rmb}} \approx I_m = 3.117 \text{ A}$$

So the total rotor winding losses p_{Corotor} are

$$p_{\text{Corotor}} = I_{\text{rmf}}^2 \frac{R_{\text{rm}}^h}{2} + I_{\text{rmb}}^2 \frac{R_{\text{rm}}^h}{2} = (3.117^2 + 1.586^2) \frac{4.1835}{2} = 25.58 \text{ W}$$

The stator copper losses p_{cos} are

$$p_{\text{cos}} = I_{m0}^2 R_{\text{sm}}^h = 3.117^2 \times 2.85 = 27.69 \text{ W}$$

The total load losses Σp from loss segregation are

$$\Sigma p = p_{\text{cos}} + p_{\text{Corotor}} + p_{\text{iron}} + p_{\text{mec}} = 27.69 + 25.58 + 24.7 + 1.5 = 79.47 \text{ W}$$

The losses calculated from the direct load test are

$$\left(\Sigma p \right)_{\text{load}} = P_{\text{in}} - P_{\text{out}} = 205 - 123 = 82 \text{ W}$$

There is a small difference of 2 W between the two tests, which tends to validate the methods of loss segregation. However, it is not sure that this is the case for most designs.

It is recommended to back up loss segregation by direct shaft loading tests whenever possible.

It is possible to define the stray load losses as proportional to stator current squared and then to use this expression to determine total losses at various load levels

$$\text{stray load losses} = (\Sigma p)_{\text{load}} - \Sigma p_{\text{segregation}} \approx R_{\text{stray}} \left[I_m^2 + (I_a a)^2 \right] \text{file:///file/id=7329707.54}$$

R_{stray} may then be lumped into the stator resistances R_{sm} and R_{sa}/a^2 .

14.3 THE CASE OF CLOSED ROTOR SLOTS

In some single-phase IMs (as well as three-phase IMs), closed rotor slots are used to reduce noise.

In this case, the rotor slot leakage inductance varies with rotor current due to the magnetic saturation of the iron bridges above the rotor slots.

The short-circuit test has to be done now for quite a few values of voltage (Figure 14.4).

In this case, the equivalent circuit should contain additionally a constant emf E_{0sc} , corresponding to the saturated closed rotor upper iron bridge (Figure 14.5).

Only the segment $AA'-E_{0sc}$ (on Figure 14.5) represents the voltage drop on the rotor and stator constant leakage reactances.

$$X_{sm} + X_{rm} = \frac{V_{sc} \sin \phi_{sc} - E_{0sc}}{I_{msc}} \quad (14.13)$$

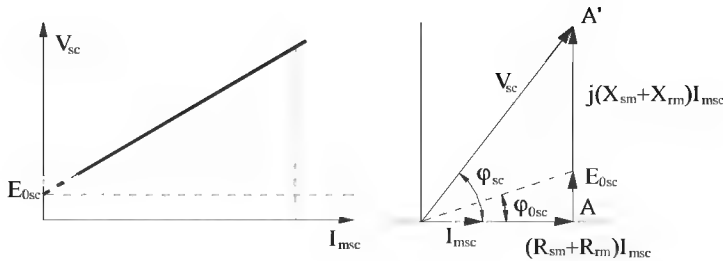


FIGURE 14.4 Short-circuit characteristics.

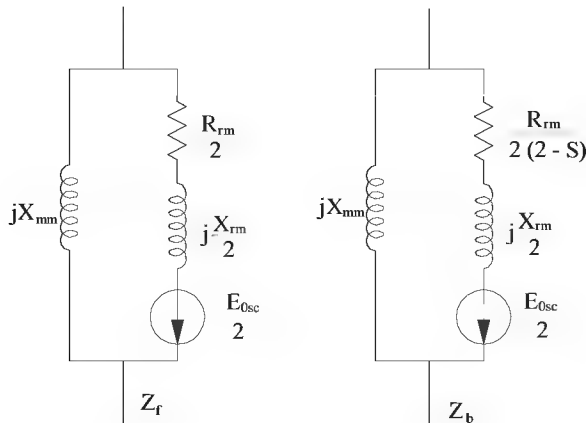


FIGURE 14.5 The forward (f, b) impedances for the closed slot rotor.

14.4 LOSS SEGREGATION IN PERMANENT CAPACITOR IMS

The loss segregation for the permanent capacitor IM may be performed as for the single-phase IM, with only the main winding ($I_a = 0$) activated.

The auxiliary winding resistance and leakage reactance R_{sa} and X_{sa} can be measured, in the end, by a short-circuit (zero speed) test performed on the auxiliary winding:

$$P_{sca} \approx (R_{sa} + R_{ra}) I_{sca}^2 \quad (14.14)$$

$$X_{sa} + X_{ra} = \sqrt{\left(\frac{V_{sca}}{I_{sca}}\right)^2 - (R_{sa} + R_{ra})^2} \quad (14.15)$$

When $X_{ra} = a^2 X_{rm}$ (with a and X_{rm} known, from (14.13)), it is possible to calculate X_{sa} with measured input power and current. With R_{sa} D.C.-measured, R_{ra} can be calculated from 14.14.

The capacitor losses may be considered through a series resistance R_c (see Figure 14.1). The value of R_c may be measured by separately supplying the capacitor from an A.C. source.

The active power in the capacitor P_c and the current through the capacitor I_c are directly measured:

$$R_c = \frac{P_c}{I_c^2} \quad (14.16)$$

Once all parameters are known, the equivalent circuit shown in Figure 14.1 allows for the computation of both stator currents, I_m and I_a , under load with both phases on, and given slip value, S . Consequently, the stator and rotor winding losses may then be determined.

From this point on, we repeat the procedure in the previous paragraph to calculate the total losses by segregation method and from direct input and output measurements, with the machine shaft loaded.

For cage-PM-reluctance rotor capacitor split-phase self-synchronizing IMs, a similar segregation process of loss may be applied [4].

14.5 SPEED (SLIP) MEASUREMENTS

One problem encountered in the load tests is the slip (speed) measurement.

Unless a precision optical speedometer (with an error in the range of 2 rpm or less) is available, it is more convenient to measure directly the slip frequency Sf_1 .

The “old” method of using a large diameter circular short-circuited coil with a large number of turns, to track the axial rotor leakage flux by a current Hall probe (or shunt), may be used for the scope.

The coil is placed outside the motor frame at the motor end which does not hold the cooling fan. Coil axis is concentric with the shaft.

The acquired current signal contains two frequencies Sf_1 and $(2-S)f_1$. An offline digital software (or a low pass hardware) filter may be applied to the coil current signal to extract the Sf_1 frequency component.

The slip computation error is expected to be equivalent to that of a 2 rpm precision speedometer or better.

14.6 LOAD TESTING

There are two main operation modes to test the single-phase IM on load: the motor mode and the generator mode.

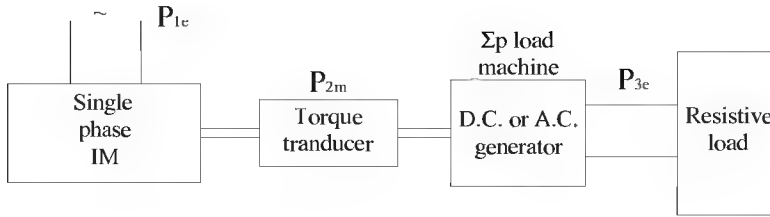


FIGURE 14.6 Load tests with a torque-meter.

Under the motor mode, the electric input power, P_{1e} , and the output mechanical power, P_{2m} , are measured. P_{2m} is in fact calculated indirectly from the measured torque T_{shaft} and speed n :

$$P_{2m} = T_{\text{shaft}} \cdot 2\pi n \quad (14.16a)$$

The torque is measured by a torque-meter (Figure 14.6). Alternatively, the load machine may have the losses previously segregated such that at any load level, the single-phase IM mechanical power P_{2m} can be calculated:

$$P_{2m} = P_{3e} - \sum P_{\text{loadmachine}} \quad (14.17)$$

The load machine may be a PM D.C. or A.C. generator with resistive load, a hysteresis or an eddy current D.C. brake.

14.7 COMPLETE TORQUE–SPEED CURVE MEASUREMENTS

Due to magnetic saturation mmf space harmonics, slotting, and skin effect influences on harmonic rotor currents, it seems that direct torque measurements at various speeds (below rated speed) are also required.

Such a measurement may be performed by using a D.C. or an A.C. generator with power-converter energy retrieval to the power grid (Figure 14.7).

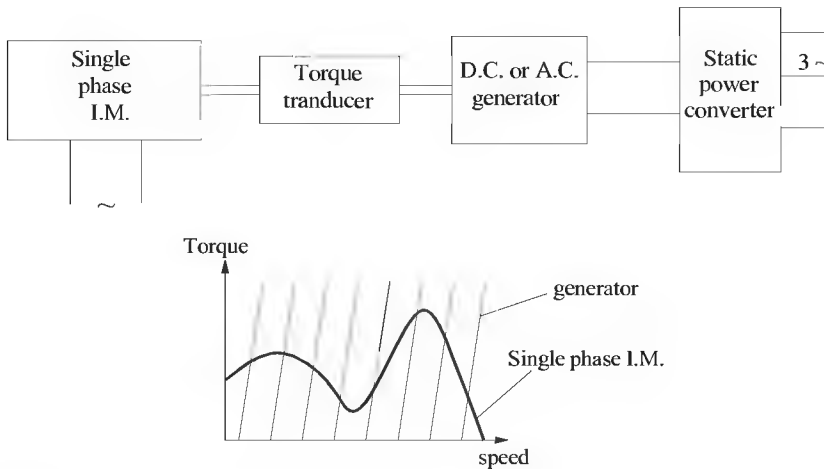


FIGURE 14.7 Torque/speed measurements with a torque-meter.

Alternatively a slow acceleration test on no load may be used to calculate the torque-speed curve. For slow acceleration, the supply voltage may be lowered from V_{sn} to V_s .

The core losses are considered proportional to voltage squared. They are measured by the loss segregation method:

$$p_{iron} = (p_{iron})_{V_{sn}} \left(\frac{V_s}{V_{sn}} \right)^2 \quad (14.18)$$

The mechanical losses are proportional to speed squared:

$$p_{mec} = (p_{mec})_{n_0} \left(\frac{n}{n_0} \right)^2 \quad (14.19)$$

With R_{sm} , R_{rm} , R_{sa} , X_{sm} , X_{ra} , X_{rm} , and X_{mm} known and with I_m and I_a and input power P_1 measured, torque can be calculated as for steady state around rated speed:

$$T_e = \frac{p_{mec}}{2\pi n} + T_{shaft} = \frac{[P_1 - p_{iron} - R_{sm} I_m^2 - R_{sa} I_a^2 - R_{rm} (I_{mf}^2 + I_{mb}^2) - R_c I_a^2]}{2\pi n} \quad (14.20)$$

The computation of T_{shaft} is to be done offline. An optical speedometer could be used to measure the speed during the slow acceleration test and during a free deceleration (after turn-off).

With $p_{mec}(n)$ known, the free deceleration test yields

$$J = - \frac{p_{mec}(n)}{2\pi \frac{dn}{dt}} \quad (14.21)$$

The moment of inertia J is thus obtained.

With J known and speed n acquired during the no-load slow acceleration test, the torque is

$$T_e(n) = J 2\pi \frac{dn}{dt} + \frac{p_{mec}}{2\pi n} \quad (14.22)$$

The torque for rated voltage is considered to be

$$(T_e(n))_{V_{sn}} \approx (T_e(n))_{V_s} \left(\frac{V_{sn}}{V_s} \right)^2 \quad (14.23)$$

The speed derivative in (14.22) may be obtained offline, with an appropriate software filter, from the measured speed signal.

The two values of torque, (14.20) and (14.22), are then compared to calculate a measure of stray losses:

$$p_{stray} \approx [(T_e)_{Equation(28.22)} - (T_e(n))_{Equation(28.20)}] \cdot \left(\frac{V_{sn}}{V_s} \right)^2 2\pi n \quad (14.24)$$

Temperature measurements methods are very similar to those used for three-phase IMs. Standstill D.C. current decay tests may also be used to determine the magnetization curve $\Psi_{mm}(I_{mm})$ and even the resistances and leakage reactances, as done for three-phase IMs.

14.8 SUMMARY

- The single-phase IM testing aims to determine equivalent circuit parameters in order to segregate losses and to measure the performance on load, and even to investigate transients.
- Due to the backward field (current) component, even at zero slip, the rotor current (loss) is not zero. This situation complicates the loss segregation in no-load tests.
- The no-load test may be done with the auxiliary phase open $I'_a = 0$, after the motor starts.
- The short-circuit (zero speed) test is to be performed, separately for the main and auxiliary phases, to determine the resistances and leakage reactances.
- Based on these results, the no-load test (with $I_a = 0$) furnishes data for loss segregation and magnetization curve ($X_{mm}(I_{mm})$), provided it is performed for quite a few voltage levels below rated voltage.
- The calculation of rotor resistance R_{rm} from the zero speed test (at 50 (60) Hz) produces a value that is acceptable for computing on-load performance, because it is measured at an average of forward and backward rotor current frequencies: $\frac{Sf_i + (2-S)f_i}{2} = f_i$.
- The difference between total losses by segregation method and by direct input/output measurements under load is a good measure of stray load losses. The stray load losses tend to be smaller than in three-phase motors because the ratio between full-load and no-load current is smaller.
- Instead of single (main)-phase no-load and short-circuit tests, symmetrical two voltages supplying for the same tests have also been proved to produce good results.
- The complete torque–speed curve is of interest also. Below the breakdown torque–speed value, there may be a deep in the torque–speed curve around 33% of no-load ideal speed (f_i/p_i) due to the third space mmf harmonic. A direct load method may be used to obtain the entire torque versus speed curve. Care must be exercised that the load machine had a rigid torque/speed characteristic to handle the statically unstable part of the single-phase IM torque–speed curve down to standstill.
- To eliminate direct torque measurements and the load machine system, a slow free acceleration at reduced voltage and a free deceleration test may be performed. With the input power, speed, and stator currents and voltage measured and parameters already known (from the short-circuit and single-phase no-load tests), the torque may be computed after loss subtraction from input at every speed (slip). The torque is also calculated from the motion equation with inertia J determined from free deceleration test. The mechanical power difference in the two measurements should be a good measure of the stray load losses caused by space harmonics. The torque in the torque/speed curve thus obtained is multiplied by the rated to applied voltage ratio squared to obtain the full voltage torque–speed. It is recognized that this approximation underestimates the influence of magnetic saturation on torque at various speeds. This is to say that full voltage load tests from $S = 0$ to $S = 1$ are required for magnetic saturation complete consideration.
- D.C. flux (current) decay tests at standstill, in the main and auxiliary winding axes, may also be used to determine resistances, leakage inductances, and the magnetization curve. Frequency response standstill tests may be applied to single-phase IM in a manner very similar to one applied to three-phase IMs (Chapter 13).
- Temperature measurement tests are performed as for three-phase IMs, in general (Chapter 13).
- For more single-phase IM testing, see Ref. [5].

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